

**SPRING 2009. FINAL EXAM
MATH 130**

Your name

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Student ID number

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For full credit it is sufficient to solve **6 problems**. You can try to do more problems but please note that there will be no partial credit for extra problems.

1	2	3	4	5	6	7	8	9	total

1. Let ABC be a triangle in a Hilbert plane and M be the midpoint of BC . Prove that $2AM < AB + AC$.

2. Let $ABCD$ be a cyclic quadrilateral in an Euclidean plane and let the angle bisectors of the angles ABD and ACD meet in M . Prove that M lies on the circle passing through A, B, C, D .

- 3.** (a) Given two acute angles α and β in a Hilbert plane, prove that there exists a triangle with angles α and β .
- (b) Give a construction of such triangle by Hilbert tools.

4. Let $\Pi_{\mathbb{Q}}$ be the Cartesian plane over the field of rational numbers. Which of the congruence axioms (C1)-(C6) hold in $\Pi_{\mathbb{Q}}$?

5. Let A be a P -point and l be a P -line in a Poincare plane. Give a ruler and compass construction (in the ambient Euclidean plane) of a P -line limiting parallel to l and passing through A .

6. Let l and m be two distinct lines in a hyperbolic plane which are parallel but not limiting parallel. Prove that there exists a unique reflection r such that $m = r(l)$.

7. Let a be a real root of the polynomial $x^4 + 5x^2 - 1$.
- (a) Is a constructible by ruler and compass?
 - (b) Is a constructible by Hilbert tools?

8. Let ABC be a triangle in a Hilbert plane, M be the midpoint of AB and N be the midpoint of AC . Show that if $BC = 2MN$, then the plane is semi-Euclidean. (Hint: construct a Saccheri quadrilateral with defect $\delta(ABC)$ and prove that it is a rectangle.)

9. Let AB be a P -segment in a Poincare plane and $\alpha(AB)$ be its angle of parallelism. Show that

$$\tan \alpha = \frac{2\mu(AB)}{\mu^2(AB) - 1},$$

where $\mu(AB)$ is the multiplicative distance.