Mathematics 54.2
Midterm 2, 20 March 2013
50 minutes, 50 points

NAME: ______________________________ **ID:** ______________________________

GSI: ____

INSTRUCTIONS:
You must justify your answers, except when instructed otherwise.
All the work for a question should be on the respective sheet.
This is a CLOSED BOOK examination, NO NOTES and NO CALCULATORS are allowed.
NO CELL PHONE or EARPHONE use is permitted.
Please turn in your finished examination to your GSI before leaving the room.

Q1	
Q2	
Q3	
Q4	
Q5	
Tot	
Ltr	

1. TRUE-FALSE Questions (18 points) $\forall$

Circle the correct answer. No justification needed.

Correct answers carry 1.5 points, wrong ones carry 1.5 points penalty.

You may leave any question blank.

You will not get a negative total on any group of six questions.

T F Eigenvectors for distinct eigenvalues of a real symmetric matrix are orthogonal

T F The determinant of any lower-triangular square matrix is the product of the diagonal entries

T F The eigenvalues of a non-singular matrix are all real

T F If A and B are square matrices and $\det A = 1, \det B = 2$, then $\det(A + B) = 3$

T F A change-of-coordinates matrix is always invertible

T F Any orthogonal matrix is diagonalizable over $\mathbf{R}$

T F The determinant of an orthogonal matrix is always ± 1

T F For an $m \times n$ matrix A, vectors in the column space of A are orthogonal to vectors in the left nullspace

T F If a real symmetric 4×4 matrix has exactly two eigenvalues, then one of the eigenspaces has dimension 2 or more

T F If A is an orthogonal matrix, then the linear map $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one and onto.

T F If every row of the square matrix A is a linear combination of the rows of the square matrix B, then $\det A = \det B$

T F If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals that from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u} \perp \mathbf{v}$

Question 2. (7 pts)
Show that, for a real matrix A with linearly independent columns, the matrix $A^T A$ is invertible.

Question 3. (10 pts, 7+3)
Write down the matrix P implementing the orthogonal projection from $\mathbf{R}^3$ onto the subspace defined by $x - y + z = 0$.
Find the eigenvalues of P, and dimensions of the eigenspaces.
Hint: The second question requires no calculation, if you think of what P does geometrically.

Question 4. (7 pts, 5+2)
Find a vector $\mathbf{v} \in \mathbf{R}^2$ such that the sequence $A^n\mathbf{v}$ has a non-zero limit as $n \to \infty$.

$$A = \begin{bmatrix} 7 & 4 \\ -9 & -5 \end{bmatrix}$$

Is A diagonalizable?

Question 5. (8 pts)
Find the linear function $y = \ell(x) = ax + b$ giving the best fit to the data points

$$(x, y) = (-1, 1); (0, 1); (1, 0); (2, 2),$$

in the sense of minimizing the sum of square errors $\sum(\ell(x) - y)^2$. Explain your steps.

THIS PAGE IS FOR ROUGH WORK (not graded)