**PHYSICS 7B, Lecture 2 – Spring 2014**
**Midterm 2, C. Bordel**
**7pm-9pm, Tuesday, April 8, 2014**

**Make sure you show all your work and justify your answers.**

***<u>Problem 1</u> – Electrical conduction (20 pts)***
*Remember: always write your symbolic solution before the numerical solution.*

A current $I$ is passed through a cylindrical Ag wire of cross-sectional area $A$ and resistivity $\rho$.
**a) Estimate the magnitude of the electric field $E$ in the wire.**

Assume Ag has a mass density $d$, an atomic mass $m_{Ag}$, and 1 free electron per atom.
**b) Estimate the magnitude of the drift velocity, $v_d$, of the conduction electrons.**

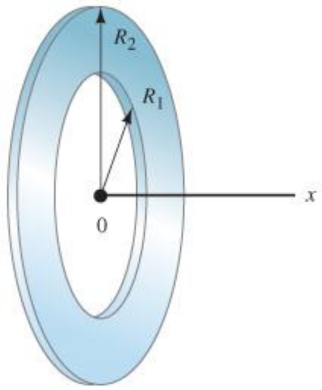
Assume that the free electrons, of mass $m_e$, can be treated as an ideal gas.
**c) Estimate the rms speed of the conduction electrons at room temperature.**
**d) Comment on the many orders of magnitude difference between $v_d$ and $v_{rms}$.**

I = 5 A ; A = $10^{-6}$ m$^2$ ; ρ= $3\times10^{-8}$ Ωm ; e= $2\times10^{-19}$ C ; d=$10^4$ kg/m$^3$ ;
$m_{Ag}$ = $2\times10^{-25}$ kg/at. ; $k_B$ = $10^{-23}$ J/K ; $m_e$=$10^{-30}$ kg.

***<u>Problem 2</u> – Electric field (20 pts)***

A flat ring of inner radius $R_1$ and outer radius $R_2$ carries a non-uniform surface charge density $\sigma(r) = \dfrac{\beta}{r}$, where $\beta$ is a positive constant, and $r$ is the radial distance measured from the center of the ring.

**Calculate the electric field produced by such a charge distribution at any point on the symmetry axis.**



***Problem 3* – *Electric field (20 pts)***

A solid insulating cylinder of infinite length and radius $R$ carries a non-uniform volume charge density $\rho(r) = \frac{\alpha}{r} e^{-\frac{r}{a}}$, where $\alpha$ and $a$ are both positive constants, and $r$ is the radial distance measured from the symmetry axis of the cylinder.

**Calculate the electric field produced at any point by the charge distribution and draw the electric field vector at 2 different points of your choice.**

***Problem 4* – *Electric potential (20 pts)***

A solid metallic sphere of radius $R_1$ and carrying some charge $Q$ is in electrostatic equilibrium.

a) **Calculate the electric field and electric potential at any point, using V(∞)=0 as the reference.**
b) **Draw the electric field lines and equipotential lines around the sphere.**

A second metallic sphere of radius $R_2$, carrying no net charge, is placed far from the first one.

c) **Calculate the electric charge of each sphere if they are electrically connected to each other by a very thin wire.**
d) **Draw the field lines corresponding to the new charge distribution.**


***Problem 5* – *Capacitor (20 pts)***

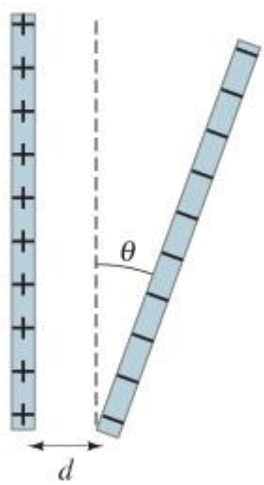
Let's consider a parallel-plate capacitor, whose plates carry some charge $+Q$ and $-Q$, have a surface area $L^2$, and are separated by a constant distance $d$, with $d \ll L$.

a) **Calculate the electric field created between the 2 charged plates.**

Now one of the 2 plates is slightly tilted by a small angle $\theta$ ($\theta \ll d/L$).

b) **Determine the capacitance $C$ and the potential energy $U$ stored by this capacitor.**

*Hint: you may consider this capacitor as a combination of infinitesimal capacitors.*



$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\vec{p} = Q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\Delta U = Q\Delta V$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{dQ}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$Q = CV$$

$$C_{eq} = C_1 + C_2 \text{ (In parallel)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \text{ (In series)}$$

$$\epsilon = \kappa\epsilon_0$$

$$U = \frac{Q^2}{2C}$$

$$U = \int \frac{\epsilon_0}{2}|\vec{E}|^2 dV$$

$$I = \frac{dQ}{dt}$$

$$\Delta V = IR$$

$$R = \rho\frac{l}{A}$$

$$\rho(T) = \rho(T_0)(1 + \alpha(T - T_0))$$

$$P = IV$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = nQ\vec{v_d} = \frac{\vec{E}}{\rho}$$

$$R_{eq} = R_1 + R_2 \text{ (In series)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ (In parallel)}$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha - 1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z} \\ &\text{(Cylindrical Coordinates)} \end{aligned}$$

$$\begin{aligned} \vec{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial \phi}\hat{\phi} \\ d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \\ &\text{(Spherical Coordinates)} \end{aligned}$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}}\sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$