PHYS 7C SPRING '05 | BATTAGLIA MT1 SOLS.

1

(i) AMPERE'S LAW:

17 pts.

$$\frac{d}{dt}\left(\epsilon_0 \vec{E}\right) = \vec{\nabla} \times \frac{\vec{B}}{\mu_0}$$

ARBITRARY — ORIENTATION IN Y-Z PLANE NOT GIVEN

$$\vec{\nabla} \times \vec{B} = -\mu_0 \epsilon_0 \omega E_0 \cos(kx-\omega t)\,\hat{y}$$

$$= -\frac{\cancel{\omega}}{\cancel{c}}\frac{E_0}{c}\frac{1}{\cancel{k}}\frac{d}{dx}\sin(kx-\omega t)\,\hat{y}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\hat{x} \times \hat{z} = -\hat{y}$$

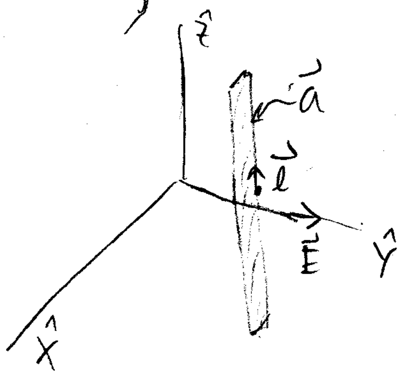
$$\Rightarrow \vec{B} = \frac{E_0}{c}\sin(kx-\omega t)\,\hat{z}$$

DIRECTION NOT REQUIRED

INTEGRAL METHOD:

$$\frac{d}{dt}\int (\epsilon_0 \vec{E})\cdot d\vec{a} = \oint \frac{\vec{B}}{\mu_0}\cdot d\vec{l}$$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \int \left(\epsilon_0 \mu_0 \frac{d}{dt}\vec{E}\right)\cdot d\vec{a}$$



SO $\vec{B}$ IS $\propto \hat{x}$ OR $\hat{z}$

NEXT →

BUT

$$\oint \vec{E} \cdot d\vec{l}' = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}'$$

SO $\vec{B} \propto \hat{y}$ OR $\hat{z} \Rightarrow \vec{B} \propto \hat{z}$

$\Rightarrow \begin{pmatrix} \text{BOUNDARY} \\ \text{DERIVATIVE} \end{pmatrix} \vec{B} = \epsilon_0 \mu_0 \frac{d}{dt} \vec{E}$.

$\Rightarrow$ GIVING SAME RESULT $\vec{B} = -\int dx\, \epsilon_0 \mu_0 \frac{d}{dt} \vec{E}$

(ii) $\nabla^2 \vec{E} = +\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E}$

8 pts.

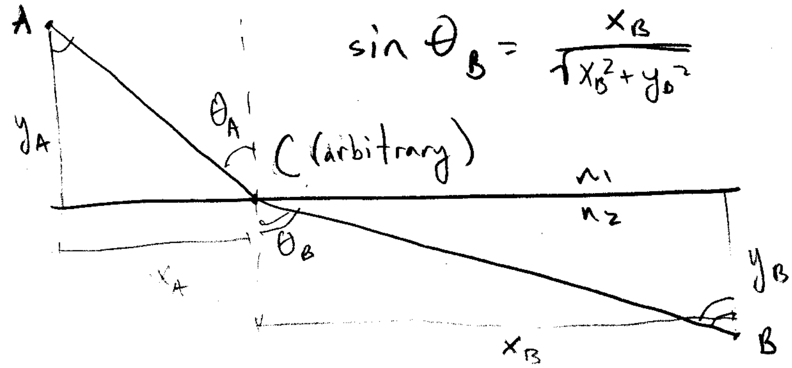
$$-k^2 E_0 \sin(kx-\omega t) = +\frac{1}{c^2}(-\omega^2) E_0 \sin(kx-\omega t)$$

$$c^2 = \frac{\omega^2}{k^2} \quad \checkmark$$

$$\sin\theta_A = \frac{x_A}{\sqrt{x_A^2+y_A^2}}$$

$$\sin\theta_B = \frac{x_B}{\sqrt{x_B^2+y_B^2}}$$

2.) $v = \frac{c}{n}$



$$t = \frac{d_A}{v_A} + \frac{d_B}{v_B} = \frac{n_1}{c}\sqrt{x_A^2+y_A^2} + \frac{n_2}{c}\sqrt{x_B^2+y_B^2}$$

For fixed A, B, the quantities $x \equiv x_A + x_B$, y_A, and y_B are fixed.

