Key.

let $x(t) = u(t)$

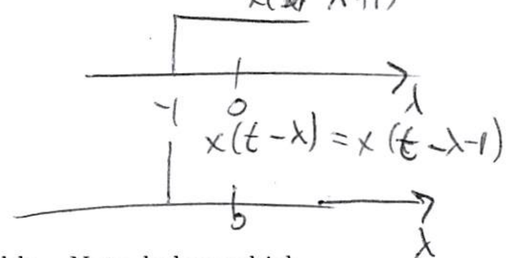
Problem 1 LTI Properties (21 pts)

[15 pts] Classify the following systems, with input $x(t)$ and output $y(t)$. In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).

let $x(t) * x(t) = y(t)$
$x(t-1) * x(t-1) = y(t-2)$.

$x(t+1) * x(t+1) = y(t+2)$

$x(\lambda+1)$



$x(t-\lambda) = x(t-\lambda-1)$

System	Causal	Linear	Time-invariant
a. $y(t) = x(t)\cos(2\pi t)$	yes	yes	no
b. $y(t) = x(t) * u(t-2)$	~~no~~ yes	yes	yes
c. $y(t) = 3x(t+1) + 1$	no	no	yes
d. $y(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$	~~yes~~ no	no	no
e. $y(t) = x(t) - \frac{1}{2}\frac{dy(t)}{dt}$	yes	~~yes~~ ?	yes

[6 pts] Two of the systems above (a,b,c,d,e) are not BIBO stable. Note below which systems are not BIBO stable, and then find a bounded input $x(t)$ which gives rise to an unbounded output $y(t)$ for each of these systems.

depends on init condx

1 System 1: b

2 Bounded input $x(t) =$ $u(t)$

1 System 2: d

2 Bounded input $x(t) =$ $u(t)$.

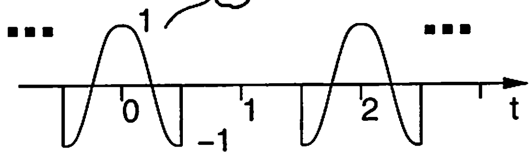
EE120 MT #1
~~Fall 2~~ Spring 2014
$\bar{x} = 64$ $\sigma_x = 18.6$
$N = 62$

Key

Problem 2 Fourier Series (27 pts)

You are given a periodic function $x(t)$ as shown, where the shape is one period of a cosine:

$\cos 4\pi t \quad -1/2 \le t \le 1/2$



$T_o = 2$

$x(t)$ can be represented by a Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jk\omega_o t}$$

[2 pts] a. What is the fundamental frequency $\omega_o =$ π

[8 pts] b. Find $x_k =$ $X_k = \dfrac{\sin \frac{\pi}{2}(2-k)}{\pi(2-k)}$

integration limits -2

algebra error -2

close guess not algebra error -3

depends on t -6

$$X_k = \frac{1}{T_o}\int_{T_o} x(t) e^{-jk\omega_o t}\, dt = \frac{1}{2}\int_{-1/2}^{1/2} \cos 2\pi t\, e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} \cos\left[\pi t(2-k)\right] dt$$

$$= \frac{1}{2} \left.\frac{\sin \pi t(2-k)}{\pi(2-k)}\right|_{t=-1/2}^{1/2} = \frac{1}{2}\left[\frac{\sin \frac{\pi}{2}(2-k)}{\pi(2-k)} + \frac{\sin \frac{\pi}{2}(2-k)}{\pi(2-k)}\right]$$

$$= X_k.$$

Problem 2, continued. $\omega_o = \frac{2\pi}{T_o} = \pi$

A periodic signal $c(t)$ with period $T_o = 2$ seconds has complex Fourier series coefficients c_k where:

$$c_k = \frac{1}{2}\left[\frac{\sin\frac{(1-k)\pi}{2}}{\pi(1-k)} + \frac{\sin\frac{(1+k)\pi}{2}}{\pi(1+k)}\right]$$

[2 pts] c. What is the time average DC power in $c(t)$? $= |c_0|^2 = \frac{1}{\pi^2}$

[3 pts] d. What is the time average power in the fundamental frequency component ω_o? $|c_1|^2 + |c_{-1}|^2 = 2\cdot\left\{\frac{1}{2}\left[\frac{1}{2}+0\right]\right\}^2 = \frac{1}{8}$

The signal $c(t)$ is passed through a filter with frequency response $H(j\omega)$, with:

$$H(j\omega) = 1 - e^{-j\omega},$$

and the output of the filter is $d(t)$, where $d(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\pi t}$.

