

## Problem 1: 33 points

### 1a:

Delta-Y transformation

+1 point

Delta-Y transform + KCL/KVL or only KCL/KVL

+2 points

Correct answer

+5 points

$$I_X = -7 \text{ A}$$

1a) -7 A

1b) doesn't matter ; 0

### 1b:

1st part:

Wrote down power expression

+1 point

Correct answer

+2 points

$$R_C = \text{doesn't matter}$$

2nd part:

Correct answer

+2 points

$$R_B = 0 \Omega$$

### 1c:

Didn't know how to find  $V_3$

- 4 points

Knew how to find  $V_3$ , but didn't know current  $i_x = \frac{V_{in}}{R_1}$ .

- 2 points

Knew how to find  $V_3$  and  $i_x$ , but had wrong signs in the answer

- 1 point

$$V_3 = V_y + i_x = V_y + \frac{V_{in}}{R_1}$$

### 1d:

Supplying/absorbing wrong

- 3 points

Calculated power instead of energy

- 2 points

Calculated energy, but reported unit of power

- 1 point

Calculated energy, reported correct units, but had math error

- 1 point

Absorbing power;  $Energy = 1 \text{ V} * \int_{q_0}^{q^1} dq = q(t = 1) - q(t = 0) = 3 * (e^4 - 1) \text{ J}$

### 1e:

Didn't get the correct answer

- 3 points

$$V_{ab} = 0 \text{ V}$$

### 1f:

$V_{th} = V_1$  (+3points) else 0 point

$I_n = \infty$  (+3points) else

Knowing  $R_{th} = 0$  (+1point)

Knowing  $I_n = V_{th}/R_{th}$  (+1point)

