EXAM 2 (100 POINTS)

GS WESTON
PHY 7B
8/5/96

(10 POINTS EACH)

1. Two positive charges each $+q_1$, and a negative charge, $-q_2$, are fixed at the verticies of an equilateral triangle, with sides r in length. Find the magnitude of the force on the negative charge. Hint: Write your answer in terms of k, q_1, q_2, r, and sin $30°$ or cos $30°$.

2. Each of the four uncharged capacitors in Figure 1 has a capacitance of 100 μF. A potential difference of 4000 V is established when the switch S is closed. How much charge then passes through the meter A?

3. A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with two conductors being held one in each hand. If the electrician's resistance is 2000 Ω, what might the fatal voltage be?

(12 POINTS)

4. What current, in terms of V and R, does the ammeter A in Figure 3 read? Assume that A has zero resistance.

(18 POINTS)

5. a) Calculate the electric field, $\vec{E}$, at point P, a distance z above the midpoint of a straight line segment of length 2L, which has a uniform charge density, lambda = charge/length as shown in Figure 2. Assume that lambda is a constant.
   b) Find the electric field, E, for z >> L.

(20 POINTS EACH)

6. Figure 4 shows a parallel plate capacitor of plate area A and plate separation d. A potential difference, V_0, is applied between the plates. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant k that partially fills the space is placed beween the plates as shown.
   a) Calculate the free charge (in terms of ϵ_0, A, d and V_0) that appears on the plates.
   b) Calculate the electric field, $\vec{E}_0$, in the gaps between the plates and the dielectric slab.
   c) Calculate the electric field, $\vec{E}$, in the dielectric slab.
   d) What is the potential difference between the plates after the slab has been introduced?
   e) What is the capacitance with the slab in place?

7. Two concentric spherical conducting shells, as shown in Figure 5, with radii a and b have +Q and -Q respectively.
   a) Calculate the electric field, $\vec{E}$, using Gauss' law for r < a.
   b) Calculate the electric field, $\vec{E}$, using Gauss' law for a < r < b.
   c) Calculate the electric field, $\vec{E}$, using Gauss' law for r > b.
   d) Calculate the electric potential, V, between the spheres.
   e) Calculate the capacitance between the spheres.

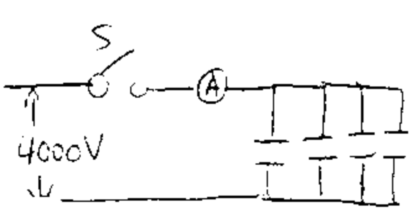

Figure 1

Figure 2

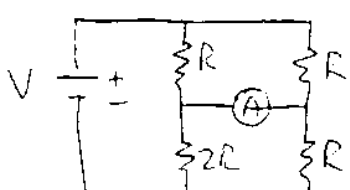

Figure 3

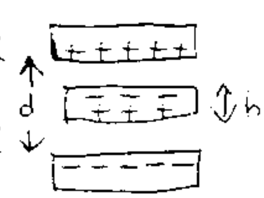

Figure 4

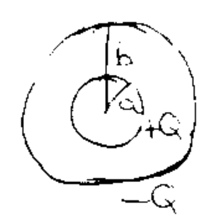

Figure 5

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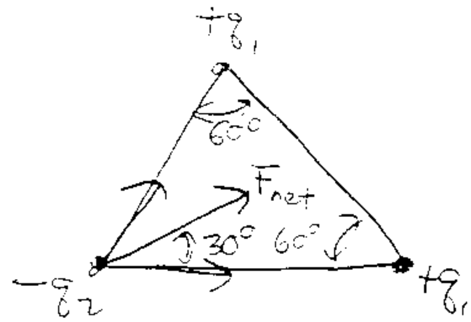
Exam 2 Solutions

