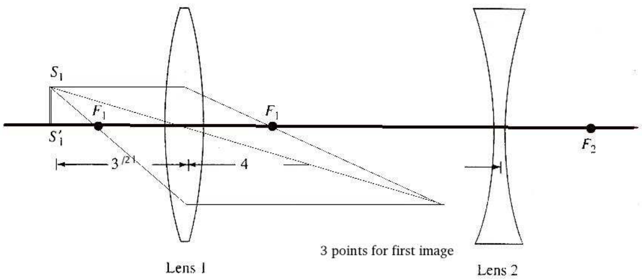
Professor M. Aganagic's Midterm 1: Tuesday, October 4 2005

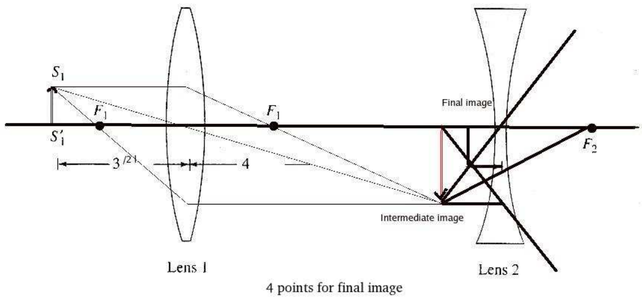
1 Problem 1

(15 pts) A converging lens with a focal length of $f_1 = l$ and a diverging lens of focal length $f_2 = -l$ are placed $4l$ apart. An object of height $l/10$ sits $3l/2$ in front of the converging lens.

i) [7 pts] Using ray tracing find the position and height of the image (approximately).

ii) [8 pts] Calculate the image position and height, and compare to your results in (i).





ii) Use the lens equation with a real object and a converging lens to find the initial image, which will serve as the object (caution! pay attention to the separation of the lenses) for the second lens.

$$\begin{aligned} \frac{1}{o_1} + \frac{1}{i_1} &= \frac{1}{f_1} \\ \frac{1}{i_1} &= \frac{1}{l} - \frac{2}{3l} \\ i_1 &= 3l \end{aligned}$$

2pts

This is a real image, 3l to the right of the first lens, and therefore $4l - 3l = l$ to the left of the second lens. It is a real object.

$$\begin{aligned} \frac{1}{o_2} + \frac{1}{i_2} &= \frac{1}{f_2} \\ \frac{1}{i_2} &= \frac{1}{-l} - \frac{1}{l} \\ i_1 &= -l/2 \end{aligned}$$

2 pts

The total magnification is just the product of the magnification due to each lens.

$$\begin{aligned} M &= m_1 \cdot m_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) \\ &= \frac{3l}{3l/2} \cdot \frac{-l/2}{l} \\ h_i &= Mh_o = -h_o = -l/10 \end{aligned}$$

Partial credit giving for carryingthrough correctly on later portions even if the first result was incorrect. Sign errors in the magnification -1pt.

2pts for getting m_1

2pts for getting h_final

2 Problem 2

(20 pts) A small mirror of area A faces a monochromatic light source at a distance $L >> A^{1/2}$.

3 pts In a time T, how much energy is incident on the mirror if the amplitude of the incident electric field at the mirror is measured E_0 ?

2 pts What is the power output of the source if it radiates uniformly in all directions? (Power output is energy emitted per unit time.)

3 pts If the incident electric field is $\vec{E}_{in} = E_0 sin(kx - \omega t)\hat{y}$, what is the incident magnetic field?

3 pts What is the $\vec{E}_{ref}(x,t) = E_{ref}(x,t)\hat{y}$? (You know this!)

3 pts Could we have had $\vec{E}_{in} = E_0 sin(kx - \omega t)\hat{x}$? Explain.

2 pts What about $\vec{E}_{in} = E_0 sin(kx^2 - \omega t)\hat{y}$? Explain.

