Department of Physics, University of California, Berkeley

Mid-term Examination 2 Physics 7B, Section 2, Prof. Smoot

6:00 pm - 8:00 pm, 6 April 2004

Name:________________________________________

SID No:________________________________________

Discussion Section:______________________________

Name of TA:___________________________________

| | |
|---|---|
| Problem 1 | |
| Problem 2 | |
| Problem 3 | |
| Problem 4 | |
| Problem 5 | |
| Problem 6 | |
| Problem 7 | |
| Problem 8 | |

Score:________________________________________

Answer all problems. Write clearly and explain your work. Partial credit will be given for incomplete solutions provided your logic is reasonable and clear. Cross out any parts that you don't want to be graded. Enclose your answers with boxes. **Express all numerical answers in SI units**. Answers with no explanation or disconnected comments will not be credited. If you obtain an answer that is questionable, explain why you think it is wrong.

### Constants and Conversion factors

| | |
|---|---|
| Charge of electron $e$ | $1.6 \times 10^{-19}$ C |
| Avogadro number, $\mathrm{N_A}$ | $6.022 \times 10^{23}$ |
| Permittivity of vacuum, $\epsilon_0$ | $8.85 \times 10^{-12}$ F·m$^{-1}$ |
| Universal gas constant, R | $8.315$ J·mol$^{-1}$·K$^{-1}$ = 1.99 cal·mol$^{-1}$·K$^{-1}$ |
| Boltzmann constant, k | $1.381 \times 10^{-23}$ J·K$^{-1}$ |
| Stefan-Boltzmann constant, $\sigma$ | $5.67 \times 10^{-8}$ W·m$^{-2}$·K$^{-4}$ |
| Acceleration due to gravity, g | 9.8 m·s$^{-2}$ |
| Specific heat of water | 1 kcal·kg$^{-1}$·°C$^{-1}$ |
| Heat of fusion of water | 80 kcal·kg$^{-1}$ |
| 1 atm | $1.013 \times 10^{5}$ N·m$^{-2}$ |
| 1 kcal | $4.18 \times 10^{3}$ J |
| 1 hp | 746 W |
| 1 liter | $10^3$ cm$^3$ |

### Some useful equations

$$
\begin{aligned}
\textit{Coulomb's law}: \ \mathbf{F} &= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}} \\
\textit{Electric field}: \ d\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \\
\textit{Electric dipole}: \ \mathbf{p} &= q\mathbf{d} \\
\textit{Torque on a dipole}: \ \vec{\tau} &= \mathbf{p}\times\mathbf{E} \\
\textit{Potential energy of a dipole}: \ U &= -\mathbf{p}\cdot\mathbf{E} \\
\textit{Gauss's law}: \ \oint \mathbf{E}\cdot d\mathbf{A} &= \frac{q_{encl}}{\epsilon_0} \\
\textit{Potential difference}: \ V_{ab} &= -\int_a^b \mathbf{E}\cdot d\mathbf{l} \\
\textit{Potential}: \ dV &= \frac{1}{4\pi\epsilon_0}\frac{dq}{r} \\
\textit{Potential energy}: \ U_{ab} &= qV_{ab} \\
\textit{Electric field and potential}: \ \mathbf{E} &= -\nabla V \\
\textit{Capacitance}: \ C &= \frac{q}{V_{ab}}
\end{aligned}
$$

$$
\begin{aligned}
Capacitors\ in\ series:\ \frac{1}{C_{eq}} &= \sum \frac{1}{C_i} \\
Capacitors\ in\ parallel;\ C_{eq} &= \sum C_i \\
Energy\ stored\ in\ a\ capacitor:\ U &= \frac{1}{2}CV^2 \\
Energy\ density:\ u &= \frac{1}{2}\epsilon_0 E^2 \\
Current:\ I &= \frac{dq}{dt} \\
Current\ and\ current\ density:\ dI &= \mathbf{j}\cdot d\mathbf{A} \\
Ohm's\ law:\ V &= IR \\
Ohm's\ law:\ \mathbf{j} &= \sigma\mathbf{E} \\
Resistivity\ and\ resistance:\ R &= \rho\frac{l}{A} \\
Electric\ power\ (DC):\ P &= VI \\
Average\ electric\ power\ (AC):\ \overline{P} &= \frac{1}{2}I_0^2 R \\
rms\ current:\ I_{rms} &= \frac{1}{\sqrt{2}}I_0 \\
Resistors\ in\ series:\ R_{eq} &= \sum R_i \\
Resistors\ in\ parallel:\ \frac{1}{R_{eq}} &= \sum \frac{1}{R_i} \\
Kirchhoff\ current\ rule:\ \sum I &= 0\ \ at\ a\ node \\
Kirchhoff\ potential\ rule:\ \sum \varepsilon &= \sum IR\ \ around\ a\ loop \\
\nabla V &= \frac{\partial V}{\partial x}\hat{\mathbf{x}} + \frac{\partial V}{\partial y}\hat{\mathbf{y}} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \\
\nabla V &= \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{\mathbf{z}} \\
\nabla V &= \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}
\end{aligned}
$$

Some useful Integrals:

$$\int \frac{dx}{\sqrt{x^2+a^2}} = log\left(x+\sqrt{x^2+a^2}\right) \quad \text{or} \quad sinh^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}arctan(\frac{x}{a}) \qquad \int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

1. [25 points] Short Questions

(a) [5 points] Circle (black in) T or F for True or False

T F (i) Unlike electric charges attract and like charges repel.

T F (ii) The net amount of charge produced in any process is zero.

T F (iii) The electric field intensity anywhere is the sum of the electric field intensity from all the contributing charges.

T F (iv) Electric charge is quantized.

