Midterm 2
EE40
Spring 2014

NAME: Solutions.

Instructions

Read all of the instructions and all of the questions before beginning the exam.

There are 4 problems in this exam. The total score is 100 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

IMPORTANT

- **If you do not put your answers within the boxes labeled 'Solution' THEY WILL NOT BE COUNTED (no matter how correct they may be in the bottom left back corner of the third to last page of the exam.)**
- **If you have more than one solution in the box, that box will be given zero points.**

Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam.

Be sure to provide units where necessary.

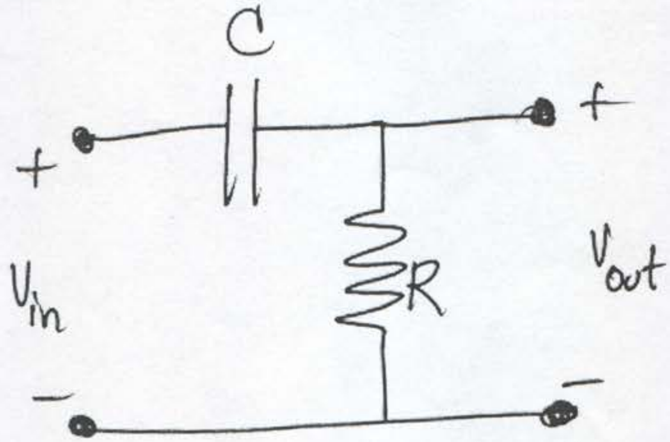
GOOD LUCK!

PROBLEM	POINTS	MAX
1		20
2		25
3		30
4		25

Donnie: *Why are you wearing that stupid bunny suit?*
Frank: *Why are you wearing that stupid man suit?*
- Donnie Darko, 2001

Problem 1 *Warm-up (20 points)*

Consider the circuit below.



a) If v_{in} is a DC source equal to 1 V, what is the steady-state value of v_{out}?

Solution: $V_{out} = 0\,V$

b) If instead, if $v_{in} = \begin{cases} 0\,V \text{ for } t < 0 \\ 1\,V \text{ for } t \geq 0 \end{cases}$ what is $v_{out}(t)$ for $t > 0$

Solution: $e^{-t/RC}$ Volts.

method 1: $V_{out}(t) = i(t)R = RC\dfrac{d}{dt}(1 - v_{out})$

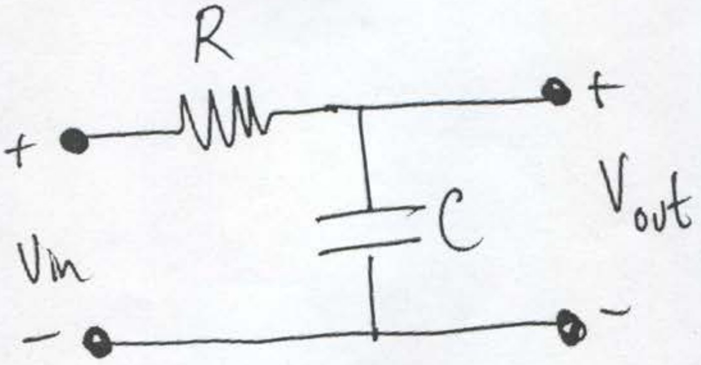
$\therefore \dfrac{dv_{out}}{v_{out}} = -\dfrac{dt}{RC} \Rightarrow v_{out}(t) = a_0 e^{-t/RC}$ V

$v_{out}(t=0) = 1V \Rightarrow v_{out}(t) = e^{-t/RC}$ V

Alternatively: $V_{out}(t) = V_{out}(t=\infty) + [V_{out}(t=0) - V_{out}(t=\infty)]\, e^{-t/RC}$

$v_{out}(t) = e^{-t/RC}\ (V)$

Consider the circuit below.



c) If v_{in} is a DC source equal to 1 V, what is the steady-state value of v_{out}?