4 pts How would the expression for $\vec{E}_{in}(x,t)$ need to be modified if the mirror were immersed in water? (This is the only modification in the setup, in particular, the source is unmodified).

a) $S = \sqrt{\frac{\epsilon_0}{\mu_0}} \langle E^2 \rangle = \sqrt{\frac{\epsilon_0}{\mu_0}} \left\langle \frac{E_0^2}{2} \right\rangle = \frac{Power}{Area}$

$\rightarrow energy = STA = \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{E_0^2}{2} TA$

b) Power at source = (Power at mirror) $\left[\frac{4\pi L^2}{A}\right] = \left(\sqrt{\frac{\epsilon_0}{\mu_0}} \frac{E_0^2}{2} A\right) \left[\frac{4\pi L^2}{A}\right] = 2\pi L^2 E_0^2 \sqrt{\frac{\epsilon_0}{\mu_0}}$

c) $\vec{B} = \frac{1}{c} E_0 \sin(kx - \omega t)\hat{z} \quad (\hat{x} = \hat{y} \times \hat{z})$

d) The reflected wave changes directions: $\sin(kx - \omega t) \rightarrow \sin(kx + \omega t)$ and undergoes a π phase shift, while retaining its former polarization, so

$$\vec{E}_{ref} = -E_0 \sin(kx + \omega t)\hat{y}$$

e) This "electric field" would point in the direction of propagation, which does not satisfy Maxwell's equations, so it is not a valid wave function.

f) This function does not satisfy the wave equation, so it is not a valid solution.

g) In the function $\sin(kx - \omega t)$, the periodicity is λ, so k can be expressed as $k = \frac{2\pi}{\lambda}$. Immersing the setup in water changes $\lambda \rightarrow \lambda / n_{water}$. So the new incident wave is

$$\vec{E}_{in}(x,t) = E_0 \sin(nkx - \omega t)\hat{y}$$

3 Problem 3

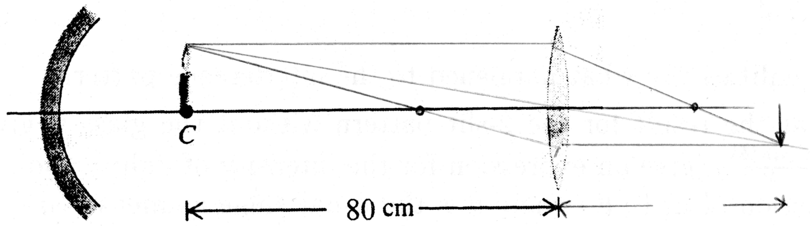
(15 pts) A candle is at the center of curvature of a concave mirror, whose focal length is $10cm$. There is a converging lens of focal length $30cm$ placed $80cm$ to the right of the candle. The lens forms two images of the candle. The first is formed from the light passing directly through the lens. The second is formed from the light that goes from the candle to the mirror, is reflected, and then passes through the lens.

i) 5 pts For each of the two images draw a principal ray diagram that locates the image.

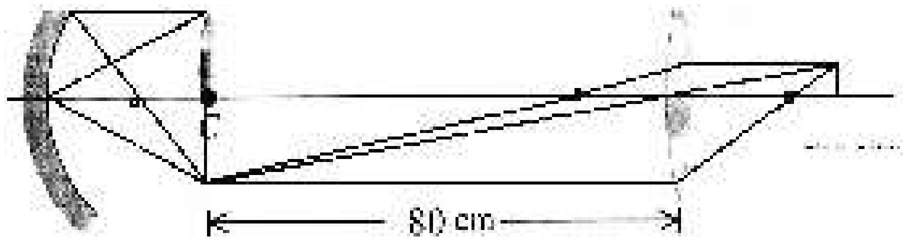
ii) 5 pts For each image answer:

- Where is the image?
- Is it real or virtual?
- Is it erect or inverted with respect to original?