T F (v) The electric potential of a system of two unlike charges is positive. ($V(\infty) = 0$)

T F (vi) If there is no charge in a region of space, the electric field on a surface surrounding the region must be zero everywhere.

T F (vii) In electrostatic equilibrium, the electric field inside a conductor is zero.

T F (viii) If the net charge on a conductor is zero, the charge density must be zero at every point on the surface of the conductor.

T F (ix) The result that E = 0 inside a conductor can be derived from Gauss's Law.

T F (x) One can induce a net charge on a conductor without touching.

T F (xi) Resistance increases as temperature increases.

(b) [5 points] Circle (black in letter of) correct answer

(i) Metals are in general electrical
(A) nonconductors.
(B) conductors.
(C) semiconductors
(D) insulators..
(E) conductance depends upon configuration.

(ii) What is the maximum voltage that a sphere of radius 5 cm can hold in air?
(A) $3 \times 10^6$ V.
(B) $1.5 \times 10^6$ V.
(C) $3 \times 10^5$ V.
(D) $1.5 \times 10^5$ V.
(E) $3 \times 10^4$ V.

(iii) How much voltage is necessary to accelerate a proton so that it has just sufficient energy to touch the surface of an iron nucleus? An iron nucleus has a charge of 26 times that of a proton ($e$) and its radius is about $4.0 \times 10^{-15}$m. Whereas the proton has a radius of about $1.2 \times 10^{-15}$m. Assume the nucleus is spherical and uniformly charged.
(A) 13.6 V.
(B) 354 V
(C) 7000 V.
(D) 500,000 V.
(E) 7,000,000 V.

(iv) A current of 10 amps in a 2-mm diameter copper wire is the result of an electron drift velocity of about
(A) $2.5 \times 10^{-6}$ m/s
(B) $2.5 \times 10^{-4}$ m/s
(C) $2.5 \times 10^{-2}$ m/s
(D) $2.5 \times 10^{0}$ m/s
(E) $2.5 \times 10^{2}$ m/s

(v) The electric flux from a cubical box 28 cm on a side immersed in water (dielectric constant = 80) is 1.45 $\times 10^3 N \cdot m^2/C$ for a Gaussian surface in the water. What is the net charge enclosed in the box?
(A) 1 pC
(B) 1 nC
(C) 1 $\mu$C
(D) 1 mC
(E) 1 C.

(c) [5 points] The figure just below supposedly shows the electric field lines near an irregular-shaped positively-charged conductor. There are five distinctly different errors in the figure. Make a list of errors and give a brief reason why each is an error.
Multiple instances of the same error are **not** different.

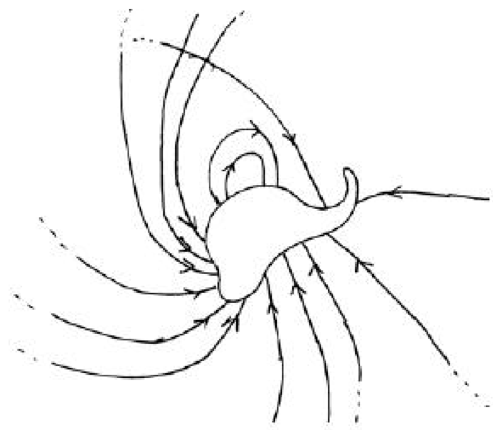


Figure 1: Figure for problem 1(c) showing field lines and conductor

(1)

(2)

(3)

(4)

(5)

(d) [5 points] Circle correct answer

(i) At a point high in the atmosphere, $He^{++}$ ions in a concentration of $2.8 \times 10^{12}/m^3$ move north with a speed of $2 \times 10^6 m/s$. The current density at that point is

(A) $5.6 \times 10^{18} A/m^2$ north
(B) $11.2 \times 10^{18} A/m^2$ north
(C) $1.8 A/m^2$ north
(D) $3.6 A/m^2$ north
(E) none of the above as it is going south

(ii) A proximity (don't have to touch only get very near ) button makes use of what effect?

(A) Ease of sensing the residual electric field on human body.
(B) Heat sensing due to the blood in the finger tips..
(C) Sensing of the electrical currents in finger tips.
(D) Sensing the potential difference between the body and the button.
(E) Capacitance, which changes as a conducting finger is brought near the button.

(iii) The wiring in a house must be thick enough so it does not become so hot as to start a fire. What diameter must a copper wire, $\rho = 1.68 \times 10^{-8} \Omega \cdot m$, if it is to carry a maximum current of 20 A and produce no more than 2 W of heat per meter?

(A) 0.5 mm.
(B) 1 mm.
(C) 1.5 mm.
(D) 2 mm.
(E) 2.5 mm.

(iv) At what current do humans experience severe heart fibrillation most lethally?

(A)1 mA.
(B) 10 mA.
(C) 100 mA.
(D) 1 A.
(E) 1 $\mu$A.

(v) A bird perches on a DC electric transmission line carrying 2500 A. The line has a resistance of $2.5 \times 10^{-5} \Omega$ per meter and the bird's feet are 4 cm apart. What potential difference does the bird feel?

(A) 2.5 $\mu$V.
(B) 2.5 mV.
(C) 2.5 V.
(D) 25 V.
(E) 250 V.

(e) [5 points] Circle correct answer

(i) Four 100 $\Omega$ resistors are connected in parallel. What is the equivalent resistance?
(A) 25 $\Omega$.
(B) 100 $\Omega$.
(C) 400 $\Omega$.
(D) 10 $\Omega$.
(E) none of the above.

(ii) Four 100 $\mu$F capacitors are connected in parallel. What is the equivalent capacitance?
(A) 25 $\mu$F.
(B) 100 $\mu$F.
(C) 400 $\mu$F.
(D) 10 $\mu$F.
(E) none of the above.

