EE 120 SIGNALS AND SYSTEMS, Spring 2012

Midterm # 1, October 3, Wednesday, 10:10-11:50 am

Name SOLUTIONS

Closed book. Two letter-size cheatsheets are allowed. Show all your work. Credit will be given for partial answers.

Problem	Points	Score Solution/Grading
1	20	Murat Arcak
2	20	Murat Arcak
3	20	Yusef Shafi
4	20	Kelvin So
5	20	Yusef Shafi
Total	100	

1. For each of the following impulse responses of LTI systems, determine whether or not the system is causal and/or stable:

a) (5 points) $h[n] = u[n] - u[n-10]$

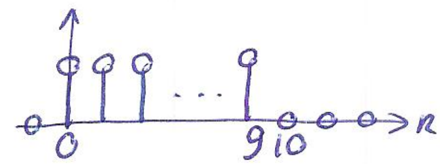
b) (5 points) $h[n] = (1/2)^{|n|}$

c) (5 points) $h[n] = \sin\left(\frac{\pi n}{3}\right) u[n]$

d) (5 points) $h[n] = (1/2)^n u[-n-1]$

a) Causal: $h[n] = 0 \quad \forall n < 0$

Stable: $\sum |h[n]| = 10 < \infty$



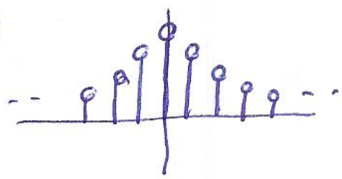
b) Non-causal: $h[n] \neq 0$ for negative n

Stable: $\sum_{-\infty}^{\infty} |h[n]| = 2 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - 1$

because $n=0$ was counted twice in $2\sum_{n=0}^{\infty}$

$= 2 \frac{1}{1-\frac{1}{2}} - 1$

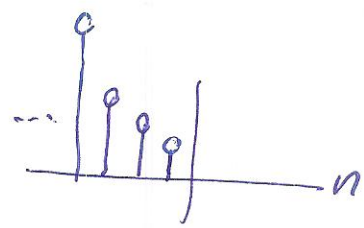
$= 3 < \infty.$



c) Causal: $h[n] = 0 \quad \forall n < 0$

Unstable: $\sum_{n=-\infty}^{\infty} |h[n]|$ doesn't converge because $h[n]$ is periodic and non-zero.

d) Non-causal $h[n] \neq 0$ for negative n:



Unstable: $\sum_{n=-1}^{-\infty} |h[n]| = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{-n} = \sum_{n=1}^{\infty} 2^n$ → doesn't converge

Common errors: 1) Applying the stability test $\sum |h[n]| < \infty$ without the absolute value.

2) Some of you wrote "$h[n]$ is stable" or "$h[n]$ is causal." It is the LTI system described by $h[n]$ that is causal or stable; $h[n]$ is a signal, not a system.

2. (20 points) Find a periodic continuous-time signal $x(t)$ that satisfies all of the following properties:

i) $x(t)$ is real and even-symmetric: $x(-t) = x(t)$

ii) $x(t)$ has period $T = 6$ and Fourier coefficients $a_k = 0$ for $k \geq 3$

iii) $x(t) = -x(t-3)$

iv) $\frac{1}{6}\int_0^6 |x(t)|^2 dt = 1$

ii) $x(t) = \sum_{k=-2}^{2} a_k e^{jk\frac{\pi}{3}t}$

i) $x(t)$ real: $a_k = a_{-k}^*$
$x(t)$ even: $a_k = a_{-k}$
$\Rightarrow a_k = a_{-k}$ and real

$$x(t) = a_0 + a_1\left(e^{jk\frac{\pi}{3}t} + e^{-jk\frac{\pi}{3}t}\right) + a_2\left(e^{jk\frac{2\pi}{3}t} + e^{-jk\frac{2\pi}{3}t}\right)$$

$$= a_0 + 2a_1 \cos\frac{\pi}{3}t + 2a_2 \cos\frac{2\pi}{3}t$$

iii) $a_0 = 0$, $a_2 = 0$ because $-x(t-3) = -a_0 + 2a_1 \cos\frac{\pi}{3}t - 2a_2\cos\frac{2\pi}{3}t$
$= x(t) = a_0 + 2a_1\cos\frac{\pi}{3}t + 2a_2\cos\frac{2\pi}{3}t$