3 pts for calculating location, 1 pt for only saying e.g., 'to the right of lens 2'



Generally, points are given for the second image if it is consistent with the first. The diagram should show relevant features such as optic axis and focal points, and the standard rays used. *2 points for each image.*



For the ray that goes directly through the lens, use the lens equation, with $f_1 = +30cm$ (converging), $o_1 = +80cm$ (real):

$$\begin{aligned} \frac{1}{o_1} + \frac{1}{i_1} &= \frac{1}{f_1} \\ \frac{1}{i_1} &= \frac{1}{+30cm} - \frac{1}{+80cm} = \frac{1}{+48cm} \end{aligned}$$

So $i_1 = 48cm$ to the right of lens 1. This is a real image, and inverted as: *2 points*

$$m = -\frac{i}{o} = -\frac{48cm}{80cm} = -0.6$$

For the ray that is reflected from the mirror first, check the image formed by the mirror. Recalling that the focal length of a spherical mirror is half the radius of curvature, and that the focal length is considered positive for a concave mirror,

$$\frac{1}{o_2} + \frac{1}{i_2} = \frac{1}{f_2}$$
$$\frac{1}{i_2} = \frac{1}{10cm} - \frac{1}{20cm} = \frac{1}{+20cm}$$

3 points

So the mirror's image is real, 20cm to the right of the mirror(directly below the candle), and 80cm to the left of the lens. It is inverted, as

$$m = -\frac{10cm}{10cm} = -1$$

Due to symmetry, it is easy to see that the image will be formed 48cm to the right of the lens, and inverted from the mirror's image, therefore upright. The total magnification is found by applying the magnification formula twice:

$$M = m_1 \cdot m_2 = -1 \cdot -0.6 = +0.6$$

2 points

And the image is real and upright, as shown in the diagram.

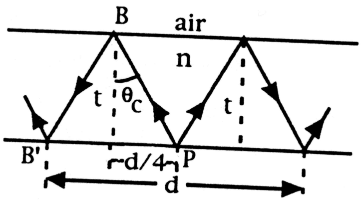
4 Problem 4

The phenomena of total internal reflection can be used to measure the index of refraction of a material via Pfund's method, as follows. A slab of thickness t is painted on one side to serve as a screen. A small hole scraped in the paint serves as a source of light. Rays striking the opposite surface will emerge if the angle is less than critical. Thus on the painted screen there will be a dark circle of diameter d, and outside of this a bright halo.

a) Derive a formula for n in terms of d and t.

b) What is the diameter of the dark circle if $n = 1.52$ and $t = 0.600 cm$?

c) If white lights is used, the critical angle depends on color caused by dispersion. Is the inner edge of the halo tinged red or violet? Explain.



a) The dark circle is formed because light hitting the clear surface will escape if the incident angle is less than θ_c. The condition for total internal reflection is: *4 points*

$$
\begin{aligned}
n_{air} sin(\pi/2) &= n_{material} sin(\theta_c) \\
\text{or } sin\theta_c &= \frac{1}{n_m}
\end{aligned}
$$

From the diagram, $tan\theta_c = \frac{d/4}{t}$ or: *4 pts*

$$sin\theta_c = \frac{d/4}{\sqrt{t^2 + (d/4)^2}}$$

Then *2 pts*

$$\boxed{n = \frac{\sqrt{(4t)^2+d^2}}{d}}$$

b) Rearranging gives: *2 pts given if consistent with part a*

$$
\begin{aligned}
dn &= \sqrt{(4t)^2 + d^2} \\
d^2n^2 &= 16t^2 + d^2 \\
d^2(n^2 - 1) &= 16t^2
\end{aligned}
$$

$$d = \frac{4t}{\sqrt{n^2 - 1}} = \frac{4 \cdot 0.600cm}{\sqrt{1.52^2 - 1}} = 2.10cm$$

c) See Giancoli page 825 for description of prisms and dispersion. Light of shorter wavelengths is refracted more strongly than light of longer wavelenth, therefore θ_c is smaller for violet, and closer to $\pi/2$ for red. A smaller θ_c results in a smaller ring and violet will be on the inside. *3 pts*