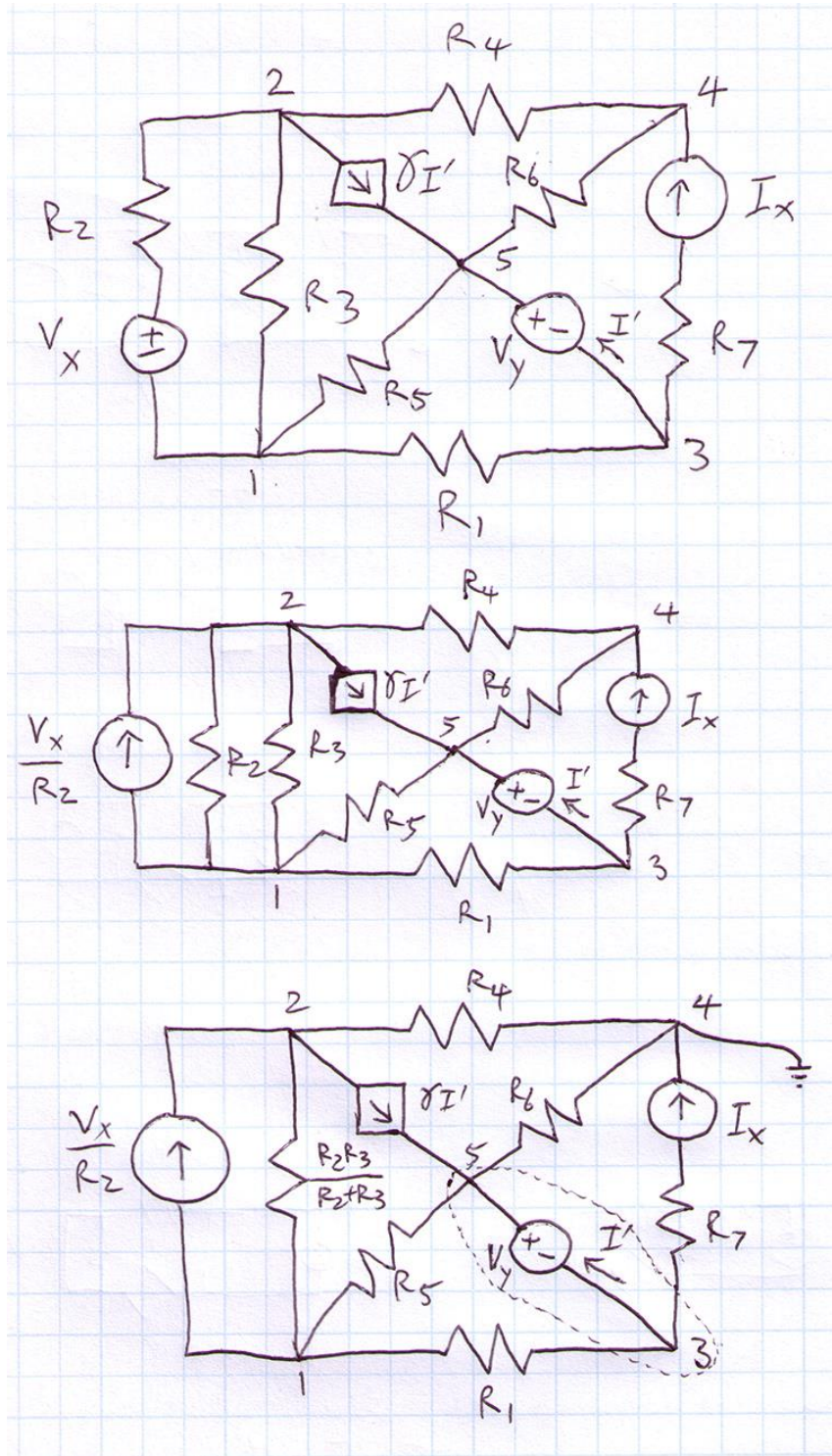
**1g:**

Write down:  $I_{R1} = 1.5A$  **(+1point)**

Get two KCL equations right:  $I_o = 2I_x$  and  $I_o = -(I_{R1} + I_x)$  **(+3points)**. I give some points for people doing mesh analysis and get the equation right.

Get:  $I_o = -1A$  **(+1point)**

**Problem 2: 25 points**



## Method 1: Nodal Analysis with Super Node

### Step 1: Set Up Equations

$$\text{Node 1: } (V1-V2)*(R2+R3)/(R2*R3) + Vx/R2 + (V1-V5)/R5 + (V1-V3)/R1 = 0$$

$$\text{Node 2: } (V2-V1)*(R2+R3)/(R2*R3) - Vx/R2 + \gamma I' + V2/R4 = 0$$

$$\text{Supernode (3&5): } V5/R6 + (V5-V1)/R5 + (V3-V1)/R1 - \gamma I' + Ix = 0$$

$$\text{Supernode Aux: } V5 = V3 + Vy$$

### Step 2: Simplify

$$\text{Node 1: } V1*[(R2+R3)/(R2*R3) + 1/R5 + 1/R1] + V2*[-(R2+R3)/(R2*R3)] + V3*[-1/R1] + V5*[-1/R5] = -Vx/R2$$

$$\text{Node 2: } V1*[-(R2+R3)/(R2*R3)] + V2*[(R2+R3)/(R2*R3) + 1/R4] + V3*0 + V5*0 = Vx/R2 - \gamma I'$$

$$\text{Supernode (3&5): } V1*[-1/R5 + -1/R1] + V2*0 + V3*[1/R1] + V5*[1/R6 + 1/R5] = \gamma I' + Ix$$

$$\text{Supernode Aux: } V1*0 + V2*0 + V3*[-1] + V5*1 = Vy$$

$$\text{Node 5: } V5/R6 + (V5-V1)/R5 = \gamma I' + I'$$

$$I'*(\gamma+1) = V5(1/R6 + 1/R5) - V1/R5$$

### Step 3: I' substitution

$$\text{Node 2: } V1*[-(R2+R3)/(R2*R3)] + V2*[(R2+R3)/(R2*R3) + 1/R4] + V3*0 + V5*0 = Vx/R2 - \gamma*[V5(1/R6 + 1/R5) - V1/R5]/(\gamma+1)$$

$$V1*[-(R2+R3)/(R2*R3) - \gamma/[(\gamma+1)*R5]] + V2*[(R2+R3)/(R2*R3) + 1/R4] + V3*0 + V5*[\gamma/(\gamma+1)]*[1/R6 + 1/R5] = Vx/R2$$

$$\text{Supernode (3&5): } V1*[-1/R5 + -1/R1] + V2*0 + V3*[1/R1] + V5*[1/R6 + 1/R5] = \gamma*[V5(1/R6 + 1/R5) - V1/R5]/(\gamma+1) + Ix$$

$$V1*[-1/R5 + -1/R1 + \gamma/[(\gamma+1)*R5]] + V2*0 + V3*[1/R1] + V5*[1/R6 + 1/R5]/(\gamma+1) = Ix$$

