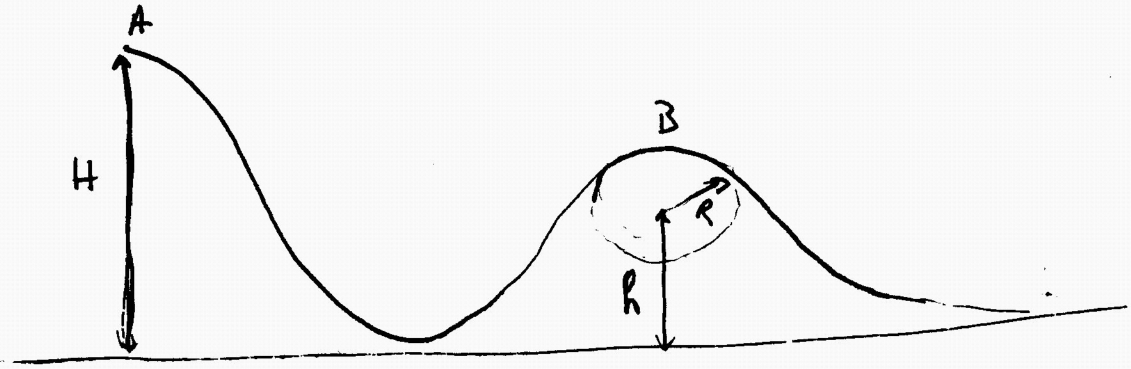
Problem 1



* Total energy at point A :

$\epsilon_A = mgH$ because $V_A = 0$ so $K_A = 0$

* Total energy at point B :

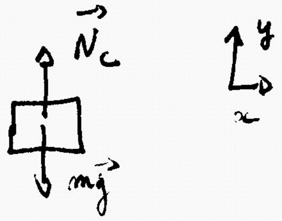
$\epsilon_B = \frac{1}{2} m v_c^2 + mg(h+R)$

- Conservation of energy $\epsilon_A = \epsilon_B$

$\Leftrightarrow mgH = \frac{1}{2} m V_c^2 + mg(h+R)$

$\Leftrightarrow \boxed{V_c^2 = 2g(H-h-R)}$

- Force diagram at C :



Newton 2nd law $\Rightarrow m\vec{a} = \Sigma \vec{F}$

Projection on the $\vec{y}$ axis $\Rightarrow ma_y = N_c - mg$

$a_y = -\frac{v_c^2}{R}$ because centripetal motion

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2) Calculate the force by subtracting the gravitational force from the removed sphere from that of the original:

$$F_{rem} = \frac{G M_{rem} m}{r_{rem}^2} \Rightarrow$$

$$\rho_{sph} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow M_{rem} = \rho_{sph} \cdot \frac{4}{3}\pi r^3 = M\left(\frac{r}{R}\right)^3$$

$$\Rightarrow \frac{GMm}{(X-a)^2}\left(\frac{r}{R}\right)^3$$

$$F_{tot} = \frac{GMm}{X^2} - F_{rem}$$

$$= GMm\left(\frac{1}{X^2} - \frac{\left(\frac{r}{R}\right)^3}{(X-a)^2}\right)$$

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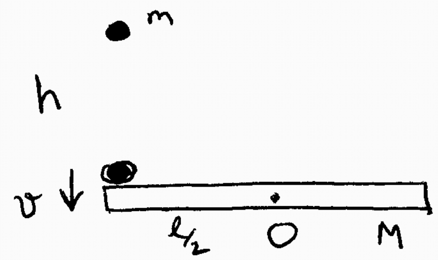
#3



