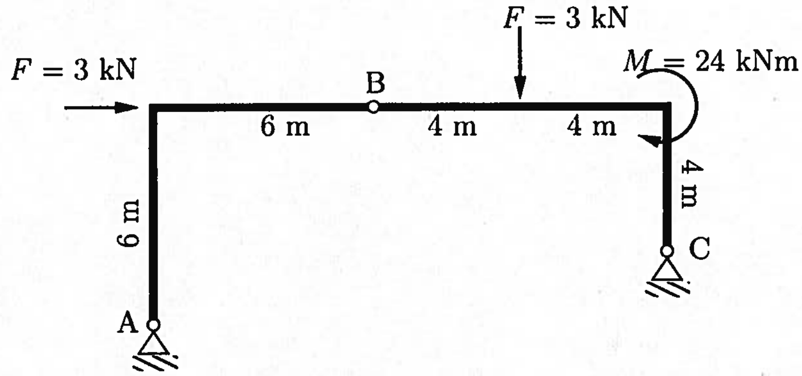
Problem 1 (12+15+8 points)

Consider a three-pinned arch of the figure below.



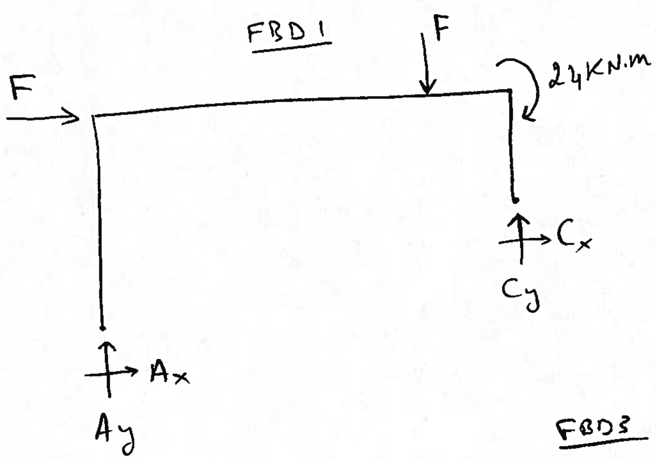
(a) Draw the free-body diagram of the three-pinned arch, as well as the free body diagrams of the subbodies AB and BC.

(b) Find the external reactions at points A and C.

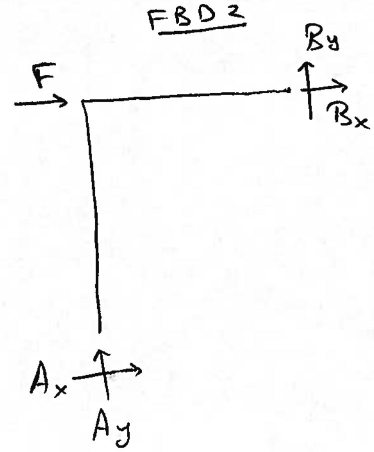
(c) Find the horizontal and vertical forces transmitted through the pin at B.

a)

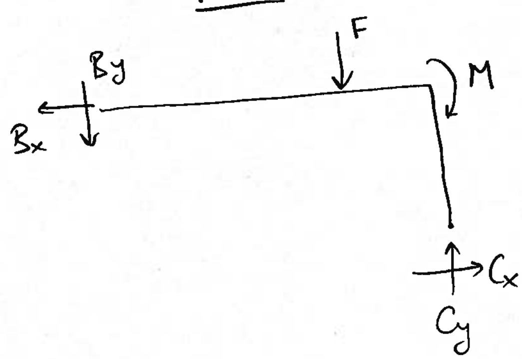
FBD 1



FBD 2



FBD 3



b)

FBD1 : $\circlearrowleft^{+}\ \Sigma M_c = 0 \Rightarrow \quad 2A_x - 14A_y - 4(3) + 4(3) - 24 = 0$

$$2A_x - 14A_y = 24 \Rightarrow A_x - 7A_y = 12 \quad (*)$$

FBD2 : $\circlearrowleft\ \Sigma M_B = 0 \Rightarrow \quad -6A_y + 6A_x = 0 \Rightarrow A_x = A_y \quad (**)$