ω dependence -3

[8 pts] e. Find d_k (leave as an expression) = _________

$$d_k = c_k H(jk\omega_o) = c_k\left[1 - e^{-jk\pi}\right] = \begin{cases} 0 & k \text{ even} \\ 2c_k & k \text{ odd} \end{cases}$$

[4 pts] f. Complete the table for specific frequency components, simplifying if possible:

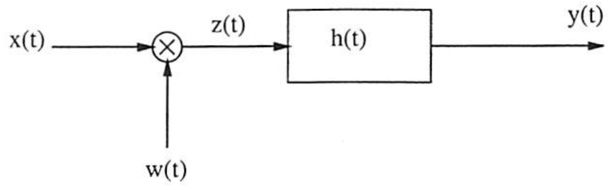
k	d(k)
0	0
1	1/2
2	0
3	0

$c_1 = \frac{1}{2}\left(\frac{1}{2}+0\right) = \frac{1}{4} \Rightarrow d_1 = \frac{1}{2}$

$c_3 = \frac{1}{2}\left[\frac{\sin(-\pi)}{\pi(-2)} + \frac{\sin\frac{4\pi}{2}}{4\pi}\right] = 0$

Key.

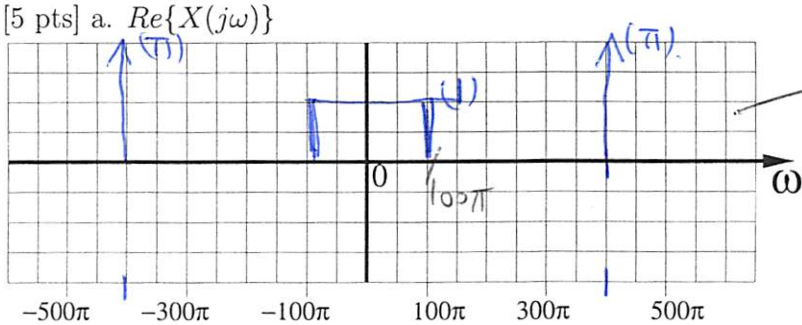
Problem 3. Fourier Transform (25 pts)
For each part below, consider the following system:



Where $x(t) = \cos(400\pi t) + \frac{\sin 100\pi t}{\pi t}$, $\quad w(t) = \frac{\sin 50\pi t}{\pi t}$, $\quad h(t) = \frac{\sin 100\pi t}{\pi t}$

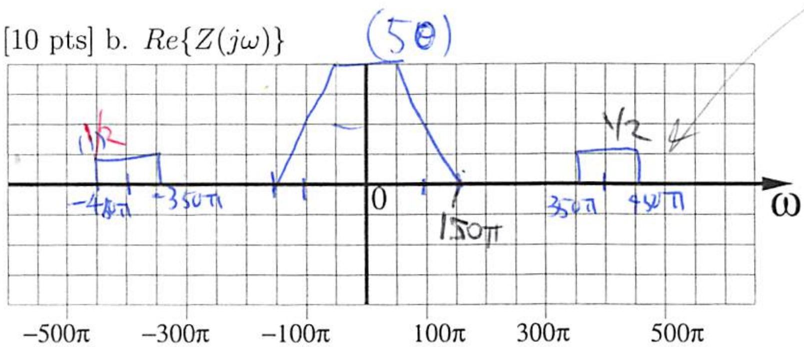
Sketch $ReX(j\omega), ReZ(j\omega), ReY(j\omega)$ labelling height/area, center frequencies, and key zero crossings for $-500\pi \leq \omega \leq 500\pi$:

[5 pts] a. $Re\{X(j\omega)\}$



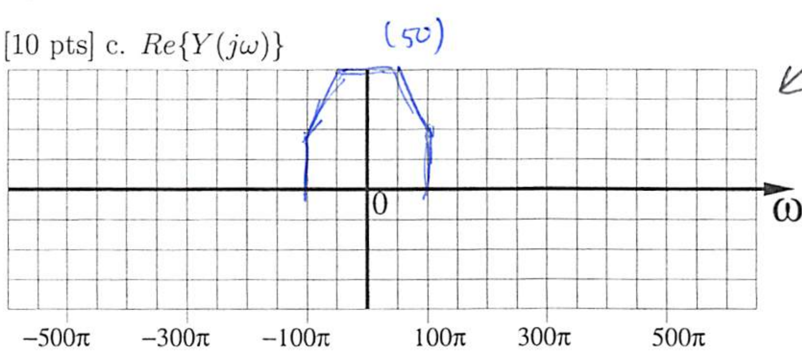
missing shape −3

[10 pts] b. $Re\{Z(j\omega)\}$



missing/extra shape −4 5
missing height −1
missing width −1
missing center −1
slightly wrong shape −2
width convolution error −3
wrong filter width −3

[10 pts] c. $Re\{Y(j\omega)\}$



Key

Problem 4. DTFT (27 points)

A causal LTI system with input $x[n]$ and output $y[n]$ is described by the difference equation:

$$y[n] - y[n-1] = x[n]$$

[4 pts] a. Find $H(e^{j\omega})$ the transfer function for the system = $\frac{1}{1-e^{-j\omega}}$

[2 pts] b. Find the impulse response $h[n]$, that is, the time response of the system to input $x[n] = \delta[n]$.

$h[n] =$ $u[n]$

~~2cos(1/2 πn)~~

[10 pts] c. If $x[n] = 2\cos(\frac{1}{2}\pi n)$ find $y[n]$. $y[n] =$ ______ $= \cos\frac{\pi n}{2} + \sin\frac{\pi n}{2}$

$$x[n] = e^{j\pi/2\, n} + e^{-j\pi/2\, n}$$

$$y[n] = e^{j\frac{\pi}{2}n} H(e^{j\pi/2}) + e^{-j\pi/2\, n} H(e^{-j\pi/2})$$

$$= \frac{e^{j\pi/2 n}}{1-e^{-j\pi/2}} + \frac{e^{-j\pi/2 n}}{1-e^{+j\pi/2}}$$

[4 pts] d. Show that $y[n]$ is real in part c (above).

let $z = \frac{e^{j\pi/2\, n}}{1-e^{+j\pi/2}}$, then $z^* = \frac{e^{-j\pi/2\, n}}{1-e^{-j\pi/2}}$

$y[n] = z + z^* = 2\,\mathrm{Re}\{z\}$. Thus $y[n]$ is real.

$$\frac{e^{j\pi n/2}}{1+j} + \frac{e^{-j\pi n/2}}{1-j} = \frac{(1-j)e^{j\pi n/2} + (1+j)e^{-j\pi n/2}}{2}$$

$j(j\sin\frac{\pi n}{2})$
$-j(-j\sin\frac{\pi n}{2})$

$$= \cos\frac{\pi n}{2} + \sin\frac{\pi n}{2}$$

Key.

Problem 4, continued.

[4 pts] e. An LTI system has transfer function $K(e^{j\omega}) = \frac{L(e^{j\omega})}{M(e^{j\omega})} = 2\cos(\omega) + 2j\sin(2\omega)$.

Write the difference equation for this system with input $m[n]$ and output $l[n]$:

$l[n] =$ ____________________

$m[n-1] + m[n+1]$
$+ m[n+2] - m[n-2]$

$K(e^{j\omega}) = e^{+j\omega} + e^{-j\omega} + e^{+2j\omega} - e^{-2j\omega}$

Impulse response

$k[n] = \delta[n+1] + \delta[n-1] + \delta[n+2] - \delta[n-2]$

$l[n] = m[n] * k[n]$

-1 sign error
-2 if not LDE

[3 pts] f. A signal $x[n]$ has DTFT $X(e^{j\omega})$.
Another signal $y[n] = x[-2n+4]$. Find the DTFT of $y[n]$ in terms of $X(e^{j\omega})$.

$Y(e^{j\omega}) =$ ____________________

for CT. $x(-2t+4)$
$= x(-2(t-2)) = x(-2t) * \delta(t-2) \longrightarrow \frac{1}{2} X\left(\frac{-j\omega}{2}\right) e^{-2j\omega}$

for DT, could sample $x(t)$ then $x[n] = x(nT)$
and $X(j\omega) \rightarrow X(e^{j\omega})$.

Thus $Y(e^{j\omega}) = \frac{e^{-2j\omega}}{2} X(e^{-j\omega/2})$