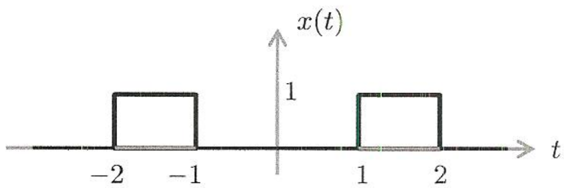
Theefoe $x(t) = 2a_1 \cos\frac{\pi}{3}t$

iv) Parseval: $a_{-1}^2 + a_1^2 = 2a_1^2 = 1$

$a_1 = \frac{1}{\sqrt{2}}$

$$\boxed{x(t) = \sqrt{2}\cos\frac{\pi}{3}t}$$

3. a) (15 points) Determine the Fourier transform of the continuous-time signal $x(t)$ depicted below:



b) (5 points) Sketch the phase plot for $X(j\omega)$.

Recall from lecture/book:

(a) Let $y(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & o/w \end{cases}$ $Y(j\omega) = \int_{-1/2}^{1/2} e^{-j\omega t}$

$= \dfrac{\sin(\omega/2)}{\omega/2}$ (sinc function)

$x(t) = y(t - 3/2) + y(t + 3/2)$

$X(j\omega) = e^{-j\omega 3/2} Y(j\omega) + e^{j\omega 3/2} Y(j\omega)$

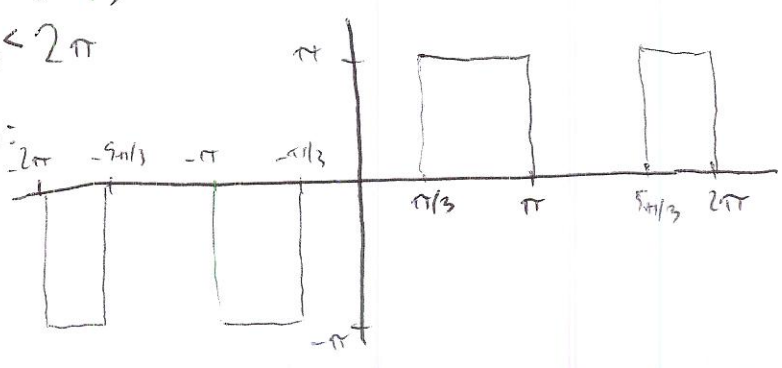
$X(j\omega) = \dfrac{\sin(\omega/2)}{\omega/2}\left[2\cos(3\omega/2)\right]$

$= \dfrac{2}{\omega}\left[\sin 2\omega - \sin \omega\right].$

(b) $X(j\omega) = |X(j\omega)| \underbrace{e^{j\measuredangle X(j\omega)}}_{\text{accounts for sign changes}}$

$\measuredangle X(j\omega) = \begin{cases} 0, & |\omega| < \pi/3 \\ \pm\pi, & \pi/3 \le |\omega| < \pi \\ 0, & \pi \le |\omega| < 5\pi/3 \\ \pm\pi, & 5\pi/3 < |\omega| < 2\pi \end{cases}$

Phase is odd, so plot is:



Additional workspace for Problem 3.

4. An ideal lowpass filter has impulse response $h_{lp}[n]$ and frequency response:

$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| < 0.2\pi \\ 0, & 0.2\pi \leq |\omega| \leq \pi. \end{cases}$$

a) (10 points) A new filter is defined by the equation:

$$h_1[n] = 2h_{lp}[n]\cos(0.5\pi n).$$

Plot the frequency response $H_1(e^{j\omega})$ and determine whether the filter is lowpass, bandpass, bandstop, or highpass.

b) (10 points) A second filter is defined by the equation:

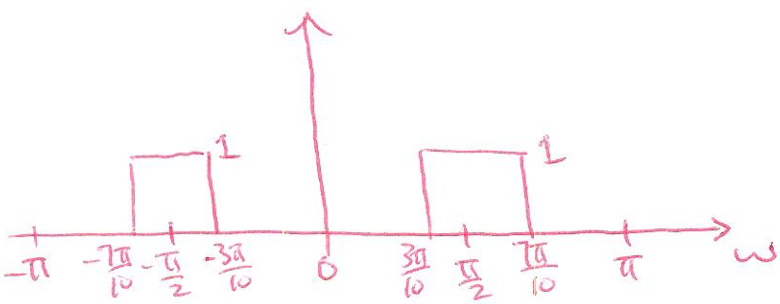
$$h_2[n] = \frac{\sin(0.1\pi n)}{\pi n} h_{lp}[n].$$