Solution: $V_{out} = 1V$

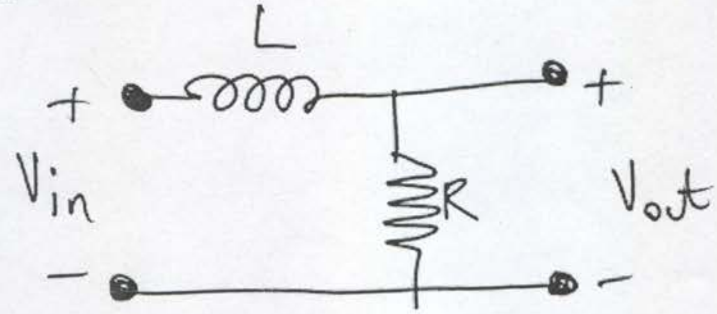
$V_{out}(t=0) = 0\ V$
$V_{out}(t=\infty) = 1V$

d) If instead, if $v_{in} = \begin{cases} 0\ V\ for\ t < 0 \\ 1\ V\ for\ t \geq 0 \end{cases}$ what is $v_{out}(t)$ for t >0

Solution: $1 - e^{-t/RC}$ Volts.

$V_{out} = 1 + (0-1)e^{-t/\tau}$
$V_{out}(t) = 1 - e^{-t/\tau}$

Consider the circuit below.



e) If v_{in} is a DC source equal to 1 V, what is the steady-state value of v_{out}?

Solution: $V_{out} = 1V$

$v_{out}(t=0) = 0 \ V$

$v_{out}(t=\infty) = 1V$

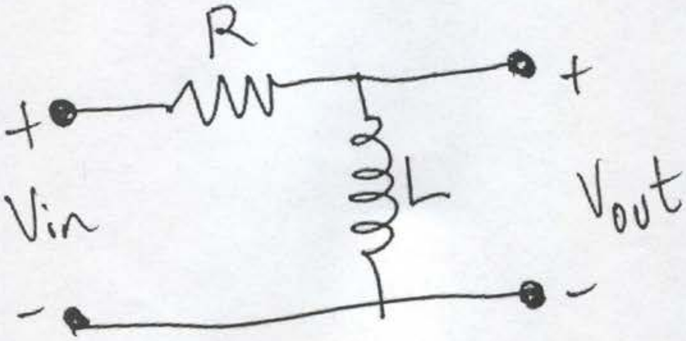
f) If instead, if $v_{in} = \begin{cases} 0\,V\ for\ t < 0 \\ 1\,V\ for\ t \geq 0 \end{cases}$ what is $v_{out}(t)$ for t >0

Solution: $1 - e^{-tR/L}$ Volts.

$$v_{out}(t) = v_{out}(t=\infty) + [v_{out}(t=0) - v_{out}(t=\infty)]e^{-t/\tau}$$
$$= 1 - e^{-t/\tau}$$

$$\tau = L/R$$

Consider the circuit below.



g) If v_{in} is a DC source equal to 1 V, what is the steady-state value of v_{out}?

Solution: $V_{out} = 0V$

$v_{out}(t=0) = 1V$

$V_{out}(t=\infty) = 0V$

h) If instead, if $v_{in} = \begin{cases} 0\,V\ for\ t < 0 \\ 1\,V\ for\ t \geq 0 \end{cases}$ what is $v_{out}(t)$ for t >0

Solution: $e^{-tR/L}$

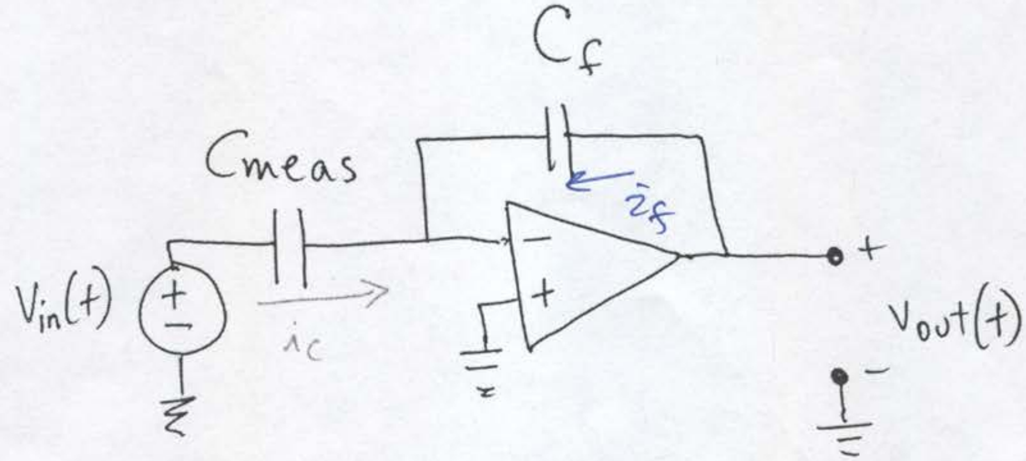
$v_{out}(t) = 0 + (1-0)e^{-t/\tau}\ ;\ \tau = L/R.$