### Step 4: Numeric substitution and simplify

$$4V1 - 2V2 - V3 - V5 = -2$$

$$5V1 - 6V2 - 2V5 = -4$$

$$15V1 - 10V3 - 10V5 = 1$$

$$V3 - V5 = -1$$

## Method 2: Pure KCL/KVL

### Step 1: Set-up equalities and put variables on RHS

Currents:

$$\text{Node 1: } 40*V1 - 20*V2 - 10*V3 - 10*V5 = -20$$

$$\text{Node 2: } -20*V1 + 30*V2 + 0*V3 + 0*V5 = 20 - I'$$

$$\text{Node 3: } -10*V1 + 0*V2 + 10*V3 + 0*V5 = -1 - I'$$

$$\text{Node 5: } -10*V1 + 0*V2 + 0*V3 + 20*V5 = 2*I'$$

*Voltages:*

$$V_5 = V_3 + V_y: 0 \cdot V_1 + 0 \cdot V_2 - V_3 + V_5 = 1$$

*Step 2:*

Keep 1 and 5

Eliminate  $I'$  from 2,3,4 and reduce it to 2 nodal equivalents

$$40V_1 - 20V_2 - 10V_3 - 10V_5 = -20$$

$$-10V_1 + 30V_2 - 10V_3 = 21$$

$$10V_2 + 10V_5 = -1$$

$$-V_3 + V_5 = 1$$

**Note:**

Any combination of this equation and others below is okay. For example,  $30V_1 + 20V_2 - 20V_3 = 0$  is acceptable, since this equation is a linear combination of three equations above.

**Rubric:**

+5 points: Work showing at least one correct KCL/KVL equation or correct supernode analysis.

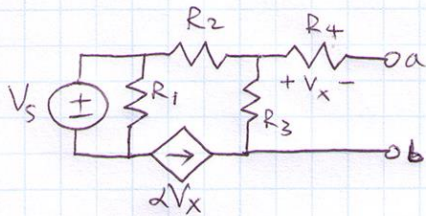
+5 points (x4): Each linearly independent equation equivalent to a linear combination of the listed equations. All the equations that satisfy  $(V_1, V_2, V_3, V_5) = (-0.8, 0.05, -1.15, -0.15)$  will be accepted.

+2.5 points (up to x4): Partial credit was awarded in the cases where students made small math or sign errors: an incorrect sign, a factor of 10, in one box of an otherwise correct equation.

### Problem 3: 15 points

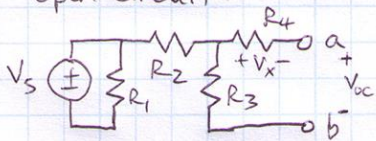
EE40 SPI4 MT1 #3

$R_1 = R_2 = 1\text{ k}\Omega$     $R_3 = R_4 = 100\ \Omega$     $V_s = 100\text{ mV}$     $\alpha = 0.01$



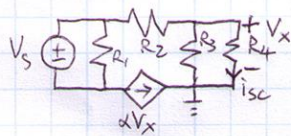
Find  $R_{th}$  across terminals a and b as a numerical answer with units

Open Circuit:



No current through  $R_4$ :  $V_x = 0$   
 $\alpha V_x = 0$ : Current source  $\rightarrow$  open circuit  
 No current through  $R_3$ :  $V_a = V_b$   
 $V_{oc} = 0$