$$\Rightarrow t = \frac{n_1}{c}\sqrt{x_A^2+y_A^2} + \frac{n_2}{c}\sqrt{(x-x_A)^2+y_B^2}$$

Minimum time occurs when $\frac{dt}{dx_A} = 0$.

$$\Rightarrow 0 = \frac{n_1 x_A}{\sqrt{x_A^2+y_A^2}} - \frac{n_2(x-x_A)}{\sqrt{(x-x_A)^2+y_B^2}}$$

$$= \frac{n_1 x_A}{\sqrt{x_A^2+y_A^2}} - \frac{n_2 x_B}{\sqrt{x_B^2+y_B^2}}$$

$$= n_1 \sin\theta_A - n_2 \sin\theta_B$$

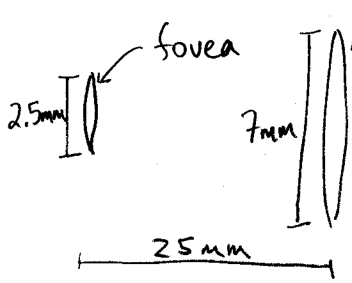
$$\Rightarrow \boxed{n_1 \sin\theta_A = n_2 \sin\theta_B}$$

For reflection, calculations are the same, but the 2nd line is also in medium 1, so $n_1 \sin\theta_A = n_1 \sin\theta_B$

$$\Rightarrow \sin\theta_A = \sin\theta_B$$

$$\Rightarrow \boxed{\theta_A = \theta_B}$$

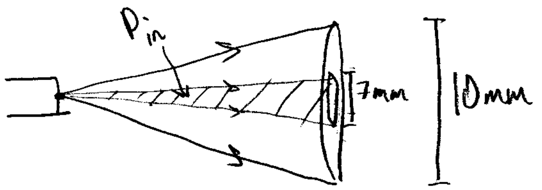
3.)



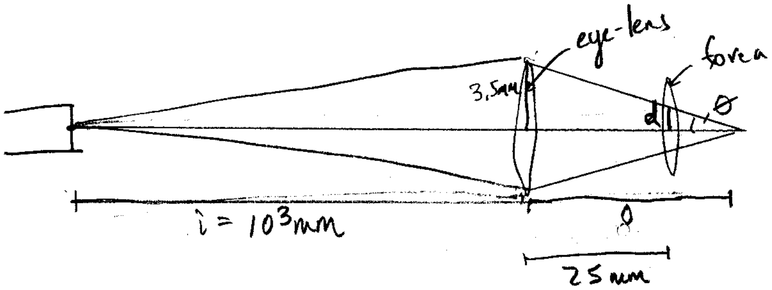
i.) lens-makers formula: $\frac{1}{f} = \left(\frac{n_{lens}}{n_{outside}} - 1\right)\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

For $\frac{1}{r_1} = -\frac{1}{r_2}$, $\frac{1}{f} \cong \left(\frac{1.5}{1} - 1\right)\left(\frac{2}{r_2}\right) = \frac{1}{r_2} \Rightarrow \boxed{r_2 \cong f = 25\,mm}$

ii.) $\tau \cong .25\,s$



$P_{in} = 3\,mW \left(\frac{\pi (7\,mm)^2/4}{\pi (10\,mm)^2/4}\right) \cong 1.47\,mW$



(not drawn to scale!)

$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \Rightarrow o = \frac{if}{i-f} \cong 25.64\,mm$

$\tan\theta = \frac{3.5\,mm}{o} = \frac{d}{o - 25\,mm} \Rightarrow d = \left(1 - \frac{25\,mm}{o}\right)3.5\,mm = .0875\,mm$

$\Rightarrow d < 1.25\,mm$ (radius of fovea), so all of "P_{in}" focuses to spot of radius d

$\Rightarrow$ irradiance $= \frac{P_{in}}{\pi d^2} \cong 61.1\,mW/mm = \boxed{6.1 \times 10^4\,W/m^2}$

iii.) If the entire fovea weighs .1g, the spot weighs $(.1g)\left(\frac{\pi d^2}{\pi r^2}\right) \cong (.1g)\left(\frac{.0875}{1.25}\right)^2 \cong 4.9 \times 10^{-4} g$

$\Rightarrow \Delta T = \frac{\Delta E}{mC} = \frac{P_{in}\Delta t}{mC} = \frac{(1.47\,mW)(.25\,s)}{(4.9 \times 10^{-4} g)(4\,J/g/C)} \cong \boxed{.2\,°C}$

Or, if spot weighs .1g, $\Delta T = \frac{(1.47\,mW)(.25\,s)}{(.1g)(4\,J/g/C)} = \boxed{9.18 \times 10^{-4}\,°C}$

Grading Rubric:

#2) 25 pts. total

- 7 pts. for finding time
- 18 pts. for deriving refl. & refraction laws <u>using Fermat's principle</u>
  (8 pts. for reflection; 10 for refraction)

#3) 25 pts. total

i.) 5 pts.

ii.) 5 pts. for getting "P_{in}"

5 pts. for finding spot size and calculating irradiance

iii.) 5 pts.