Otto West: *"Apes don't read philosophy."*
Wanda: *"Yes they do, Otto. They just don't understand it. Now let me correct you on a couple of things, OK? Aristotle was not Belgian. The central message of Buddhism is not 'Every man for himself.' And the London Underground is not a political movement. Those are all mistakes, Otto. I looked them up."*

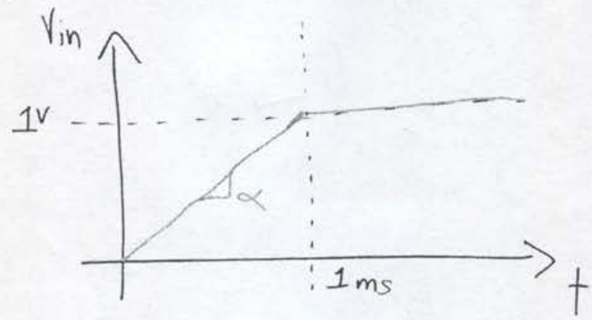
<u>A Fish Called Wanda</u>, 1988

Problem 2 (25 points)

Consider the circuit below; it is designed to measure C_{meas}.



$V_{in}(t)$ is 0 for $t < 0$ and follows the following graph for $t \geq 0$.

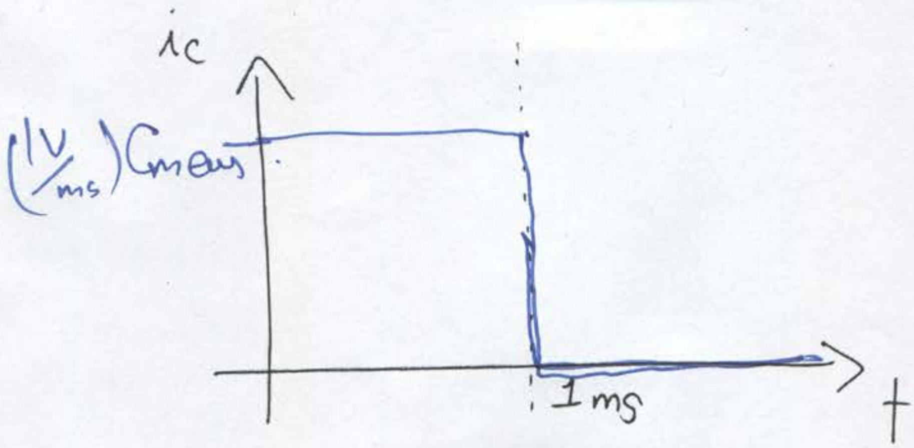


a) Provide a plot of $i_C(t)$ on the axes below.

$\underline{0 \leq t \leq 1\,ms}$: $\quad i_c(t) = C_{meas} \dfrac{dV_{in}}{dt}$

$$i_c(t) = C_{meas}\,\alpha = \left(\frac{1V}{ms}\right) C_{meas}.$$

$\underline{1\,ms < t}$: $\quad \dfrac{dV_{in}}{dt} = 0 \quad \therefore\ i_c(t) = 0.$



b) Provide an expression for $v_{out}(t)$ for $0 < t \leq 1$ ms.

