

2/26/14

Midterm Examination 1

Prof. C. Chang-Hasnain

- If you need additional conditions to solve a problem, please write down your assumptions.
- Answer in the space below the problem. If you need additional space, you can use the overflow pages in the back. Please indicate which page the answer continues in the lower-right corner.
- Please put a square box around your final answers for each problem.

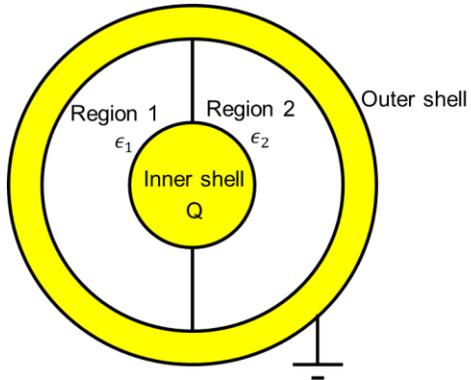
Your Name	
Student ID #	
Signature	

Problem	Points	Points Earned
1)	20	
2)	30	
3)	20	
4)	30	
Total	100	

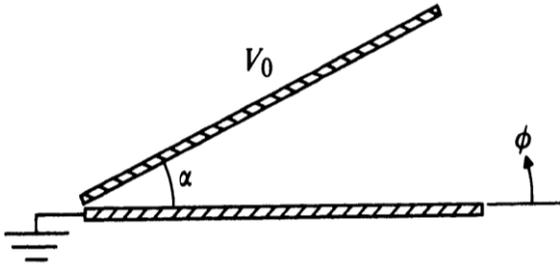
Laplace Equation:

Cartesian Coordinates	$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$
Cylindrical Coordinates	$\Delta f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} = 0$
Spherical Coordinates	$\Delta f = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\rho^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2} = 0.$

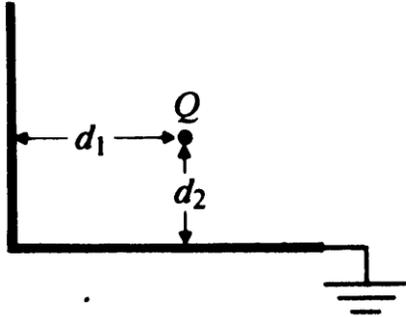
1. **(Gauss's Law and Boundary conditions)** We have two concentric **spherical shells** made of perfect **conductors**. Two different dielectric are filled in between the two shells, as shown below. On the left side the permittivity is ϵ_1 , and on the right side permittivity is ϵ_2 . The inner shell carries a total amount of charge Q , whereas the outer shell is grounded. Please find the electric field in region 1 and region 2.



2. **(Laplace equation and Boundary conditions)** Two infinite conducting planes maintained at potentials 0 and V_0 form a wedge-shaped configuration, as shown in the figure. Determine the potential distributions for the regions: (a) $0 < \phi < \alpha$ (b) $\alpha < \phi < 2\pi$.



3. **(Columb's law and image method)** A positive point charge Q is located at distance d_1 and d_2 , respectively, from two grounded perpendicular conducting half-planes (infinitely long), as shown in the figure. **Determine the forces on Q caused by the charges induced on the planes.**



4. **(Magnetic potential and Laplace equation)** A coaxial transmission line shown below has an inner conductor with radius a and outer conductor between $r=5a$ and $r=6a$. The inner and outer conductors both carry current I in opposite directions, \hat{z} and $-\hat{z}$, respectively. The magnetic potential $A = 0$ at $r = 5a$. **Find the vector magnetic potential** between two conductors, i.e. $5a < r < 6a$. (**hint: A only has r dependence.**)