(iii) A 1200 watt hair dryer runs on 120 V AC power in your house. What is the peak current through the hair dryer?
(A) 1 A.
(B) 1.4 A.
(C) 10 A.
(D) 14 A.
(E) none of the above

(iv) The EV-1 electric car is powered by 26 batteries, each 12 V and 52 A-hr. Assume the car is on the level moving at 40 km/hr so that has an average retarding force of 240 N. After how many kilometers must the batteries be recharged?
(A) 2.5 km
(B) 25 km.
(C) 250 km.
(D) 2500 km.
(E) none of the above.

(v) A capacitor is often used in electronics to keep energy powering a circuit even if there is a momentary loss of power from the electric company. What capacitance would be required for a TV, operating at an internal voltage of 120 V at 150 W to provide sufficient energy during a 0.1 second lapse in power?
(A) 2 pF = $2 \times 10^{-12}$ F
(B) 2 nF = $2 \times 10^{-9}$ F
(C) 2 $\mu$F = $2 \times 10^{-6}$ F
(D) 2 mF = 2000 $\mu$F = $2 \times 10^{-3}$ F
(E) 12 F
(F) 2000 F

2. [15 points] A touted breakthrough in high fidelity (Hi-Fi) was the development capacitance speakers. There are large parallel plates (1.5 m by 0.3 m), one of which is fixed in a fairly rigid frame and the other separated by 2mm of a thin soft foam with dielectric constant $\epsilon = 1.2\epsilon_0$. When the plates are charged, they attract, compressing the foam.
(a) [4 points] How much charge and electrical energy is stored in the capacitor at 600 volts if the plates did not get closer together?

(b) [6 points] When a voltage $V = 600$ Volts is applied across the plates how much does the separation of the plates change, if the spring constant of the foam is $k = 10^3 N/m$? What is the force on the plate? How much energy is stored in the compression and how does it compare to the stored electrical energy?

(c) [5 points] If the audio signal is at a frequency 10 kHz, how much electrical power goes to the speakers? How much goes to sound = moving air and the speaker plates?

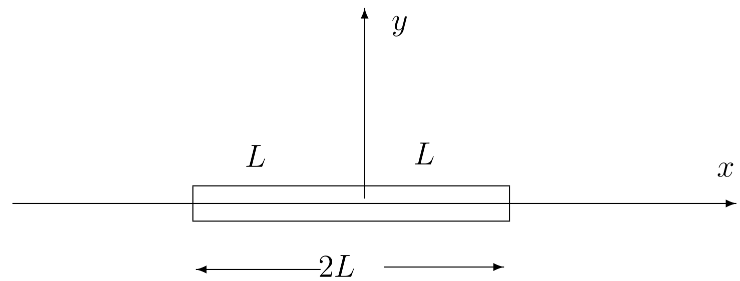
3. [15 points] The HCl molecule has a dipole moment of about $3.4 \times 10^{-30} C \cdot m$. The two atoms are separated by about $1.0 \times 10^{-10} m$.
(a) [4 points] What is the net charge on each atom?

(b) [3 points] Is this equal to an integer multiple of $e$? If not, explain.

(c) [4 points] What maximum torque would this dipole experience in a $2.5 \times 10^4 N/C$ electric field?

(d) [4 points] How much energy would be needed to rotate one molecule $45°$ from its equilibrium position of lowest potential energy?

4. [20 points] A thin rod of length $2L$ is centered on and lying on the $x$ axis. The rod carries




a uniformly distributed charge $Q$.

(a) [5 points] Determine the potential $V(y)$ as a function of $y$ (perpendicular direction to rod) for points on the $y$-axis. Let $V = 0$ at infinity.

(b) [5 points] Determine the potential $V(x)$ for points along the $x$ axis outside the rod.

(c) [5 points] What is the electric field along the $y$ axis?

(d) [5 points] Place a sphere of radius $R$ at $y = d$, $x = 0$ with $d < R < L$. What is the electric flux through the sphere?

5. [15 points] Suppose that a person's internal body resistance is 1100 Ω.

(a) [3 points] What current passes through the body when that person is accidentally connected to 110 V?

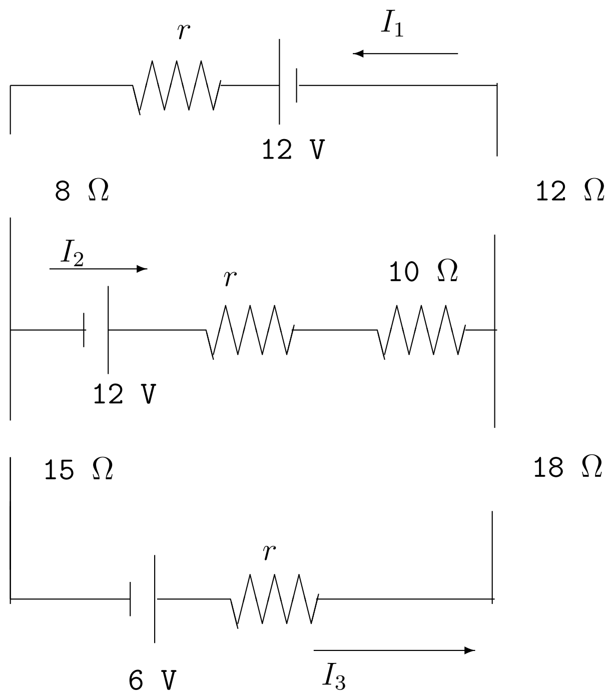
(b) [3 points] If there is an alternative path to ground with resistance of 40 Ω, what current passes through the person?

(c) [3 points] If the voltage source can produce at most 1.5 A, how much current passes through the person in case (b)?