Solution: $-\left(1V/_{ms}\right) \dfrac{C_{meas}}{C_f} t$

$$i_c(t) + C_f \frac{dv_{out}}{dt} = 0$$

$$C_f \frac{dv_{out}}{dt} = -\alpha C_{meas}.$$

$$\therefore \; v_{out}(t) = -\alpha \frac{C_{meas}}{C_f} t$$

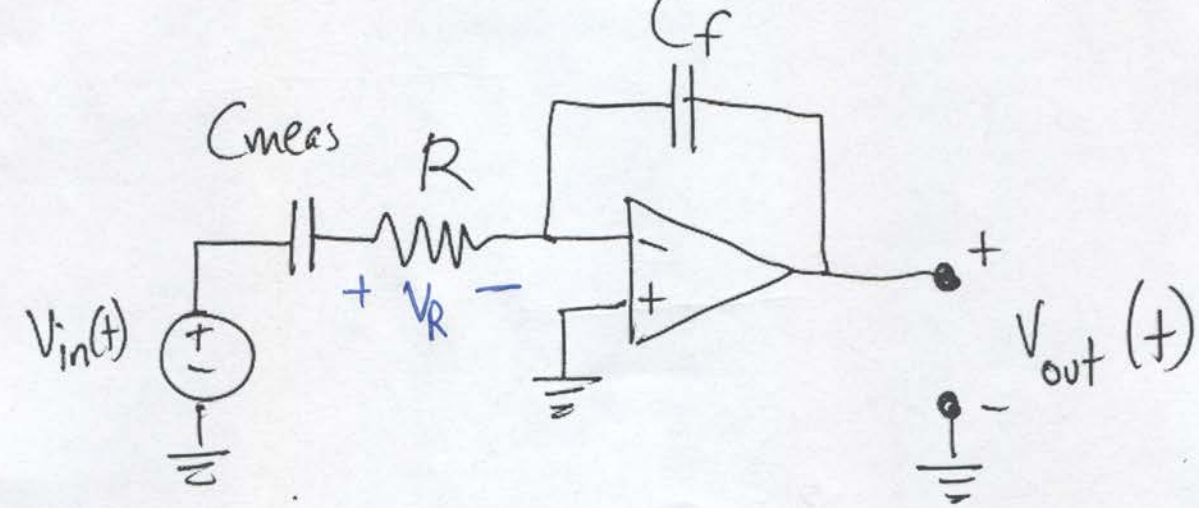
$$V_{out}(t) = -\left(1V/_{ms}\right) \frac{C_{meas}}{C_f} t$$

c) What is the value of $v_{out}(t)$ at 1 ms?

Solution: $-C_{meas}/C_f$ Volts.

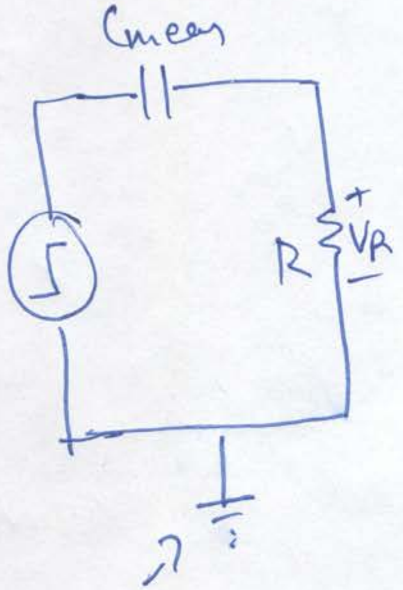
$$V_{out}(t = 1ms) = -\frac{1V}{ms} \times 1ms \times \frac{C_{meas}}{C_f}$$

Now consider the following circuit.



d) For the circuit above, assume $v_{in}(t)$ changes so fast at $t = 0$ that it can be considered a step function. (In other words, assume $v_{in} = \begin{cases} 0\ V\ for\ t < 0 \\ 1\ V\ for\ t \ge 0 \end{cases}$). Provide an expression for $v_{out}(t)$ for $t > 0$.

Solution: $v_{out}(t) = \dfrac{-C_{meas}}{C_f}\left(1 - e^{-\frac{t}{RC_{meas}}}\right)$ Volts.