① Electrical Forces

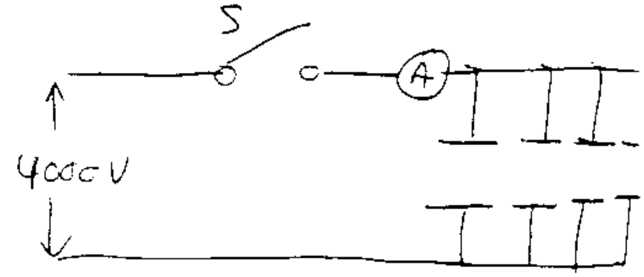


$$F_{net} = \Sigma F \cos 30°$$

$$F = \frac{k q_1 q_2}{r^2}$$

$$\Rightarrow F_{net} = \frac{2 k q_1 q_2 \cos 30°}{r^2}$$

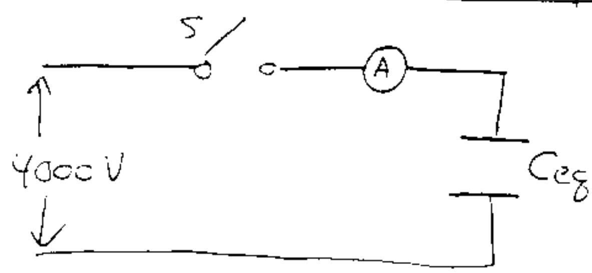
② Capacitors in Parallel



$$C_{eq} = 4C = 4(100\,\mu F) = 400\,\mu F$$

$$Q_{eq} = C_{eq} V = (400\,\mu F)(4000\,V)$$

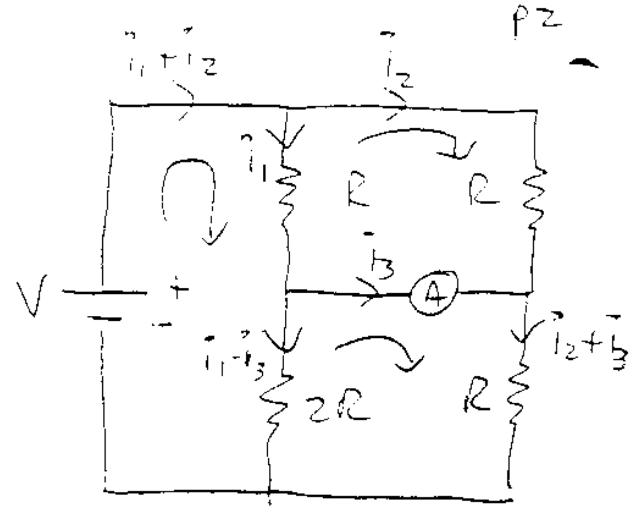
$$\boxed{Q_{eq} = 1.6\ C}$$



③ Ohm's Law

$$V = IR = (50 \times 10^{-3} A)(2000\,\Omega) = \boxed{100\,V = V}$$

④ Multiloop Circuit



(1) $V - i_1 R - (i_1 - i_3)(2R) = 0$

(2) $-i_2 R + i_1 R = 0$

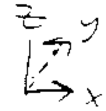
(3) $-(i_2+i_3)R + (i_1 - i_3)(2R) = 0$

(2) $\Rightarrow i_1 = i_2$

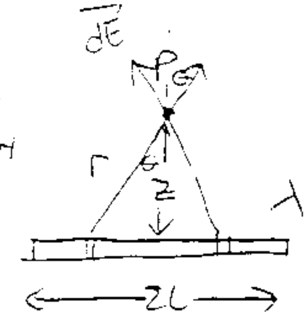
(3) $\Rightarrow -(i_1+i_3)R + (i_1-i_3)(2R) = 0 \Rightarrow i_1 R - 3 i_3 R = 0 \Rightarrow i_1 = 3 i_3$

(1) $\Rightarrow V - (3i_3)R - (3i_3 - i_3)(2R) = 0 \Rightarrow V = 7 i_3 R \Rightarrow \boxed{i_3 = V/7R}$