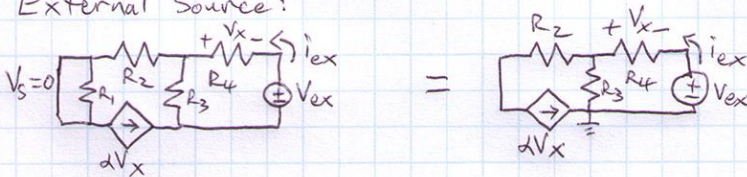
Short Circuit:



Assign ground:  $i_{sc} = V_x / R_4$   
 Ground node:  $\alpha V_x + \frac{V_x}{R_3} + \frac{V_x}{R_4} = 0$   
 $V_x \left( \alpha + \frac{1}{R_3} + \frac{1}{R_4} \right) = 0 \implies V_x = 0$   
 $i_{sc} = V_x / R_4 = 0$

Both  $V_{oc}$  and  $i_{sc}$  are 0 so  $R_{th}$  would be undefined.  
 Another solution method is necessary: external source.

External Source:



$R_1$  shorted out by zero'd independent source

Ohm's Law:  $V_x = -i_{ex} R_4$

Nodal:  $i_{ex} = \alpha V_x + (V_{ex} + V_x) / R_3 = -\alpha i_{ex} R_4 + V_{ex} / R_3 - i_{ex} R_4 / R_3$

$V_{ex} = i_{ex} (R_3 + R_4 + \alpha R_3 R_4)$

$R_{th} = 300\ \Omega$

Assign ground,

$V_{ex} / i_{ex} = R_{th} = R_3 + R_4 + \alpha R_3 R_4$

**Rubric:**

Concept - 5 points

Execution - 5 points

Correctness - 5 points

*Concept:*

This problem could not be done using the open circuit and short circuit method of equivalent circuits. If the student wrote that they explicitly realized this after attempting this method, that the answer was undefined and something was wrong, proceeded to do the external source method, or began with the external source method and attempted to execute it, they received the points. If the student arrived at a numerical answer that was 0, infinity, or anything other than undefined using this method, they received no points.

*Execution:*

If the student did the analysis either symbolically or numerically using either an external voltage or external current source, and showed using any combination of KCL, KVL, and Ohm's Law that they solved for the equivalent resistance without any errors concerning passive sign convention or execution of KCL and KVL, they received the points. If the student made minor errors such as copying the wrong values for the given variables, inverting the Ohm's Law relationship for equivalent resistance (reciprocal answer), not zero'ing the independent sources (still correct answer), or sign errors that did not directly relate to passive sign convention or KCL and KVL book-keeping, they received the points.

*Correctness:*

If the student gave the correct numerical answer with units, they received the points.

If you feel that your grading of this problem did not follow the above rubric, please submit a regrade request.

### Problem 4: 27 points

Note: We have been generous in this grading. Each correct concept or expression got points however trivial with minimum (-1) penalty for wrong units or no units. The rubric is enclosed for your information and full points were given for correct work irrespective of the methodology used. Re-grade requests on this problem has a higher probability of returning lower grades.

#### 4a: (maximum 6 points)

No : 3 points

Resistor can't deliver power or equivalent reasoning : 6 points

#### 4b: (maximum 21 points)

Yes: 3 points ( only if there is no other work and 4a answer is "No" or "bank" )

Solution : 21 points

Nodal 1:  $(V_s - V_x) / R_s + (V_1 - V_x) / R_{b1} + (V_2 - V_x) / R_{b2} = 0$  : 3 pts

Substitution:  $0.01 ( 1 - V_x ) = 0.002 * ( V_x - 0.5 )$  1 pt

$V_x = 11/12 \text{ V}$  ;  $I_{b1} = I_{b2} = 5/12 \text{ mA}$  6 pts

Nodal 2:  $b_2 * I_{b2} + b_1 * I_{b1} - I_L = 0$  3 pts

Substitution:  $I_L = 5/12 ( 0.1 + b_1 ) \text{ mA}$  1 pt

Power =  $I_L^2 * R_L$  2 pts

Substitution:  $(5/12 ( 0.1 + b_1 ) 0.001)^2 * 144 \cdot 10^6 = 1 \text{ W}$  2 pts

Solve  $b_1 = 0.1$  3 pts