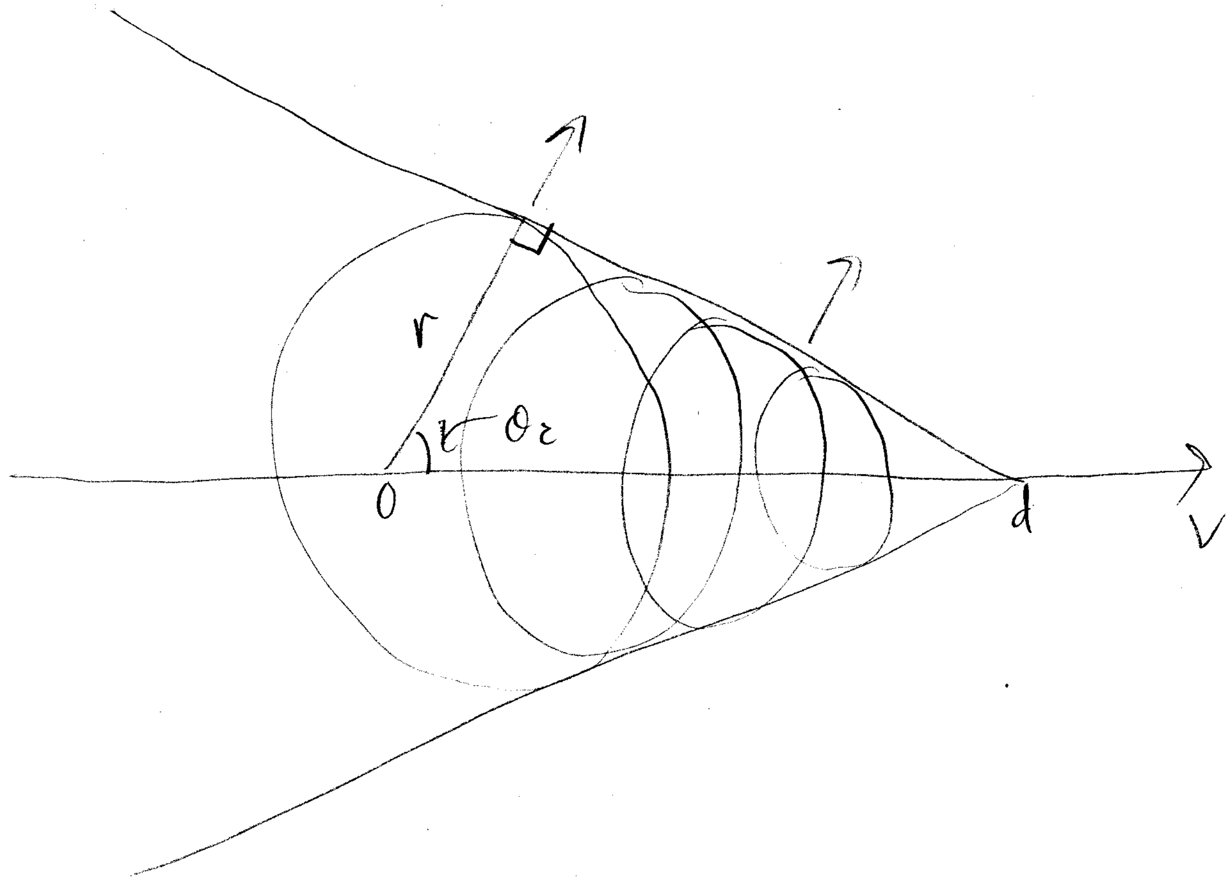
graded independently (missing one piece doesn't directly affect the others)

+ 5 pts. for correctly putting parts together and getting correct final result

(note: students could assume either spot size or entire fovea weighs 1 g for part iii.; see solutions)

4

(i)
8 pts.



$$\cos\theta_c = \frac{r}{d} = \frac{\frac{d}{v}\cdot\frac{c}{n}}{d} = \frac{c}{vn} \checkmark$$

(ii) $\frac{c}{vn} < 1 \Rightarrow \left\lceil \frac{v}{c} > \frac{1}{n} \right\rfloor$
4 pts.