(d) [3 points] Does the person goes into heart disrupting fibrillation in case (b)? in case (c)? (Ventricular fibrillation is a very fast irregular heart rhythm in the lower chambers of the heart. During ventricular fibrillation the heart quivers, and provides little or no blood flow to the body. If not treated immediately ventricular fibrillation will cause sudden cardiac arrest.)

(e) [3 points] If a defibrillator is applied to reset and restart the heart of a person suffering ventricular fibrillation, it requires a short pulse of about one ampere. The defibrillator must deliver between 100 and 360 Joules (maximum) of energy to the person. A capacitor is charged with the appropriate amount of energy, say 200 Joules, the doctor shouts clear, and a switch connects the capacitor to the patient to make a circuit. In order to obtain 1 amp current, what voltage and what capacitance are needed?

6. [21 points] For the circuit in the accompanying schematic assume that all three batteries have internal resistance $r = 1\Omega$.
(a) [5 points] Draw a circuit with the resistances reduced to an equivalent minimum number and show their values.



(b) [5 points] Write the appropriate equations and solve for the currents $I_1$, $I_2$, and $I_3$.

(c) [3 points] What is the terminal voltage on the 6-V battery?

(d) [4 points] Suppose the 6-V battery is replaced by an unknown emf $\xi$. If the current through the 10-$\Omega$ resistor is $I_2 = -0.30$ A (current to the left), what is $\xi$?

(e) [4 points] What would be the current $I_1$, if the 18-$\Omega$ resistor were shorted out (replaced with a conductor)? (Use the original circuit as a starting place.)

7. [15 points] Twelve resistors, each of resistance $R$ are connected as the edges of a cube. Determine the equivalent resistance

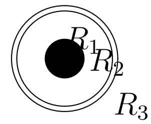
*Hint:* Apply an emf and determine currents; use symmetry at junctions.

(a) [5 points] between two ends of any resistor. (i.e. two corners of one edge).

(b) [5 points] between any two opposite corners of the same face of the cube. (i.e. the diagonal corners on a face of the cube)

(c) [5 points] between the opposite volume diagonal corners. (i.e. to get from one corner to the other one must go through at least 3 resistors.)

8. [20 points] A solid metal sphere of radius $R_1$ is surrounded by a concentric spherical shell of inner radius $R_2$ and outer radius $R_3$. Initially, there is no charge on either sphere. A charge $+Q$ is now placed on the inner sphere via a very fine insulated wire. (Assume that




the potential at infinity and grounded is zero.)

(a) [4 points] What is the potential of the inner sphere?

(b) [4 points] Suppose that the shell is momentarily grounded. Now what is the potential of the inner sphere?

(c) [4 points] Finally, after that momentary grounding, the inner sphere is momentarily grounded. What is now the potential of the shell?

(d) [4 points] A dielectric material (water with dielectric constant K = 80) now fills the space between the sphere and shell. What is the capacitance of the system?

(e) [4 points] The dielectric material has a bulk resistivity $\rho$. What is the resistance between the inner sphere and the outer shell?

End of Examination

1. Short Problems

(f) [5 points] Circle correct answer

(i) Which of these is **not** examples of charging by induction or contact

(A) You rub a balloon on your sweater and it sticks on a wall.
(B) Bring a charged plate near an electroscope causes the leaves to move apart.
(C) Spinning wheels (Windhorst machine) rub against electrodes causes sparks in the air.
(D) Chalk dust sticks to your glasses.
(E) A van deGraff generator makes hair stand on end.

(ii) One kind of hybrid electric car uses fuel cells to generate electrical energy to power the electric motor. A single methane fuel cell can produce 0.8 V with a current density of 0.7 A/m$^2$ at about 50 per cent efficiency compared to the 20 to 25 per cent for internal combustion engine. A hydrogen fuel cell combined with air produces about the same and its by product is water. Methane makes water and $CO_2$. Methane used for direct electrical production produces about 800 kJ/mole. Assume the car is on the level moving at 40 km/hr so that has an average retarding force of 240 N. What is the "mileage" in kilometer per liter and the current if the electric motor operates at 24 V at 40 km/hr? (A) 1.1 amps and 188 km/liter.

(B) 11.1 A and 144 km/liter.
(C) 111 A and 88 km/liter.
(D) 1110 A and 9 km/liter.
(E) none of the above

Answer: (C) 111 A and 88 km/liter.

$P = Fv = 240N \times 40km/hr = 2666.7w = IV = 24VI$ $I = 2666.7w/24 = 111.1A$ $\dot{C}H_4 = 800 \times 10^3/2666.7sec/mole = 300sec/mole$ In one hour the car goes 40 km and uses up $3600/300moles = 12moles = 12 \times (12+4)gm = 192gm = 192gm/424kg/m^3/hr = 4.5283 \times 10^{-4}m^3/hr = 0.4528Liters/hr$ The "mileage" is 40/0.4528 = 88 km/liter.

(iii) 240 V AC is applied to two conductors made of the same material. One conductor is twice as long and twice as thick a diameter as the second. What is the ratio of the power transformed (to heat) in the first relative to the second?

(A) 0.125
(B) 0.25
(C) 0.5
(D) 1 - they are the same
(E) 2
(F) 4
(G) 8

Answer: (E) 2

Reasoning: Resistive power loss for constant voltage is given $P = V^2/R$ so power is inversely proportional to resistance. $P_1/P_2 = R_2/R_1$ The resistance of a piece of material with resistivity $\rho$ is $R = \rho L/A$ so that $P_1/P_2 = R_2/R_1 = L_2A_1/L_1A_2 = 1 \times 2^2/2 \times 1 = 2$

(iv) A lightening flash transfers 4 C of charge and 4 MJ of energy to the Earth. What was the potential difference?

