Solutions to Midterm 1

Problem 1.

(a)

$$a_A = \frac{8}{3.6} = 2.22\ \text{m/s}^2 \qquad \downarrow$$

$$(a_B)_t = -\frac{8}{3.6} = -2.22\ \text{m/s}^2 \qquad \uparrow$$

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(100/3.6)^2}{300} = 2.57\ \text{m/s}^2 \qquad \leftarrow$$

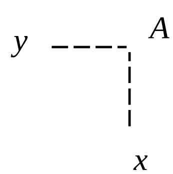
Attach a translating (x, y) frame to car A with the x-axis directed along v_A. Thus

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\Rightarrow \quad -2.22\mathbf{i} + 2.57\mathbf{j} = 2.22\mathbf{i} + \mathbf{a}_{B/A}$$

$$\Rightarrow \quad \mathbf{a}_{B/A} = -4.44\mathbf{i} + 2.57\mathbf{j}$$

$$\Rightarrow \quad a_{B/A} = 5.13\ \text{m/s}^2$$



(b) A coordinate system attached to B (with the x-axis in the direction of v_B) is a rotating system. It can be shown from rigid-body kinematics that the acceleration $\mathbf{a}_{\text{rel}}$ of car A as observed from car B is

$$\mathbf{a}_{\text{rel}} = \mathbf{a}_A - \mathbf{a}_B - \dot{\boldsymbol{\omega}} \times \mathbf{r}_{A/B} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) - 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}}$$

$$\Rightarrow \quad \mathbf{a}_{\text{rel}} \neq \mathbf{a}_A - \mathbf{a}_B = -\mathbf{a}_{B/A}$$

where $\mathbf{v}_{\text{rel}}$ is the velocity of car A as observed from car B.

Problem 2.

Sum of forces acting on the entire system:

$$\Sigma F_x = -P + 2\mu_k mg = 2m\ddot{x}$$

$$\ddot{x} = \mu_k g - \frac{P}{2m}$$

Sum of forces acting on block A (note that the critical static friction is known):

$$\Sigma F_y = -mg + N_A \cos\theta - \mu_s N_A \sin\theta = 0$$

$$\Sigma F_x = -N_A \sin\theta - \mu_s N_A \cos\theta = m\ddot{x} = \mu_k mg - \frac{P}{2}$$

Solve for N_A

$$N_A = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Solve for P

$$P = 2mg\left(\mu_k + \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = 125.44 \text{ N}$$

Problem 3.

1. $\boldsymbol{G_{A,+}} - \boldsymbol{G_{A,-}} = \boldsymbol{I_A}$; calling $r_A^2 \coloneqq L^2 + y_A^2$, we have

$$m_A v_A\left\{\left(\frac{L}{r_A}\right)\boldsymbol{E_X} + \left(\frac{y_A}{r_A}\right)\boldsymbol{E_Y}\right\} - m_A v_i \boldsymbol{E_X} = \boldsymbol{I_A}$$

$$\cdot\,\boldsymbol{E_x} \to m_A\left(v_A = \frac{r_A}{\Delta t_A}\right)\left(\frac{L}{r_A}\right) - m_A v_i = I_{A,x}$$

$$\to I_{A,x} = m_A\left(\frac{L}{\Delta t_A} - v_i\right) = 4\left(\frac{10}{3} - 4\right) = -2.667 \text{ N}$$

$$\cdot\,\boldsymbol{E_y} \to m_A\left(\frac{r_A}{\Delta t_A}\right)\left(\frac{y_A}{r_A}\right) = I_{A,y}$$

$$\to I_{A,y} = m_A\left(\frac{y_A}{\Delta t_A}\right) = 4\left(\frac{7.5}{3}\right) = 10 \text{ N}$$

so $\boldsymbol{I_A} = -2.667\boldsymbol{E_X} + 10\boldsymbol{E_y}$ N

2. $\boldsymbol{G_{B,+}} - \boldsymbol{G_{B,-}} = \boldsymbol{I_B}$; calling $r_B^2 \coloneqq L^2 + y_B^2$, and of course recognizing that $\boldsymbol{I_A}$ and $\boldsymbol{I_B}$ are equal and opposite, we have

$$\boldsymbol{G_{B,+}} - \boldsymbol{G_{B,-}} = -\boldsymbol{I_A} \to m_B v_B\left\{\left(\frac{L}{r_B}\right)\boldsymbol{E_X} - \left(\frac{y_B}{r_B}\right)\boldsymbol{E_Y}\right\} - m_B v_i \boldsymbol{E_X} = -\boldsymbol{I_A}$$

$$\cdot\,\boldsymbol{E_x} \to m_B\left(\frac{r_B}{\Delta t_B}\right)\left(\frac{L}{r_B}\right) - m_B v_i = -I_{A,x}$$

$$\to \Delta t_B = L\left(v_i - \frac{I_{A,x}}{m_B}\right)^{-1} = 10\left(4 - \frac{-2.667}{1}\right)^{-1} = 1.5 \text{ s}$$

3. Once Δt_B determined, fairly simple to find y_B

$$\cdot\,\boldsymbol{E_x} \to -m_B\left(\frac{r_B}{\Delta t_B}\right)\left(\frac{y_B}{r_B}\right) = -I_{A,y}$$

$$\to y_B = \frac{I_{A,y}\Delta t_B}{m_B} = \frac{(10)(1.5)}{1} = 15 \text{ m}$$