Virtual gnd.
assuming ideal
op-amp.

$$V_R = e^{-t/RC_{meas}}$$

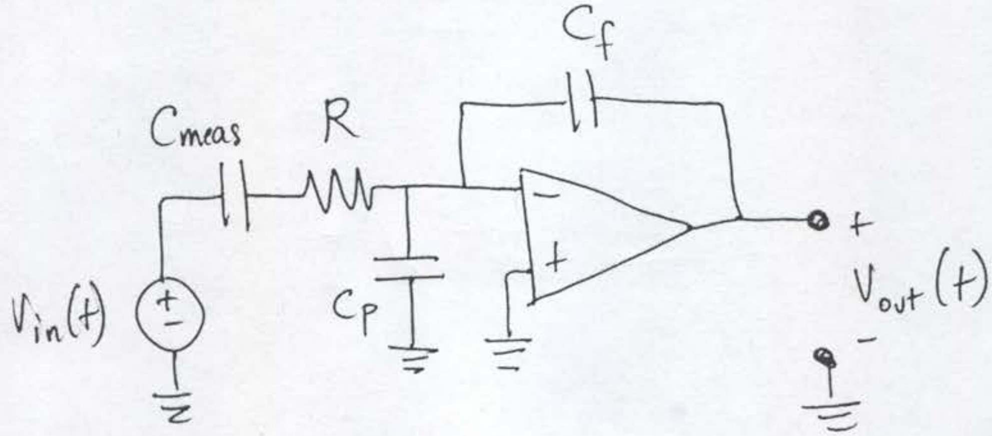
KCL:

$$\frac{V_R}{R} = -C_f \frac{dv_{out}}{dt}$$

$$\therefore v_{out} = -\frac{1}{RC_f}\int_0^t e^{-\frac{t'}{RC_{meas}}}\,dt'$$

$$v_{out} = -\frac{RC_{meas}}{RC_f}\left(1 - e^{-t/RC_{meas}}\right)$$

Now consider the following circuit.



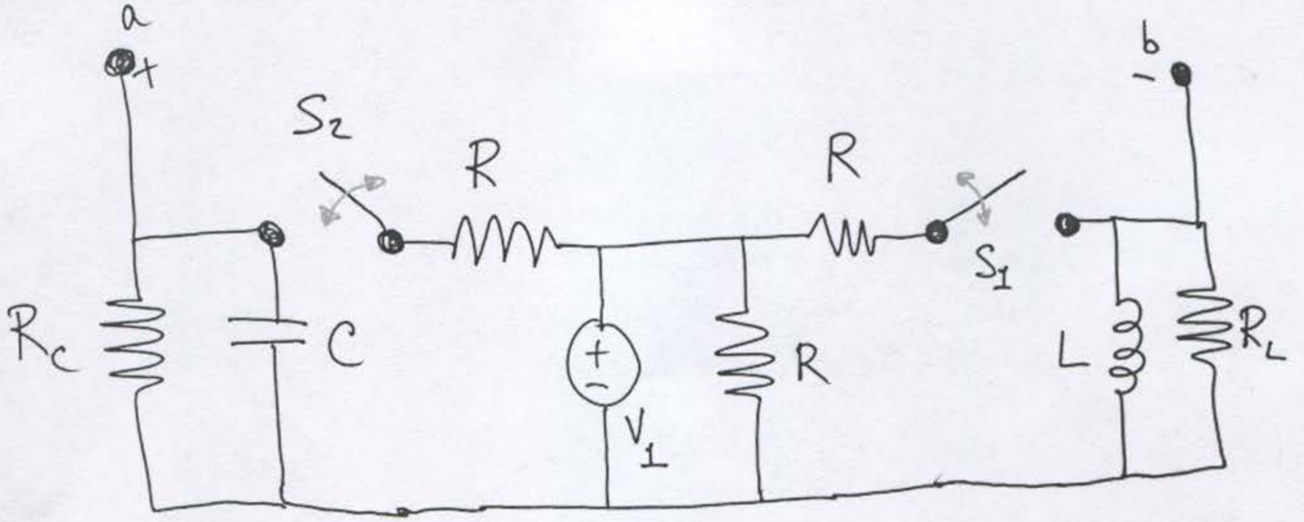
e) For the circuit above, assume $v_{in}(t)$ changes so fast at t = 0 that it can be considered a step function. (In other words, assume $v_{in} = \begin{cases} 0\ V\ for\ t < 0 \\ 1\ V\ for\ t \geq 0 \end{cases}$). Provide an expression for $v_{out}(t)$ for t > 0.

Solution: $-\frac{C_{meas}}{C_f}\left(1-e^{-t/RC_{meas}}\right)$ Volts

Still assuming ideal op-amp, C_P is shorted.

$$\therefore\ v_{out}(t) = -\frac{C_{meas}}{C_f}\left(1-e^{-\frac{t}{RC_{meas}}}\right)$$

Problem 3 (n points)
Consider the circuit below.