(A) 1000 V.
(B) 10,000 V.
(C) 100,000 V.
(D) 1,000,000V.
(E) 10,000,000 V
Answer: (D) 1,000,000V.
Reasoning: $E = QV = 4CV = 4 \times 10^6 J$
$V = 10^6 J/C = 10^6 V$

(v) The energy stored at potential $V$ in a capacitor in a parallel plate capacitor of area $A$ and plate separation $d$ is equal to
(A) $\frac{\epsilon_0 A}{d}$
(B) $\frac{\epsilon_0 A}{d} V^2$
(C) $\frac{\epsilon A}{2d} V^2$
(D) $\frac{\epsilon_0 A}{2d} V^2$
(E) $\frac{\epsilon_0 A}{d} V$

(vi) A hollow conductor of arbitrary shape carries a charge Q, the charge surface density in a region of interest is $\sigma$. If now a very small section of the conductor shell is removed, what is the electric field intensity in the hole? Assume that the hole is so small that none of the other charges and distribution were affected by introducing the hole.
(A) $2\sigma/\epsilon_0$
(B) $\sigma/\epsilon_0$
(C) $\sigma/2\epsilon_0$
(D) $A\sigma/\epsilon_0$ where A is the area of the hole
(E) 0
(F) none of above

4. [9 points] A solid metal cube has a spherical cavity at its center. At the center of the cavity is a point charge of $Q = +8\mu C$. The metal cube carries a charge of $q = -7\mu C$.

(a) [3 points] What is the charge on the surface of the spherical cavity?

Answer: This is a simple application of Gauss's law and the fact that the electric field is zero in a conductor. We make a spherical Gaussian surface just into the metal (conductor) and the net flux is zero and thus the enclosed charge is zero. Thus the charge on the inner surface must be $-Q = -8\mu C$

(b) [3 points] What is the total charge on the outer surface?

Answer: Charge is conserved so that the net charge in the metal must be $q = -7\mu C$ and so that the outer surface must have a charge of $+1\mu C$.

(c) [3 points] What is the flux through one of the faces of a cube just outside the metal surface? Assume that the length of a cube edge is $l = 1m$.

Answer: $\frac{q_{net}}{6\epsilon_0}$ This is a simple application of Gauss's law. The total flux through all the faces is $q_{net}/\epsilon_0$ and there are six (6) equivalent faces. $\Phi_E = 1\mu C/(6\times 8.85\times 10^{-12}) = 0.113\times 10^{-6} C\cdot m$

Solutions

1. [25 points] Short Questions

(a) [5 points] Circle (black in) T or F for True or False

T F (i) Unlike electric charges attract and like charges repel.

T F (ii) The net amount of charge produced in any process is zero.

T F (iii) The electric field intensity anywhere is the sum of the electric field intensity from all the contributing charges.

T F (iv) Electric charge is quantized.

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T F (ix) The result that $E = 0$ inside a conductor can be derived from Gauss's Law.

T F (x) One can induce charge on a conductor without touching.

T F (xi) Resistance increases as temperature increases.

(b) [5 points] Circle (black in letter of) correct answer

(i) Metals are in general electrical

(A) nonconductors.
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(C) semiconductors
(D) insulators..
(E) conductance depends upon configuration.

(ii) What is the maximum voltage that a sphere of radius 5 cm can hold in air?

(A) $3 \times 10^6$ V.
(B) $1.5 \times 10^6$ V.
(C) $3 \times 10^5$ V.
(D) $1.5 \times 10^5$ V.
(E) $3 \times 10^4$ V.
Answer: (D) $1.5 \times 10^5$ V.
Reason: Air breaks down at an electric field of $3 \times 10^6$ V/m. The electric field is $E = Q/(4\pi\epsilon_0 r^2) = 3\times10^6$ V/m or $Q = 4\pi\epsilon_0 r^2 E = 4\pi\times8.85\times10^{-12}\times0.05^2\times3\times10^6 = 8.34\times10^{-7}$ coulombs. Voltage is given by $V = Er = 3 \times 10^6 \times 0.05 = 1.5 \times 10^5 V$.

(iii) How much voltage is necessary to accelerate a proton so that it has just sufficient energy to touch the surface of an iron nucleus? An iron nucleus has a charge of 26 times that of a proton ($e$) and its radius is about $4.0 \times 10^{-15}$m. Whereas the proton has a radius of about $1.2 \times 10^{-15}$m. Assume the nucleus is spherical and uniformly charged.

(A) 13.6 V.
(B) 354 V
(C) 7000 V.
(D) 500,000 V.
(E) 7,000,000 V.
Answer: (E) 7,000,000 V.
Reason: Potential Energy is $eV = \frac{e26e}{4\pi\epsilon_0 d^2} = 26 \times 10^{-19}e/(4\pi \times 8.85 \times 10^{-12}\times)(1.2 + 4.0) \times 10^{-15}eV = 7.23(9.4\ if\ use\ iron\ radius) \times 10^6 eV$

(iv) A current of 10 amps in a 2-mm diameter copper wire is the result of an electron drift velocity of about

(A) $2.5 \times 10^{-6}$ m/s
(B) $2.5 \times 10^{-4}$ m/s
(C) $2.5 \times 10^{-2}$ m/s
(D) $2.5 \times 10^{0}$ m/s
(E) $2.5 \times 10^{2}$ m/s

(v) The electric flux from a cubical box 28 cm on a side immersed in water (dielectric constant = 80) is 1.45 $\times 10^3 N \cdot m^2/C$ for a Gaussian surface in the water. What is the net charge enclosed in the box?

