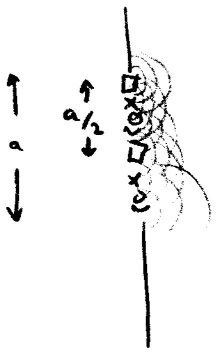
Phys 7c - Spring 2004 - Sect 1 (Lee) - MT 2 Solutions

1a)

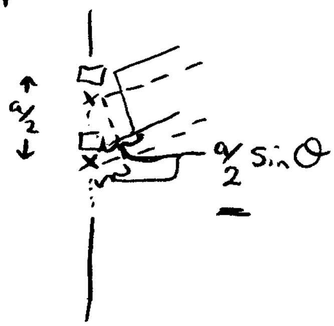


Huygen's principle - each part of the slit can be treated as a mini-radiator.

We can go down the slit adding the contribution of a Huygen's wavelet with that $a/2$ away - that way when we get to the middle, we will have taken care of all wavelets.

I.e. add E-fields from □s, ×s, Os, ~s, and we're done!

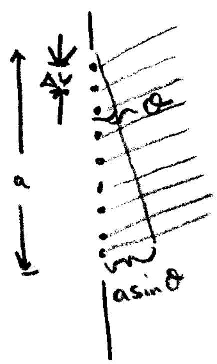
The point: For 1st order minimum



$$\frac{a}{2}\sin\theta_{min,1} = \frac{\lambda}{2}$$

$$a\sin\theta_{min,1} = \lambda$$

Long way:

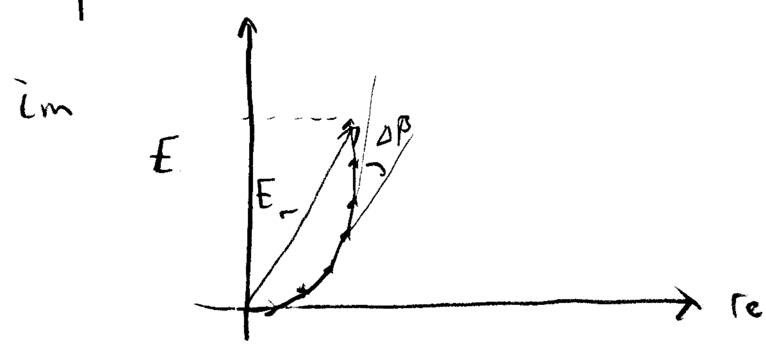


Treat as N slits.

Let phase shift from top slit be 0

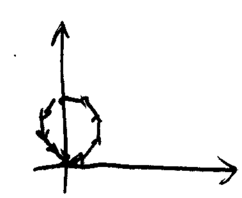
phase shift to next slit: $\Delta\beta = \frac{2\pi}{\lambda}\Delta y\sin\theta$

Thus phasor diagram looks like this:



The actual value of E is that of the resultant E-field phasor E_r projected onto Im-axis.

1st order min @

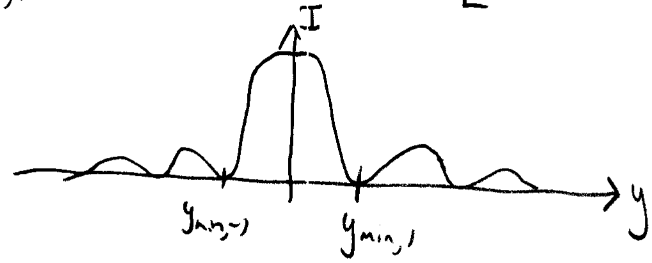


This is when $N\Delta\beta = 2\pi = N\left(\frac{2\pi}{\lambda}\Delta y \sin\theta_{min,1}\right) = \frac{2\pi}{\lambda}\underbrace{(N\Delta y)}_{a}\sin\theta_{min,1}$

$$2\pi = \frac{2\pi}{\lambda} a \sin\theta_{min,1}$$

$$\boxed{a \sin\theta_{min,1} = \lambda}$$

b) $a \sin\theta_{min,1} = \lambda \simeq a\theta_{min,1} \simeq a\frac{y_{min,1}}{L} \Rightarrow a \simeq \frac{\lambda L}{y_{min,1}}$



The question says the dist. between 1st order minima is 5mm. That's the value $2y_{min,1}$. Since otherwise this is a trivial problem, I have to take off 2 pts for using $5\,mm = y_{min,1}$ instead of $5\,mm = 2y_{min,1}$. Sorry!

$$2y_{min,1} = 5.0\,mm \Rightarrow y_{min,1} = 2.5\,mm$$

$$a = \frac{(589.3\times10^{-9}\,m)(50\times10^{-2}\,m)}{2.5\times10^{-3}\,m} = 1.2\times10^{-4}\,m = \boxed{120\,\mu m = a}$$

2 Setup. Let us put the origin of both ~~...~~ frames at the light flashing.

in other words, the light flashes at $x=0, t=0$ in S, and at $x'=0$, $t'=0$ in S'.