From (*) & (**) $\Rightarrow$ $\boxed{A_x = A_y = -2\ KN}$

FBD1 : $\Sigma F_y = 0 \Rightarrow \quad A_y - 3 + C_y = 0$

$$\Rightarrow \boxed{C_y = 5\ KN}$$

$\Sigma F_x = 0 \Rightarrow \quad A_x + 3 + C_x = 0$

$$\boxed{C_x = -1\ KN}$$

c) FBD2 : $\Sigma F_y = 0 \Rightarrow B_y + A_y = 0 \Rightarrow \boxed{B_y = 2\ KN}$

$\Sigma F_x = 0 \Rightarrow A_x + F + B_x = 0 \Rightarrow \boxed{B_x = -1\ KN}$

In summary :

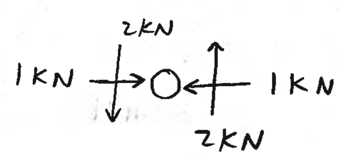
$A_x = -2\,\underline{i}$ (KN)

$A_y = -2\,\underline{j}$ (KN)

$C_x = -1\,\underline{i}$ (KN)

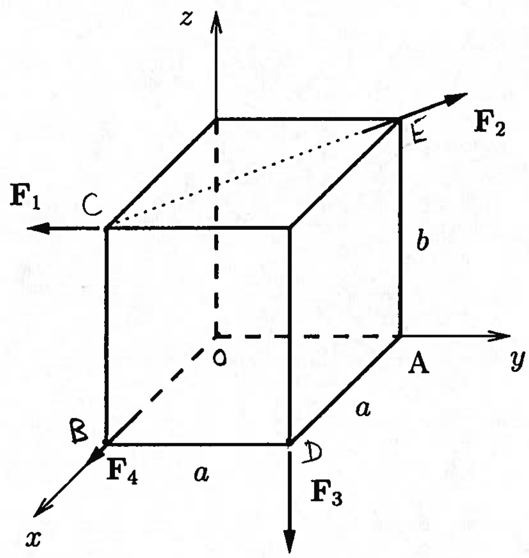
$C_y = 5\,\underline{j}$ (KN)

Pin :



Problem 2 (4+6+8+12 points)

A system of four forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$ is acting on the orthogonal parallelepiped of the figure below. Let the magnitudes of these forces be $F_1 = 5$ kN, $F_2 = 5\sqrt{2}$ kN, $F_3 = 8$ kN, and $F_4 = 5$ kN, respectively.



(a) Express each of the four forces in vector form.

(b) Find the resultant $\mathbf{R}$ of the four forces.

(c) Find the moment of $\mathbf{F}_2$ about the x-axis.

(d) What should be the relation between the lengths a and b so that the system of four forces be statically equivalent to a force that passes through point A?

(a) $\underline{F}_1 = -5\underline{j}$ kN (1)

$\underline{F}_2 = 5\sqrt{2}(-\cos 45^\circ \underline{i} + \sin 45^\circ \underline{j})$ kN $= (-5\underline{i} + 5\underline{j})$ kN (1)

$\underline{F}_3 = -8\underline{k}$ kN (1)

$\underline{F}_4 = 5\underline{i}$ kN (1)

(b) $\underline{R} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \underline{F}_4 = -8\underline{k}$ kN (3) (2) (1)

(c) $\underline{M}_x = (b\underline{k} \times \underline{F}_2 \cdot \underline{i})\underline{i} = (b\underline{k} \times (-5\underline{i} + 5\underline{j}) \cdot \underline{i})\underline{i} = [(-5b\underline{j} - 5b\underline{i}) \cdot \underline{i}]\underline{i} = -5b\underline{i}$ (4) (3) (1)

(d)

We require that the resultant moment of all forces about A must be zero.

$\therefore \underline{0} = \underline{M}_A = \underline{r}_{AC} \times \underline{F}_1 + \underline{r}_{AE} \times \underline{F}_2 + \underline{r}_{AD} \times \underline{F}_3 + \underline{r}_{AB} \times \underline{F}_4$ (6)

$= (a\underline{i} - a\underline{j} + b\underline{k}) \times (-5\underline{j}) + b\underline{k} \times (-5\underline{i} + 5\underline{j}) + a\underline{i} \times (-8\underline{k}) + (a\underline{i} - a\underline{j}) \times 5\underline{i}$

$= -5a\underline{k} + 5b\underline{i} - 5b\underline{j} - 5b\underline{i} + 8a\underline{j} + 5a\underline{k}$ (4)