$R_L = 1\ k\Omega$
$L = 1\ mH$

$R_C = 1\ k\Omega$
$C = 1\ nF$

$R = 100\ \Omega$

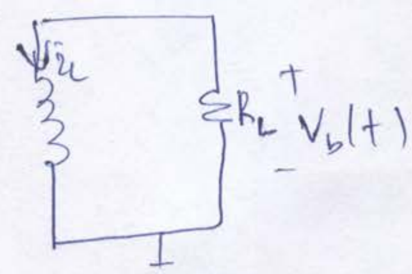
Switch 1 (S1) is closed (i.e. horizontal in the circuit above) for $t < 0$.
Switch 2 (S2) is open (i.e. raised and not connected) for $t < 0$.

S1 opens at $t = 0$.
S2 closes at $t = 1 \times 10^{-6}$ s

a) Provide an expression for $v_{ab}(t)$ for $0 < t < 1 \times 10^{-6}$ s. Note that $v_{ab}(t)$ is the potential difference between node a and node b.

Solution: $10V_1 e^{-t/1\mu s}$

$V_a = 0$;



$i_L(t=0) = V_1/R$.

$i_L(t=\infty) = 0$

$\therefore i_L(t) = \frac{V_1}{R} e^{-tR_L/L}$

$\therefore V_b(t) = -\frac{R_L}{R} V_1 e^{-t/1\mu s} = -10V_1 e^{-t/1\mu s}$

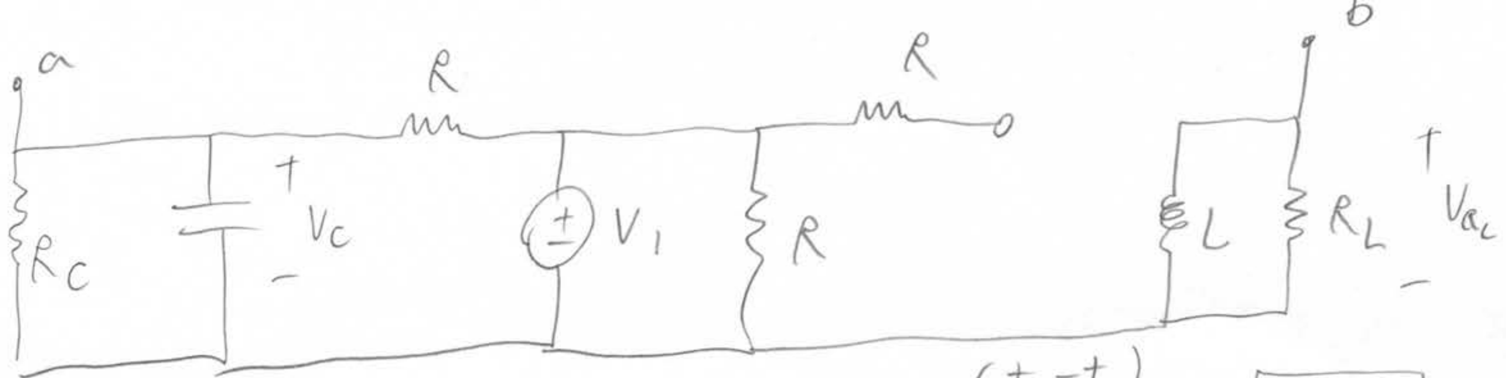
$V_{ab}(t) = 10V_1 e^{-t/1\mu s}$

Prob 3b) [15 pts total]

At $t = 1 \times 10^{-6}$ s.

$$V_C(t = 1\times 10^{-6}) = 0\ V \quad \boxed{2\text{ pts}}$$

$$V_C(t = \infty) = \frac{R_C}{R + R_C} \cdot V_1 \quad \boxed{2\text{ pts}}$$



$$V_C(t) = V_f + [V_0 - V_f]\, e^{-\frac{(t - t_1)}{\tau}} \quad \boxed{3\text{ pts}}$$

$$\tau = R_{eq} \cdot C = (R_C \parallel R) \cdot C \quad \boxed{2\text{ pts}}$$

$$V_{ab}(t) = V_C(t) - V_{R_L}(t) \leftarrow \boxed{1\text{ pt}}$$

$$= \left(\frac{R_C}{R + R_C}\right) V_1 \left(1 - e^{-\left(\frac{t - 10^{-6}}{(R_C \parallel R)\cdot C}\right)}\right) + \frac{R_L}{R} V_1 e^{-\frac{t}{10^{-6}}}$$