Thus, A' is at $x' = -\frac{L}{2}$ in S', while B' is at $x' = \frac{L}{2}$ in S'.

Let event ① be the light pulse reaching A', event ② light reaching B'.

(a) In S', the distance is L.

In S, it is length contracted, so $\frac{L}{\gamma} = \frac{4}{5} 100\, c{*}\text{min}$

(as $\gamma = 5/4$). so $\boxed{\text{length in } S = 80\, c{*}\text{min}}$

(b) The events ① and ② (arrival of light at A' and B') must occur simultaneously in S', as A' and B' are equidistant from the light source and at rest in S'.

so yes, the events are simultaneous.

(c) we know the events cannot be simultaneous in S, let's show this with Lorentz transformation.

Event ① occurs at $x' = -\frac{L}{2}$, $t' = \frac{L}{2c}$.

The Lorentz transf. we need is

$$t = \gamma\left(t' + \frac{v x'}{c^2}\right)$$

(as we are changing from S' to S, S moves at $-v$ relative to S').

so, $t_{①} = \gamma\left(\frac{L}{2c} - \frac{vL}{2c^2}\right)$.

Event ② is at $x' = \frac{L}{2}$, $t' = \frac{L}{2c}$.

$t_{②} = \gamma\left(\frac{L}{2c} + \frac{vL}{2c^2}\right)$.

so, $\Delta t = t_{②} - t_{①} = \frac{\gamma v L}{c^2} = \frac{5}{4} \cdot \frac{3}{5} \cdot 100\, {*}\text{min} = 75$ min

(as $v = 3/5$, $\gamma = 5/4$, $L = 100\, c{*}\text{min}$).

$\boxed{\Delta t = 75 \text{ min}}$

(d) at time $t=0$ in the S frame,

A is at $x = -\frac{L}{2\gamma}$
B is at $x = \frac{L}{2\gamma}$ } lorentz length contraction!

the light is at $x=0$.

so, A is $\frac{L}{2\gamma}$ from the light. the light travels at c, A travels at v, towards each other. so they cover the distance at rate $c+v$.

so, $\frac{L}{2\gamma} = (c+v)\, t_{①}$.

B is $\frac{L}{2\gamma}$ from the light, but B and the light go the same direction, (in other words, B heads away from the light's source), so they cover the gap at $c-v$.

$(c-v)t_{②} = \frac{L}{2\gamma}$.

so, $\Delta t = t_{②} - t_{①} = \frac{L}{2\gamma}\left(\frac{1}{c-v} - \frac{1}{c+v}\right)$

$= \frac{L}{2\gamma}\left(\frac{c+v-c+v}{(c-v)(c+v)}\right) = \frac{2vL}{2\gamma(c^2-v^2)} = \frac{vL}{c^2\gamma}\left(\frac{1}{1-v^2/c^2}\right)$

$\Delta t = \frac{vL}{c^2\gamma}\gamma^2 = \frac{vL\gamma}{c^2} = 75\text{ min}$ just as in (c).

so, $\boxed{\Delta t = 75\text{ min}}$.

(e) The time will be time dilated so

$$\gamma t' = t$$
$$\text{or } t' = \frac{t}{\gamma}$$

so, the time length S sees elapse on S' clocks

is $\frac{vL\gamma}{c^2\gamma} = \frac{vL}{c^2} = \boxed{60 \text{ min}}$

doesn't agree, until we include spatial separation. see next page.