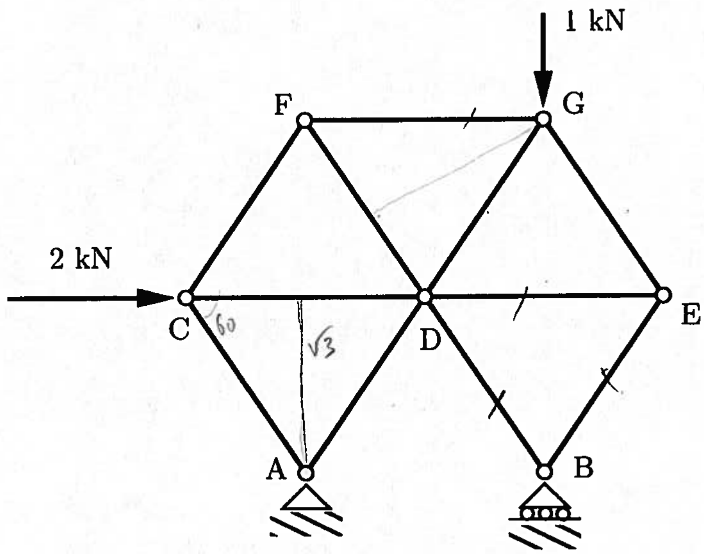
$= (8a - 5b)\underline{j}$

$\therefore \; 8a - 5b = 0$

$\therefore \; \dfrac{a}{b} = \dfrac{5}{8}$ (2)

Problem 3 (5+5+5+20 points)

In the hexagonal truss shown in the figure below all truss bars have length 2 m.



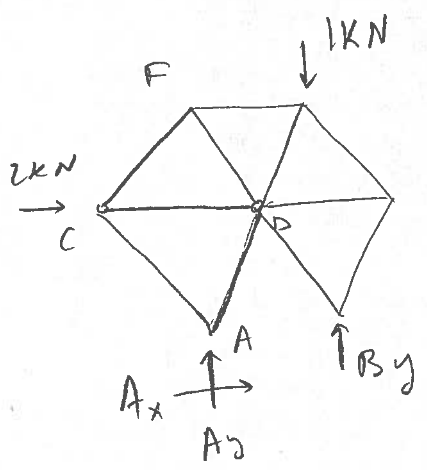
(a) Draw the free-body diagram of the truss.

(b) Argue that the truss is statically determinate.

(c) Determine the external reactions at points A and B.

(d) Determine the forces of members DE, FG, BD and BE and state explicitly if they are in tension or compression.

a)



b) $2j = n + r$?

$14 = 11 + 3$ ✓

Also, we have 3 reactions & 3 equilibrium equations.

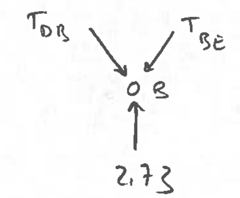
c) $\Sigma M_A = 0 \Rightarrow 2B_y - 1(2) - 2\sqrt{3} = 0$

$\Rightarrow B_y = 2.73\,kN \uparrow$

$\Sigma F_y = 0 \Rightarrow A_y = B_y - 1 = 1.73\,kN \uparrow$

$\Sigma F_x = 0 \Rightarrow A_x = -2\,kN = 2\,kN \leftarrow$

d)

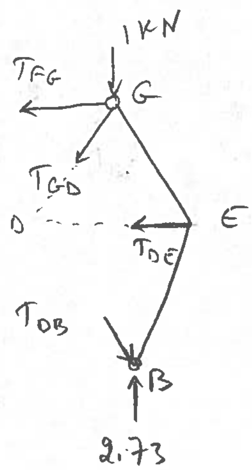


$\Sigma F_x = 0 \Rightarrow T_{DB} = T_{BE}$

$\Sigma F_y = 0 \Rightarrow -T_{DB}\frac{\sqrt{3}}{2} - T_{BE}\frac{\sqrt{3}}{2} + 2.73 = 0$

$T_{BD}\sqrt{3} = 2.73 \Rightarrow \boxed{T_{BD} = 1.576\,kN \ (C)}$

$\boxed{T_{DE} = 1.576\,kN \ (C)}$



$\Sigma M_D = 0 \Rightarrow T_{FG}(\sqrt{3}) - 1 + 2.73 = 0$

$\boxed{T_{FG} = 1\,kN \ (C)}$

$\Sigma M_G = 0 \Rightarrow -T_{DE}\sqrt{3} + T_{DB}\sqrt{3} = 0$

$\boxed{\Rightarrow T_{DE} = 1.57\,kN \ (T)}$