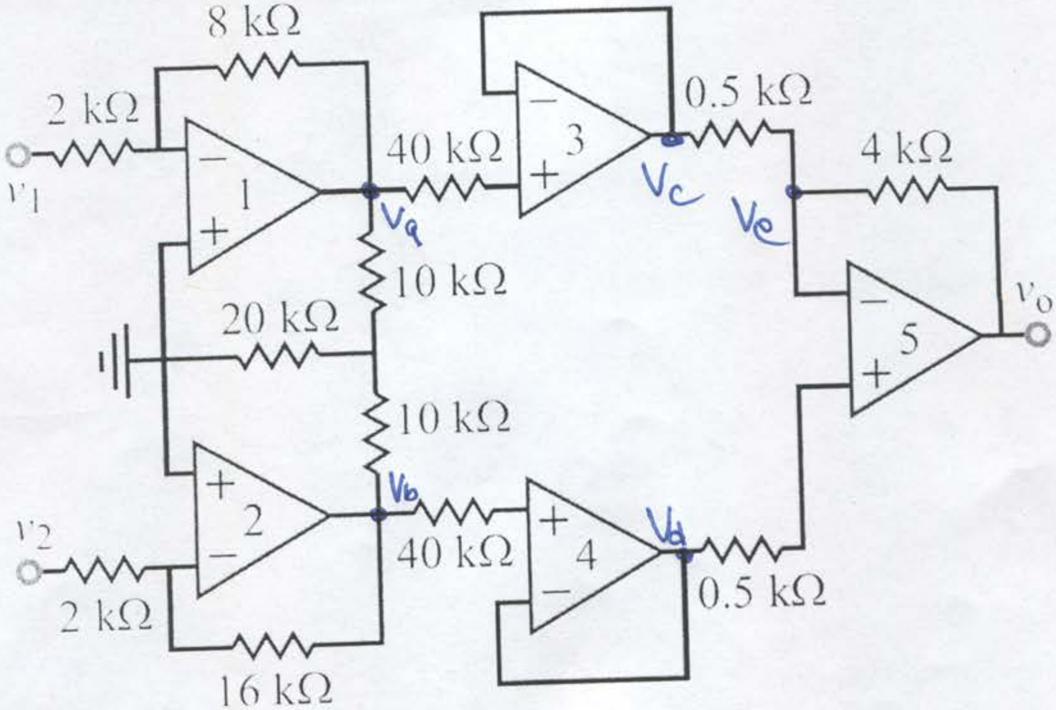
$$= \underbrace{\frac{10}{11} V_1 \left(1 - e^{-1.1\times 10^{-7}(t - 10^{-6})}\right)}_{\boxed{3\text{ pts}}} + \underbrace{10 V_1 e^{-\frac{t}{10^{-6}}}\ [V]}_{\boxed{2\text{ pts}}}$$

Extra Space

Nigel Tufnel: "The numbers all go to eleven. Look, right across the board, 11, 11, 11 and..."
Marty DiBergi: "Oh, I see. And most amps go up to 10?"
Nigel Tufnel: "Exactly."

This is Spinal Tap, 1984

Problem 4 (25 points)
Consider the circuit below.



Provide an expression for vo.

Solution: $32V_1 - 72\ V_2$

note that assuming ideal op-amps,

$V_a = -4V_1$; $V_b = -8V_2$.

$V_c = V_a = -4V_1$; $V_d = V_b = -8V_2$

$V_e = V_d = -8V_2$

KCL @ V_e : $\frac{V_o}{4} - \frac{V_e}{4} = \frac{V_e}{.5} - \frac{V_c}{.5}$

$\frac{V_o}{4} = -2V_2 - 16V_2 + 8V_1$

$\boxed{V_{out} = 32V_1 - 72V_2}$

Extra Space