(5) Electric Field due to Continuous Charge Distribution



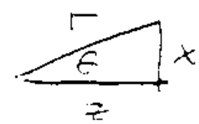
a) <u>From Symmetry</u> – Consider Segments on left & right



$$\vec{dE} = \frac{2k(\lambda dx)\cos\theta}{r^2}\,\hat{z} = dE\,\hat{z}$$

$$(\vec{E} = E\,\hat{z})$$

$\cos\theta = z/r$, $r = \sqrt{x^2+z^2}$



$$E = \int dE = k\int_0^L \frac{2\lambda z}{(x^2+z^2)^{3/2}}\,dx$$

let $x = z\tan\theta$

$dx = z\sec^2\theta\,d\theta$

$$\Rightarrow E = 2k\lambda z\int_0^L \frac{dx}{(x^2+z^2)^{3/2}} = \int \frac{2k\lambda z\,(z\sec^2\theta)\,d\theta}{z^3\sec^3\theta}$$

$$E = \frac{2k\lambda}{z}\int\cos\theta\,d\theta = \frac{2k\lambda}{z}\sin\theta\Big|_0^{\theta_0} = \frac{2k\lambda}{z}\frac{x}{\sqrt{x^2+z^2}}\Big|_0^L$$

$$\boxed{E = \frac{2k\lambda}{z}\frac{L}{\sqrt{L^2+z^2}}}$$

b) for $z \gg L$

$$\Rightarrow \sqrt{z^2+L^2} \to z \quad \Rightarrow \boxed{E = \frac{2k\lambda L}{z^2}} \text{ for } z \gg L$$

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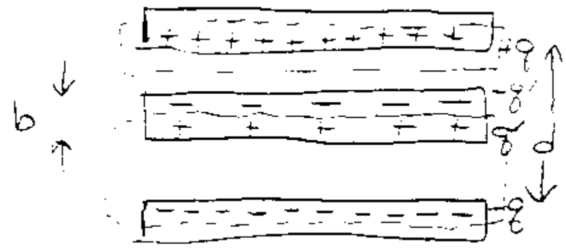
⑥ Capacitor w/ Dielectric



a) $q = C_0 V_0 = \left(\frac{\epsilon_0 A}{d}\right) V_0$

$$\boxed{q = \frac{\epsilon_0 A V_0}{d}}$$

b) $\oint \vec{E} \cdot d\vec{A} = q_{enc}/\epsilon_0 \Rightarrow E_0 A = q/\epsilon_0$

$$\Rightarrow E_0 = q/\epsilon_0 A = \boxed{V_0/d = E_0}$$

c) $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0 \kappa} \Rightarrow -EA = \frac{-q}{\epsilon_0 \kappa} \Rightarrow E = \boxed{\frac{q}{\epsilon_0 \kappa A} = \frac{E_0}{\kappa} = E}$

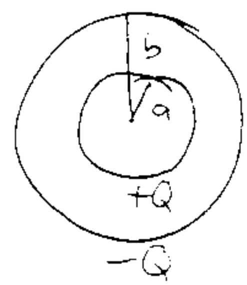
d) $V = \int_+^- E\, ds = E_0 (d-b) + E b = \frac{V_0}{d}(d-b) + \frac{V_0}{d\kappa} b$

$$\boxed{V = V_0 \left[1 + \frac{b}{d}\left(\frac{1}{\kappa} - 1\right)\right]}$$

e) $C = q/V = \frac{\epsilon_0 A}{d\left[1 + \frac{b}{d}(1/\kappa - 1)\right]} = \boxed{\frac{\epsilon_0 A}{d + b\left(\frac{1}{\kappa} - 1\right)} = C}$

⑦ Gauss Law + Capacitance of Spherical Shells

a) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$



$r<a \Rightarrow q_{enc}=0 \Rightarrow \boxed{\vec{E}=0 \quad r<a}$

b) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$\Rightarrow E(4\pi r^2) = \frac{+Q}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} \qquad a<r<b}$

c) $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad r>b \Rightarrow q_{enc}=0 \Rightarrow \boxed{\vec{E}=0 \quad r>b}$

d) $V = -\int_b^a \vec{E}\cdot d\vec{s} = \frac{-Q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\Big|_b^a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

e) $C = \frac{Q}{V} = \boxed{4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) = C}$