Determine and plot the frequency response $H_2(e^{j\omega})$.

a) $h_1[n] = 2\, h_{lp}[n] \cos\left(\frac{\pi}{2} n\right) = \underbrace{h_{lp}[n]\, e^{+j\frac{\pi}{2}n}} + \underbrace{h_{lp}[n]\, e^{-j\frac{\pi}{2}n}}$

Frequency shift in Fourier domain

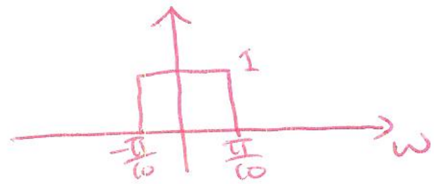
$\therefore H_1(e^{j\omega}) = H_{lp}(e^{j(\omega - \frac{\pi}{2})}) + H_{lp}(e^{j(\omega + \frac{\pi}{2})})$



Band pass filter

Additional workspace for Problem 4.

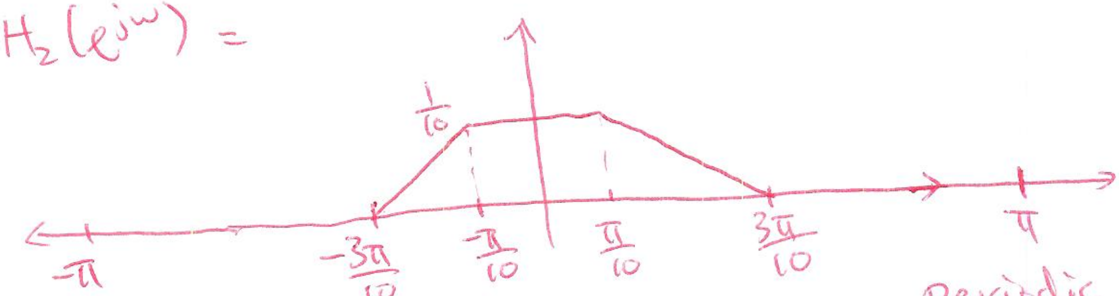
b) $\frac{\sin\left(\frac{\pi}{10}n\right)}{\pi n} \overset{\mathcal{F}}{\longleftrightarrow}$



Using Multiplication property:

$H_2(e^{j\omega}) = \frac{1}{2\pi}$

$\therefore H_2(e^{j\omega}) =$



periodic 2π

5. Consider an LTI system with frequency response:

$$H(e^{j\omega}) = \frac{1 - e^{-2j\omega}}{1 + \frac{1}{2}e^{-4j\omega}}.$$

a) (5 points) Write a difference equation that characterizes a system with this frequency response.

b) (15 points) Determine the output $y[n]$ if the input is:

$$x[n] = \sin\left(\frac{\pi n}{4}\right).$$

(a) $$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$Y(e^{j\omega})\left[1 + \frac{1}{2}e^{-4j\omega}\right] = X(e^{j\omega})\left[1 - e^{-2j\omega}\right]$$

$$y[n] + \frac{1}{2}y[n-4] = x[n] - x[n-2]$$

(b) $$X(e^{j\omega}) = \frac{\pi}{j}\left[\delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right)\right]$$

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$= \frac{\pi}{j}\left(\left[\frac{1 - e^{-2j\omega}}{1 + \frac{1}{2}e^{-4j\omega}}\right]\delta\left(\omega - \frac{\pi}{4}\right) - \delta\left(\omega + \frac{\pi}{4}\right)\right)$$

$$y[n] = \frac{1}{2\pi}\int_{2\pi} Y(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2j}\left[\frac{1 - e^{-j\frac{\pi}{2}}}{1 + \frac{1}{2}e^{-j\pi}}e^{j\frac{\pi}{4}n} - \frac{1 - e^{j\frac{\pi}{2}}}{1 + \frac{1}{2}e^{j\pi}}e^{-j\frac{\pi}{4}n}\right]$$

$$= 2\left(\frac{1}{2j}\right)\left[(1+j)e^{j\frac{\pi}{4}n} + (-1+j)e^{-j\frac{\pi}{4}n}\right]$$

$$= 2\sin\frac{\pi}{4}n + 2\cos\frac{\pi}{4}n \quad \left(= 2\sqrt{2}\sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right)\right)$$

Additional workspace for Problem 5.