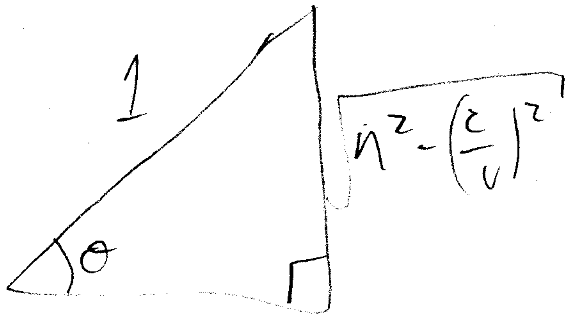
(iii) THERE IS REFRACTION:
13 pts.

$n \sin\theta_c = \sin\theta$; $\frac{R}{x_r} = \tan\theta$ dist. to screen

$$\sin\theta_c = \sqrt{1-\cos^2\theta_c} = \sqrt{1-\left(\frac{c}{vn}\right)^2}$$

$$\sin\theta = \sqrt{n^2-\left(\frac{c}{v}\right)^2}$$

NEXT →



$$\hookrightarrow \sqrt{1-n^2+\left(\frac{c}{v}\right)^2}$$

$$R = x\tan\theta = x\cdot\frac{\sqrt{n^2-\left(\frac{c}{v}\right)^2}}{\sqrt{1-n^2+\left(\frac{c}{v}\right)^2}}$$

$$= x\sqrt{\frac{\left(\frac{v}{c}n\right)^2-1}{1-\left(\frac{v}{c}\right)^2\left(n^2-1\right)}}$$

$$= 1\,\text{m}\sqrt{\frac{\left(\frac{2.5}{3}\cdot 1.5\right)^2-1}{1-\left(\frac{2.5}{3}\right)^2\cdot\left(1.5^2-1\right)}}$$

$$R \simeq 2.06\ \text{m}$$

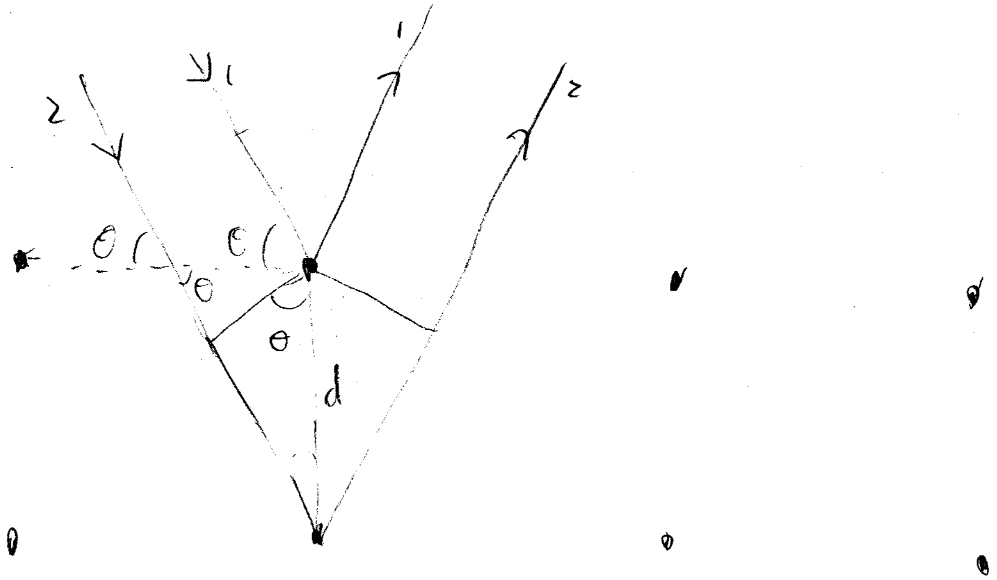
+ RADIUS AT EDGE OF GLASS: $0.75\text{cm}\cdot\tan\theta_2$

$$= 0.75\,\text{cm}\sqrt{\left(\frac{vn}{c}\right)^2-1}$$

$$\Rightarrow \ulcorner 2.07\ \text{m} \lrcorner$$

5

(i)
15 pts.



$\Delta x_{12} = 2(d\sin\theta) \Rightarrow \boxed{n\lambda = 2d\sin\theta}$

THIS IS FOR ONE LAYER, BUT STACKING LAYERS YOU GET THE SAME RELATION

(ii)
7 pts.

$\theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right)$

$n=0$: 0^0.
$n=1$: 45.7^0
$n=2$: —

$\boxed{45.7^0 \simeq 0.8\,(\text{rad})}$

(iii)
3 pts.

$\frac{\lambda}{2d} < 1 \Rightarrow \lambda < 2d$

$\lambda < 0.215\,\text{nm}$ $\boxed{\text{NO}}$