Physics H7C; Final

Monday, 5/12; 7-10 PM

Write your responses below, or on extra paper if needed. Show your work, and take care to explain what you are doing; partial credit will be given for incomplete answers that demonstrate some conceptual understanding. Cross out or erase parts of the problem you wish the grader to ignore.

Problem 1: Appetizers (4 pts each)

1a) The Schrodinger equation is non-relativistic. For the problem of a particle in a 1D box (i.e., infinite square well) for what box sizes L is it okay that we neglect special relativistic effects? Write this in the form of an inequality for L.

1b) A particle in a 1D infinite well of size L is in the $n = 3$ state. What is (to some reasonable approximation) the probability, in percent, of finding the particle within a distance $L/100$ of the center of the well?

1c) According to classical thermodynamics, the energy per particle for an ideal gas is $(3/2)kT$. For a gas of number density n, at what temperature T do we we expect to quantum effects to become important (that is, at what T can we no longer consider the gas atoms to be independent, non-overlapping particles)?

1d) The blue sky is the result of light scattered by the atoms in air. Qualitatively explain, in the classical E&M picture, 1) Why an atom scatters light, and 2) Why atoms typically scatter blue light more than red light.

1e) An electromagnetic wave described by

$$\vec{E} = \frac{E_0(\hat{y} + \hat{z})}{\sqrt{2}} e^{i(kx - \omega t)}$$

is normally incident on a sheet of birefringent material that has an index of refraction n_y for y-polarized light, and a different index of refraction of n_z for z-polarized light. What is the minimum thickness of the sheet such that light emerging on the opposite side is 100% circularly polarized?

1f) In a lab coordinate system, an event A is observed at the origin, $x^\mu_A = (ct, x) = (0, 0)$. A bit later, an event B is observed at coordinate $x^\mu_B = (1, 2)$. Draw a spacetime diagram showing the possible spacetime locations of event B in *every possible frame* moving with constant velocity with respect to the lab (and sharing the same origin). Show on your sketch that event B may occur before event A in some frames, and explain why this does not bother you.

Problem 2: Compton Scatter (15 pts)

A PHOTON MOVES IN THE POSITIVE x-direction, with a lab frame energy equal to the two times the rest mass energy of an electron. Upon hitting an electron at rest in the lab, the photon is scattered backwards in the negative x direction.

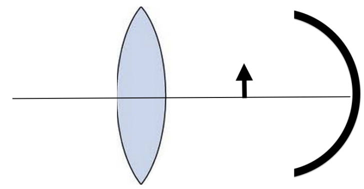
2a) Find the velocity of the electron in the lab frame after the scattering (treated in full special relativity).

2b) Determine by what factor the lab-frame wavelength of the photon is changed after the scatter.

2c) This interaction is observed by a spaceship flying by. At what speed relative to the lab frame (and in what direction) does the spaceship need to move such that, in the spaceship frame, one would observe the electron to *lose* energy to the photon in this interaction?

Problem 3: Mirror Lens (15 pts)

THE CONCAVE MIRROR AND A CONVEX LENS shown in the figure are separated by a distance $2R$. The mirror has a radius of curvature R. Each side of the lens has a radius of curvature of R, and the index of refraction is $n = 2$. An object (an arrow) is placed exactly midway between the mirror and lens.



3a) Find the location, relative to the lens, of the images (plural) produced by this system.

3b) A white screen is placed to the left of the lens at a location where multiple images form. Sketch what you would see on the screen, noting how big each image is relative to the object.

Problem 4: A Potential Universe (15 pts)

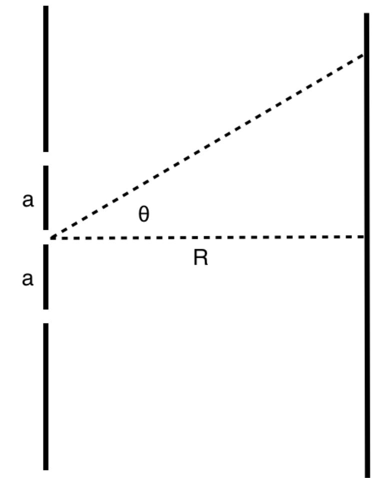
IMAGINE A UNIVERSE WHERE THE Coulomb force is modified, such that an electron in an atom is subject to a central potential[1] $V(r) = -k/r^3$. An exact solution of Schrodinger's equation for this potential is too difficult for an exam, but some essential quantities can be estimated using basic concepts in quantum mechanics.

[1] The corresponding classical force is $F = -dV/dr$.

4a) Find an expression that, up to a numerical factor, gives the energy of the ground state of hydrogen in this Universe

4b) Find an expression that, up to a numerical factor, gives the expectation value $\langle r \rangle$ of the electron in the ground state.

Problem 5: Triple Slit interference (15 pts)



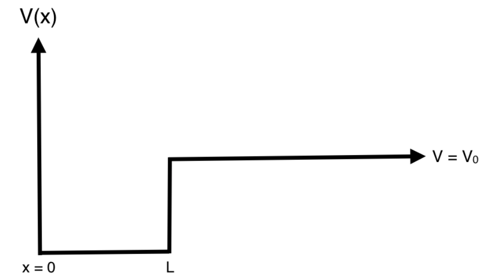
A COHERENT, MONOCHROMATIC SOURCE of light of wavelength λ is incident on a screen with three slits, each separated by a distance a. The size of each slit, d, is small ($d \ll a$ and $d \ll \lambda$). The intensity is measured on a very distant screen, $R \gg a$.

5a) If all three slits are open, how much brighter are the brightest intensity maxima compared to when only two slits are open?

5b) When all thee slits are open, at what angles θ do the brightest intensity maxima appear?

5c) Derive the expression for how the intensity, $I(\theta)$ varies as a function of θ. You can write the function relative to $I(0)$, the intensity seen at $\theta = 0°$.

Problem 6: Semi Infinite Well (15 pts)



A PARTICLE IS LOCATED in the potential well shown, which has $V \to \infty$ at $x = 0$, $V = V_0$ for $x \geq L$, and $V = 0$ inside the well. The particle has mass m and energy $E < V_0$.

6a) Sketch out the probability distribution $|\psi|^2$ for the lowest energy stationary state of a bound particle in this well.

6b) Sketch out the probability distribution for the next highest energy state of a bound particle in this well.

6c) Find an equation which relates the possible values of E to the parameters V_0, L, and fundamental constants. (It is not possible to solve this equation explicitly for E, so leave it as is).

Problem 7: Gimme (1 pt)

Is light a particle or a wave?

extra work space