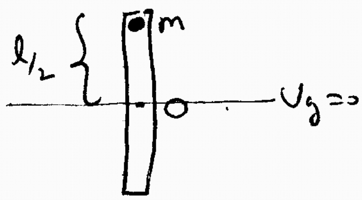
m hits rod w/ $\boxed{v = \sqrt{2gh}}$

about O:

$$\frac{d\vec{L}}{dt} = \tau_{net,O} = 0 \Rightarrow \boxed{L_i = L_f} \Rightarrow m v \frac{l}{2} = \left(m \frac{l^2}{4} + I\right)\omega$$

$$\Rightarrow \boxed{\omega = \frac{m v l/2}{m\frac{l^2}{4} + I}} = \frac{m v l/2}{m\frac{l^2}{4} + M\frac{l^2}{12}} = \boxed{\frac{6 m v}{l(3m+M)} = \omega}$$

ω should be large enough to make 'm' go to top



$$\boxed{E_i = E_f} \Rightarrow \boxed{\frac{1}{2}\left(I + m\frac{l^2}{4}\right)\omega^2 = m g \frac{l}{2}}$$

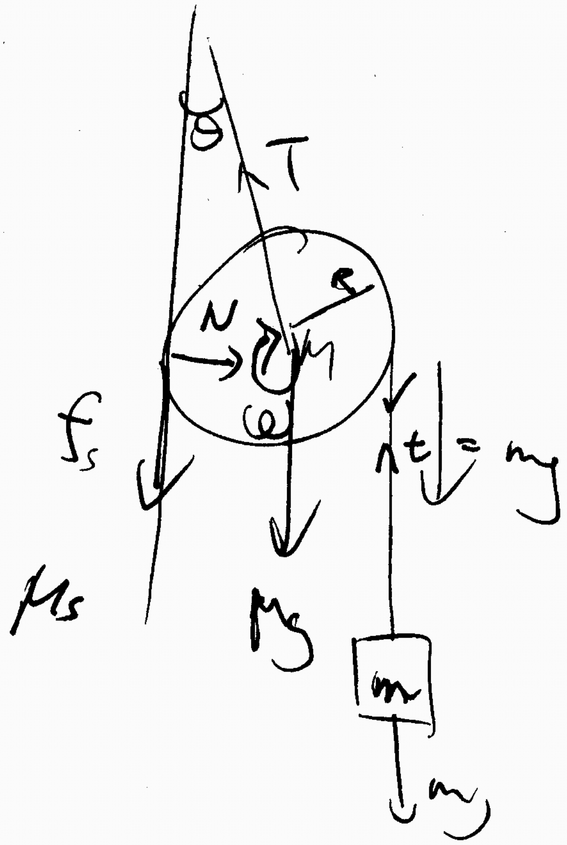
$$\cancel{\frac{1}{2}}\cancel{\left(I + m\frac{l^2}{4}\right)} \cdot \frac{\cancel{m}\, v^2\, \cancel{l^2}/4}{\left(m\frac{l^2}{4} + I\right)^{\cancel{2}}} = \cancel{m} g \cancel{\frac{l}{2}}$$

$$\frac{m \cdot 2gh \cdot l}{2 \cdot 4\left(m\frac{l^2}{4} + \frac{Ml^2}{12}\right)} = g \Rightarrow \frac{6mh}{l(3m+M)} = 1$$

$$6mh = 3ml + Ml \longrightarrow 6m\left(h - \frac{l}{2}\right) = Ml \Rightarrow \boxed{m > \frac{M}{6} \cdot \frac{l}{h - l/2}}$$

no solution for $h < l/2$

④ T_x, T_y, N, f_s, m



$$\begin{cases} T_x = N \\ T_y = f_s + (M+m)g \\ \dfrac{T_x}{T_y} = \tan\theta \\ f_s \le \mu_s N \\ -Rf_s + Rmg = 0 \Rightarrow f_s = mg \end{cases}$$