5 Problem 5

Light falls normally on a soap bubble and is reflected back. If the bubble's walls have a thickness t and index of refraction n, express the condition for constructive interference of the reflected light in terms of the incident wavelength λ, n, t. If $t = 400nm$ and $n = 1.3$, what color or colors will interfere constructively?

a) The interference pattern arises due to a phase difference between the light reflected from the front and back surfaces of the bubble. If the phase difference is $(2m-1)\pi$ for some integer m, then there is destructive interference (no light seen), while if the phase difference is $2m\pi$ there will be a bright spot.

The ray directly reflected picks up a phase $\phi_1 = \pi$ because the index of refraction of soap is higher than that of air.

The ray reflecting off the back surface picks up a phase due only to the extra path length traveled. Fractions of a wavelength traveled will give fractions of 2π as :

$$\frac{\phi_2}{2\pi} = \frac{2t}{\lambda_n} = \frac{2tn}{\lambda}$$

Thus, the condition for constructive interference is:

$$\Delta\phi = 2\pi\frac{2tn}{\lambda} - \pi = 2m\pi$$

$$\boxed{4tn = (2m-1)\lambda, \quad m = 1, 2, 3, \ldots}$$

12 pts

b) To find the colors appearing, we invert the expression above to isolate λ:

$$\lambda = \frac{4nt}{2m-1} = \frac{4(1.3)(400nm)}{2m-1} \approx \frac{2100nm}{2m-1}$$

For the values m = 1 through 4, the wavelengths obtained are:

$$\lambda = 2100nm, 700nm, 420nm, 300nm$$

Only red (700 nm) and blue (420 nm) are part of the visible spectrum. *3 pts*

6 Problem 6

In a 2-slit experiment a piece of glass with an index of refraction n and thickness L is placed in front of the upper slit.

a) Describe qualitatively what happened to the interference pattern.

b) Given that the result for the 2-slit pattern without the glass is given by: $I_\theta = I_0 cos^2(\frac{\pi d sin(\theta)}{\lambda})$, (where d is the distance between the slits and θ is the usual angle as measured from the center), give an expression for the intensity of light at points on a screen as a function of n, L, θ.

c) Write down an expression for values of θ that locate the interference maxima. Compare to results of part b.

a) The addition of the glass causes one of the rays to pick up an extra (constant) phase, causing the whole pattern to shift *4 pts*

without changing the separation between successive maxima . *-1 pt if stretched*

Because the glass is in front of the *upper* slit, the pattern will shift upwards. *1pt*

b) The usual phase is

$$\delta_{old} = \frac{2\pi}{\lambda} d \sin\theta$$

The new phase is simply

$$\delta_{new} = \delta_{old} + (n-1)L\frac{2\pi}{\lambda}$$

4 pts

The intensity pattern will then be:

$$I(\theta) = I_0 \cos^2\left(\frac{\delta_{new}}{2}\right) \text{ or } I_0 \cos^2\left(\frac{\delta + \Delta\phi_{new}}{2}\right)$$

4 pts

$$\boxed{I(\theta) = I_0 \cos^2\left(\frac{\pi}{\lambda}(d\sin\theta + (n-1)L)\right)}$$

partial credit for missing -L

c) Condition for constructive interference is :

$$\delta_{new} = 2\pi m, \quad m = 0, 1, 2, \ldots$$

4 pts

$$\frac{d\sin\theta}{\lambda} \pm (n-1)\frac{L}{\lambda} = 2\pi m$$

2 pts

Solving for $\sin\theta$ *1 pt*

$$\sin\theta = \frac{\lambda m \mp (n-1)L}{d}$$