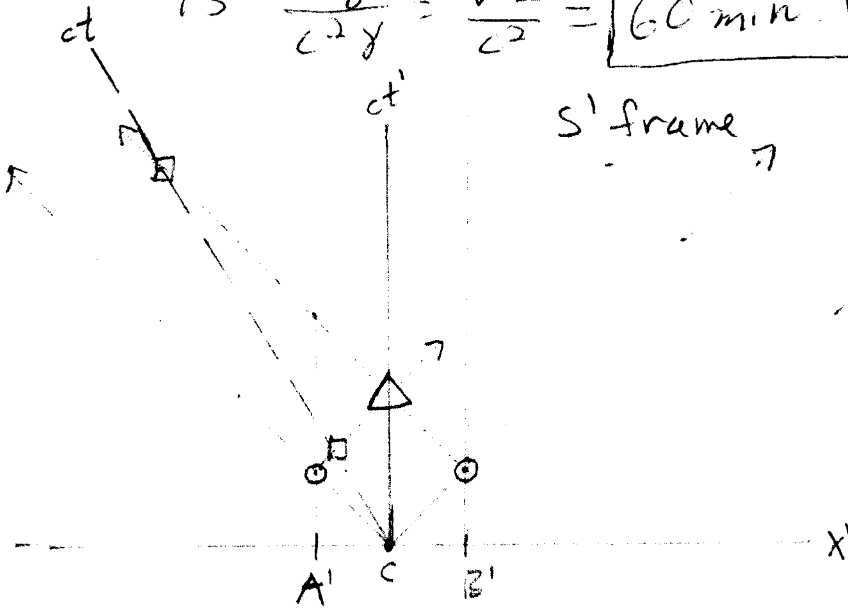


figure 1

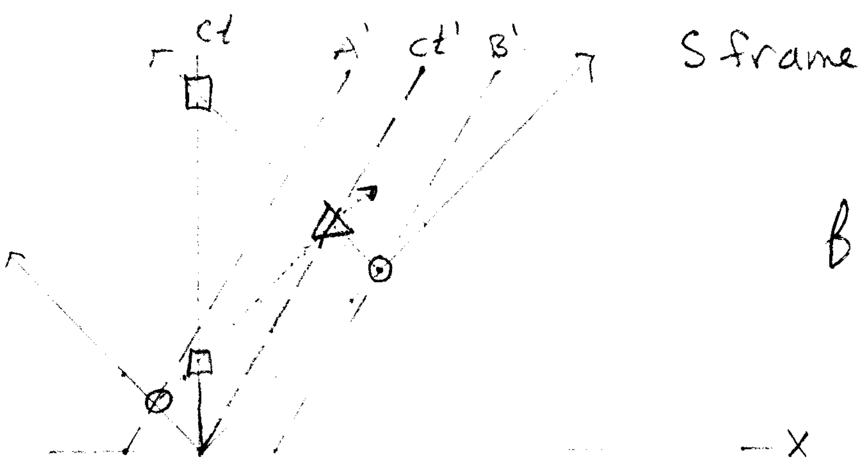


figure 2

Alternate interp:
Find $\Delta t'$ for the two events, taking into account the distance between them in S or S'.

So, $\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right)$

$\Delta x' = 100\ c\,\text{min}$, $\Delta t = 75\text{ min}$,

$v = .6c$, so,

$75\text{ min} = \frac{5}{4}\left(\Delta t' + \frac{3}{5}100\right)$

$75\text{ min} = \frac{5}{4}\Delta t' + 75\text{ min}$

$\Rightarrow \Delta t' = 0$.

this agrees with S' events.

For explanation, see next page (eventually).

Note: I present this full explanation for your edification and amusement; a minor 1-sentence expl. is all I graded for on this part (e).

Grading Notes for Lee Midterm 2, # 2.

ECF means error carried forward, could still lose 1 or 2 pts.

(a) doing the Lorentz contraction backwards got 1pt.

(b) Giving an explanation in S, and saying so, got 2pts (you were supposed to do S').

(c)
- correct answer without subtracting: -2
- not plugging in: -1
- messing up length insertion: up to -5 (less than that for minor mistakes, i.e. 100, not 50).

(d)
- method as in solution, but length contraction not taken into account: -5 (or done incorrectly).
- Not using a new method from part c (either using Lorentz transforms or doppler formula, the second lost points w/o explanation of why it might work): 5 points possible (fewer given if mistakes)
- Not resolving disagreement with c: possible -2. (shouldn't be any).

(e)
- time dilation backwards: 1pt
- no expl of 60≠0: -1pt
- Δx is the difference between where events occur, so it isn't 80; 2pts possible.
- alternate interpretation: giving 0 without calculation: 2 pts.

3.

a)

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$$

$$\theta = \pi \Rightarrow \lambda_2 = \lambda_1 + \frac{2h}{mc}$$

$$= \frac{3h}{mc} = 3\lambda_c$$

$$\Rightarrow \Delta E = \frac{h}{\lambda_1} - \frac{h}{\lambda_2} = \frac{2}{3}mc^2 = \gamma mc^2 - mc^2$$

$$\Rightarrow \gamma = \frac{2}{3}. \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5}{3}$$

$$\Rightarrow V = 0.8c.$$

b)

$$f_1' = \sqrt{\frac{1-\beta}{1+\beta}}\, f_1$$

$$f_2' = \sqrt{\frac{1+\beta}{1-\beta}}\, f_2$$

$$\Rightarrow f_1' = f_2' \Rightarrow f_1\sqrt{\frac{1-\beta}{1+\beta}} = f_2\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow 3(1-\beta) = 1+\beta$$

$$\Rightarrow \beta = 0.5 \Rightarrow V = 0.5c.$$

because $f_1 > f_2$, the observer should move in the same direction with the first photon in order to get $f_1' = f_2'$.

c). the velocity of observer is $V_0 = 0.5c$.

and because electron is at rest in Lab ref-frame.

$$V_{ei}' = -V_0 = -0.5V.$$

$$V_{ef}' = \frac{V_{ef} - V_0}{1 - \frac{V_{ef} V_0}{c^2}} = \frac{0.8c - 0.5c}{1 - \frac{0.8c\, 0.5c}{c^2}} = 0.5c$$

d). initial: $\xrightarrow{\gamma,\ f}$ $\xleftarrow{e \quad -0.5c}$

final. $\xleftarrow{\gamma,\ f}$ $\xrightarrow{e \quad 0.5c}$

The frequency of photon does not change in the reaction nor does the speed of electron, ~~fr the Neither~~ ~~energy of photon nor~~ which means no energy is transfered to electron. The energy of the system is conserved.