Max. m : $f_s = \mu_s N = mg$

$$T_x = N = \frac{mg}{\mu_s}$$

$$T_y = \frac{T_x}{\tan\theta} = \frac{mg}{\mu_s \tan\theta} = f_s + (M+m)g$$

$$\frac{m}{\mu_s \tan\theta} = M + 2m$$

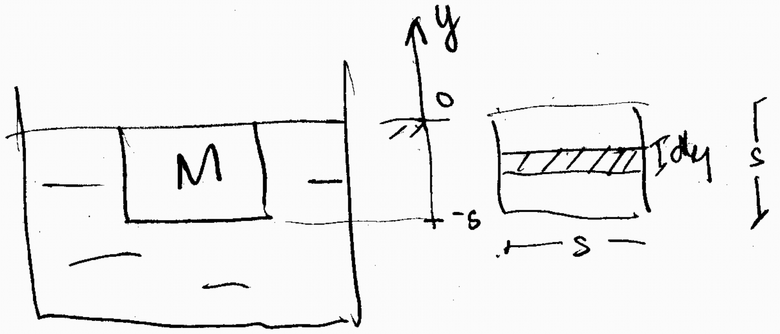
$$m\left(\frac{1}{\mu_s \tan\theta} - 2\right) = M$$

$$\boxed{m = \frac{M}{\frac{1}{\mu_s \tan\theta} - 2}}$$

min μ_s : $m = \infty \Rightarrow \dfrac{1}{\mu_s \tan\theta} = 2$

$$\boxed{\mu_s = \frac{1}{2\tan\theta}}$$

(5)



Archimedes' rule: $dF_b = \rho(y) g \, dV = \frac{\rho_0 g}{1+\frac{y}{\lambda}} s^2 dy$

Floating just submerged

$$\rightarrow F_b = \int_{-s}^{0} \frac{\rho_0 g}{1+\frac{y}{\lambda}} s^2 dy = \lambda \rho_0 s^2 g \int_{-s}^{0} \frac{dy}{\lambda + y}$$

$$= \lambda \rho_0 s^2 \ln\left|\frac{\lambda}{\lambda - s}\right| g \quad \#$$

Static condition:

$$\Sigma F = F_b - Mg = 0$$

$$\rightarrow M = \frac{F_b}{g} = \frac{\lambda \rho_0 s^2 \ln\left|\frac{\lambda}{\lambda-s}\right| g}{g} = \lambda \rho_0 s^2 \ln\left|\frac{\lambda}{\lambda-s}\right| \quad \#$$

(6)

$$F_k = -4kx \quad \Longrightarrow \quad k' = 4k$$

$$M_{total} = M + m$$

$$\omega = \sqrt{\frac{k'}{M_{tot}}} = 2\sqrt{\frac{k}{M+m}}$$

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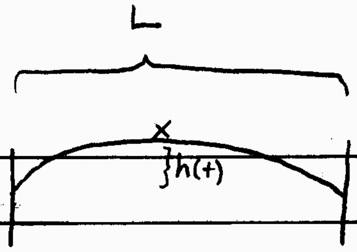

$F_T = 20N$, $M = 0.25\,kg$, $L = 2M$

We want to make sure that the acceleration due to gravity on the dust is greater than (or equal to) the maximum acceleration of the belt on the way down.
The height of the particle is

$$h(t) = A \sin(k_n L/2) \cos(\omega_n t)$$

$$a = \frac{d^2h}{dt^2} = -\omega_n^2 A \sin(k_n L/2) \cos(\omega_n t).$$

At max, $|\cos(\omega_n t)| = 1$,

$$a_M = -\omega_n^2 A |\sin(k_n L/2)|$$

Now

$$K_n = \frac{\pi n}{L}, \quad \omega_n = V \cdot K_n, \quad V = \sqrt{\frac{F_T}{\rho}} = \sqrt{\frac{F_T L}{M}}$$

$$a_M = \underbrace{\frac{F_T L}{M}}_{V^2} \underbrace{\left(\frac{\pi n}{L}\right)^2}_{K_n^2} A \times \begin{cases} 0 & \text{if } n \text{ even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

So our condition is

$$g > \frac{F_T \pi^2 n^2}{L M} \cdot A \quad \text{for } n \text{ odd}$$

$$9.8\,m/s^2 > 3.94\, n^2\, m/s^2 \quad (\text{for } n \text{ odd})$$

$$2.48 > n^2 \quad (\text{for } n \text{ odd})$$

While $2^2 > 2.48$, this is even (on node), so

$N = 3$ is first that jumps.