(A) 1 pC

(B) 1 nC
(C) 1 $\mu$C
(D) 1 mC
(E) 1 C.
Answer: (C) 1 $\mu$C
Reasoning: Use Gauss's Law $\Phi_E = q_{net}/\epsilon = q_{net}/(K\epsilon_0)$ $q_{net} = 1.45 \times 10^3 \times 8.85 \times 10^{-12} \times 80 = 1.027 \times 10^{-6} C$

(c) [5 points] The figure just below supposedly shows the electric field lines near an irregular-shaped positively-charged conductor. There are five distinctly different errors in the figure. Make a list of errors and give a brief reason why each is an error.
Multiple instances of the same error are **not** different.

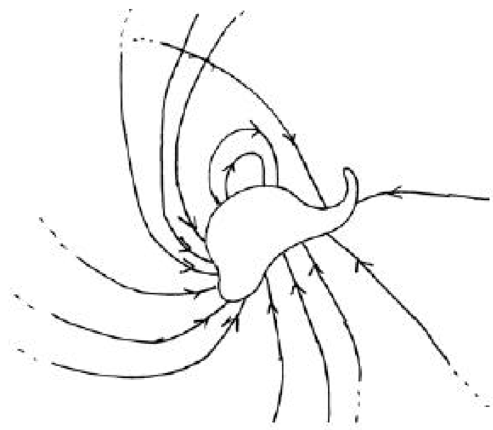


Figure 2: Figure for problem 1(c) showing field lines and conductor

(1) Field lines leave a positive charge. The field lines should all leave the conductor.

(2) The conductor has constant potential everywhere. Thus field lines cannot go from the surface to the same surface.

(3) Field lines cannot cross.

(4) There should be more field lines where the surface is more convex (smaller radius) compared to where the surface is less convex (larger radius).

(5) Field lines should be normal to the surface of the conductor.

(d) [5 points] Circle correct answer

(i) At a point high in the atmosphere, $He^{++}$ ions in a concentration of $2.8 \times 10^{12}/m^3$ move north with a speed of $2 \times 10^6 m/s$. The current density at that point is

(A) $5.6 \times 10^{18} A/m^2$ north
(B) $11.2 \times 10^{18} A/m^2$ north
(C) $1.8 A/m^2$ north
(D) $3.6 A/m^2$ north
(E) none of the above as it is going south

Answer: (C) $1.8 A/m^2$ north

Reason: $\vec{j} = nq\vec{v} = 2.8 \times 10^{12}/m^3 \times 2 \times 1.6 \times 10^{-19} C \times 2 \times 10^6 m/sec = 1.79 C/m^2 \cdot sec$

(ii) A proximity (don't have to touch only get very near ) button makes use of what effect?

(A) Ease of sensing the residual electric field on human body.
(B) Heat sensing due to the blood in the finger tips..
(C) Sensing of the electrical currents in finger tips.
(D) Sensing the potential difference between the body and the button.
(E) Capacitance, which changes as a conducting finger is brought near the button.

(iii) The wiring in a house must be thick enough so it does not become so hot as to start a fire. What diameter must a copper wire, $\rho = 1.68 \times 10^{-8} \Omega \cdot m$, if it is to carry a maximum current of 20 A and produce no more than 2 W of heat per meter?

(A) 0.5 mm.
(B) 1 mm.
(C) 1.5 mm.
(D) 2 mm.
(E) 2.5 mm.

Answer:

$R = \rho L/A = 1.68 \times 10^{-8} \Omega \cdot m \times 1m/(\pi d^2/4)$ $P = I^2 R = 20^2 \times 1.68 \times 10^{-8} \Omega \cdot m^2/(\pi d^2/4) = 2w$ $d = \sqrt{4 \times 20^2 \times 1.68 \times 10^{-8} \Omega \cdot m^2/2w\pi} = 2.068 \times 10^{-4} m = 2.068 mm$

(iv) At what current do humans experience severe heart fibrillation most lethally?

(A)1 mA.
(B) 10 mA.
(C) 100 mA.
(D) 1 A.
(E) 1 $\mu$A.

(v) A bird perches on a DC electric transmission line carrying 2500 A. The line has a resistance of $2.5 \times 10^{-5} \Omega$ per meter and the bird's feet are 4 cm apart. What potential difference does the bird feel?

(A) 2.5 $\mu$V.
(B) 2.5 mV.
(C) 2.5 V.
(D) 25 V.
(E) 250 V.

Answer: (B) 2.5 mV.
$V = RI = 0.04 \times 2.5 \times 10^{-5}\Omega \times 2500A = 2.5mV$

(e) [5 points] Circle correct answer

(i) Four 100 Ω resistors are connected in parallel. What is the equivalent resistance?
(A) 25 Ω.
(B) 100 Ω.
(C) 400 Ω.
(D) 10 Ω.
(E) none of the above.

(ii) Four 100 $\mu$F capacitors are connected in parallel. What is the equivalent capacitance?
(A) 25 $\mu$F.
(B) 100 $\mu$F.
(C) 400 $\mu$F.
(D) 10 $\mu$F.
(E) none of the above.

(iii) A 1200 watt hair dryer runs on 120 V AC power in your house. What is the peak current through the hair dryer?
(A) 1 A.
(B) 1.4 A.
(C) 10 A.
(D) 14 A.
(E) none of the above

(iv) The EV-1 electric car is powered by 26 batteries, each 12 V and 52 A-hr. Assume the car is on the level moving at 40 km/hr so that has an average retarding force of 240 N. After how many kilometers must the batteries be recharged?
(A) 2.5 km
(B) 25 km.
(C) 250 km.
(D) 2500 km.
(E) none of the above.
Answer: (C) 250 km.
$P = Fv = 240N \times 40km/hr = 2666.7w$ $Work = Fd = 240N \times d = 26 \times 12V \times 52A - hr = 16224W - hr = 5.84 \times 10^7 joules$ $d = W/F = 5.84 \times 10^7 joules/240N = 243 \times 10^3 m = 243km$

(v) A capacitor is often used in electronics to keep energy powering a circuit even if there is a momentary loss of power from the electric company. What capacitance would be required for a TV, operating at an internal voltage of 120 V at 150 W to provide sufficient energy during a 0.1 second lapse in power?
(A) 2 pF = 2 $\times 10^{-12}$ F
(B) 2 nF = 2 $\times$ $10^{-9}$ F
(C) 2 $\mu$F = 2 $\times$ $10^{-6}$ F
(D) 2 mF = 2000 $\mu$F = 2 $\times$ $10^{-3}$ F
(E) 12 F
(F) 2000 F

Answer: (D) 2 mF = 2000 $\mu$F = $2 \times 10^{-3}$ F
Reasoning: $E = P\Delta T = 150W \times 0.1sec = 15J = \frac{1}{2}CV^2 = \frac{1}{2}120^2C$
$C = 2E/V^2 = 2 \times 15J/120^2 = 2.083 \times 10^{-3}F$

2. [15 points] A touted breakthrough in high fidelity (Hi-Fi) was the development capacitance speakers. There are large parallel plates (1.5 m by 0.3 m), one of which is fixed in a fairly rigid frame and the other separated by 2mm of a thin soft foam with dielectric constant $\epsilon = 1.2\epsilon_0$. When the plates are charged, they attract, compressing the foam.
(a) [4 points] How much charge and electrical energy is stored in the capacitor at 600 volts if the plates did not get closer together?

Answer: $Q = CV = 2.39 \times 10^{-9}600 = 1.4337 \times 10^{-6}C$ $C = \epsilon A/d^2 = 1.2 \times 8.85 \times 10^{-12} \times 1.5 \times 0.3/0.002 = 2.39 \times 10^{-9}$F $E = 0.5CV^2 = 0.5 \times 2.39 \times 10^{-9}F600^2 = 4.3 \times 10^{-4}J$
(b) [6 points] When a voltage $V = 600$ Volts is applied across the plates how much does the separation of the plates change, if the spring constant of the foam is $k = 10^3N/m$? What is the force on the plate? How much energy is stored in the compression and how does it compare to the stored electrical energy?

Answer: $\vec{F} = q\vec{E} = CV\vec{E} = CV^2/(d-\delta) = \epsilon AV^2/(d-\delta)^2 = k\delta$ where $\delta$ the displacement.

$\delta \simeq \epsilon AV^2/(k(d-\delta)^2) = 1.5 \times 0.3m^2 600^2 V^2 \times 1.2 \times 8.85 \times 10^{-12}/(k = (2mm - \delta)^2 \simeq 4.3011 \times 10^1/10^3 = 4.3 \times 10^{-4}m$ $F = -kx = 10^3N/m \times 4.3 \times 10^{-4}m = 0.43N$

At the smaller separation the stored electrical energy is $EE = 0.5CV^2 = 0.5 \times (2/(2 - 0.43) \times 2.39 \times 10^{-9}F600^2 = 5.5 \times 10^{-4}J$

$PE = \frac{1}{2}kx^2 = 0.5 \times 0.043N \times 4.3 \times 10^{-4}m = 9.24 \times 10^{-6}J$

A more precise calculation takes into account the change in spacing (increases E). $F - \nabla E = -\frac{\partial}{\partial x}(\frac{1}{2}\frac{\epsilon AV^2}{x}) = \frac{1}{2}\frac{\epsilon AV^2}{x^2} = K(x_0 - x)$ The solution (iterative or otherwise) is the plate separation changes by 0.5 mm $= 5 \times 10^{-4}m$.

The force on the plate is $F = Kx = 10^3N/m \times 5 \times 10^{-4}m = 0.5N$

The energy stored in compression is $PE = 0.5Kx^2 = 0.25 \times 10^{-4}J$

The electrical energy stored is $EE = 0.5CV^2 = 0.5\epsilon AV^2/d = 0.5 \times 1.2 \times 8.85 \times 10^{-12} \times 1.5 \times 0.3 \times 600^2/1.5 \times 10^{-3} = 5.73 \times 10^{-4}J$
(c) [5 points] If the audio signal is at a frequency 10 kHz, how much electrical power goes to the speakers? How much goes to sound = moving air and the speaker plates?

Answer: The voltage must swing from +600 V to -600V regularly at a frequency of $10^4$ Hz. This is a peak-to-peak AC voltage so that the power is one half the peak energy stored in the capacitor times the frequency. Thus $P = 0.5 \times 4.3 \times 10^{-4}J \times 10^4/sec = 2.15watts$ approximation and $2.86watts$ more precisely.

$P = 9.24 \times 10^{-6} \times 10^4J/s = 0.0924watts$

3. [15 points] The HCl molecule has a dipole moment of about $3.4 \times 10^{-30} C \cdot m$. The two atoms are separated by about $1.0 \times 10^{-10} m$.

(a) [4 points] What is the net charge on each atom?

Answer: $p = qd = q \times 10^{-10} m = 3.4 \times 10^{-30} C \cdot m$ so that $q = p/d = 3.4 \times 10^{-20} C$,

(b) [3 points] Is this equal to an integer multiple of $e$? If not, explain.

Answer: No. The electron is shared between the various

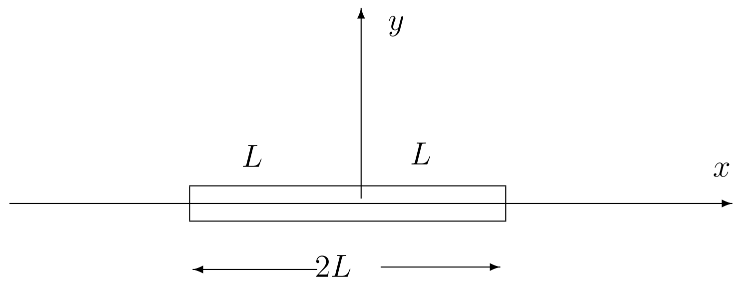
(c) [4 points] What maximum torque would this dipole experience in a $2.5 \times 10^4 N/C$ electric field?

Answer: $\vec{\tau} = \vec{p} \times \vec{E}$ so that $\tau_{max} = pE = 3.4 \times 10^{-30} C \cdot m \times 2.5 \times 10^4 N/C = 8.5 \times 10^{-26} N \cdot m$,

(d) [4 points] How much energy would be needed to rotate one molecule 45° from its equilibrium position of lowest potential energy?

Answer: $U = -\vec{p} \cdot \vec{E} = -pEcos\theta = 8.5 \times 10^{-26} N \cdot mcos\theta$ for 45° $cos(\theta) = 0.7071$ $U = 6 \times 10^{-26} J$ for the potential at 45°. or$\Delta U = 2.5 \times 10^{-26} J$ for the difference between the minimum $cos\theta = 1$ and $cos\theta = 0.7071$

4. [20 points] A thin rod of length $2L$ is centered on and lying on the $x$ axis. The rod carries




a uniformly distributed charge $Q$.

(a) [5 points] Determine the potential $V(y)$ as a function of $y$ (perpendicular direction to rod) for points on the $y$-axis. Let $V = 0$ at infinity.

Answer: $dV = dQ/4\pi\epsilon_0 r$ where $dQ = Qdx/(2L)$ and $r^2 = x^2 + y^2$ or $r = \sqrt{x^2+y^2}$
$V(y) = \frac{Q}{4\pi\epsilon_0 2L}\int_{-L}^{L}\frac{dx}{\sqrt{x^2+y^2}} = \frac{Q}{4\pi\epsilon_0 2L}arcsinh(x/y)|_{-L}^{L} = \frac{Q}{4\pi\epsilon_0 2L}(arsinh(L/y) - arcsinh(-L/y))$
which also equals $= \frac{Q}{4\pi\epsilon_0 2L}log(\frac{\sqrt{L^2+y^2}+L}{\sqrt{L^2+y^2}-L})$

(b) [5 points] Determine the potential $V(x)$ for points along the $x$ axis outside the rod.

Answer: $dV = dQ/4\pi\epsilon_0 r$ where $dQ = Qdx/(2L)$ and $r^2 = (x'-x)^2+y^2$ or $r = \sqrt{(x'-x)^2+y^2}$ and $y = 0$ $V(y) = \frac{Q}{4\pi\epsilon_0 2L}\int_{-L}^{L}\frac{dx}{x'-x} = \frac{Q}{4\pi\epsilon_0 2L}ln(x'-x)|_{-l}^{L} = \frac{Q}{4\pi\epsilon_0 2L}ln(x+L)/(x-L))$

(c) [5 points] What is the electric field along the $y$ axis?

Answer: We can derive this either of two ways:

(1) $\vec{E} = -\nabla V(y) = \int_{-L}^{L}\frac{Qy}{4\pi\epsilon_0 2L(x^2+y^2)^{3/2}}dx\hat{y}$

(2) Integrate $d\vec{E} = \frac{dQ\hat{r}}{4\pi\epsilon_0 r^2} = \frac{dQ\hat{r}}{4\pi\epsilon_0(x^2+y^2)} = \frac{Q}{4\pi\epsilon_0 2L}\int\frac{dx}{x^2+y^2}\hat{r} = \frac{Q}{4\pi\epsilon_0 2L}\int\frac{ydx}{(x^2+y^2)^{3/2}}\hat{y}$ since only the *haty* direction adds coherently the $\hat{x}$ has canceling components and there is no *hatz* component along the $y$ direction by symmetry.

$\vec{E} = \frac{Q}{4\pi\epsilon_0 2L}\frac{yx}{y^2\sqrt{x^2+y^2}}|_{-L}^{L} = \frac{Q}{4\pi\epsilon_0 2L}\left(\frac{L}{y\sqrt{y^2+L^2}} - \frac{-L}{y\sqrt{y^2+L^2}}\right) = \frac{Q}{4\pi\epsilon_0}\frac{1}{y\sqrt{y^2+L^2}}$

Note that this has the correct dependence: it goes down as $1/r$ near ($d < L$) the rod and as $1/r^2$ far from the rod.

(d) [5 points] Place a sphere of radius $R$ at $y = d$, $x = 0$ with $d < R < L$. What is the electric flux through the sphere?

Answer: The easiest way to work this problem is to use Gauss's law $\Phi_E = Q_{enclosed}/\epsilon_0$ We can find the enclosed charge by multiplying the linear charge density times the length of the rod enclosed by the sphere. By simple trigonometry $\ell = \sqrt{R^2-d^2}$ is half the length in the sphere. $Q_{enclosed} = \lambda(2\ell) = \frac{Q}{2L}2\sqrt{R^2-d^2} = Q\sqrt{R^2-d^2}/L$ Thus the flux is

$\Phi_E = Q_{enclosed}/\epsilon_0 = \frac{Q}{\epsilon_0}\frac{\sqrt{R^2-d^2}}{L}$

5. [15 points] Suppose that a person's internal body resistance is 1100 Ω.

(a) [3 points] What current passes through the body when that person is accidentally connected to 110 V?

Answer: $I = V/R = 110V/1100\Omega = 0.1A$

(b) [3 points] If there is an alternative path to ground with resistance of 40 Ω, what current passes through the person?

Answer: $I = V/R = 110V/1100\Omega = 0.1A$, the same because the same voltage is across the same resistance.

(c) [3 points] If the voltage source can produce at most 1.5 A, how much current passes through the person in case (b)?

Answer: $V = IR = I\frac{1}{1/R_1+1/R_2} = 1.5A/(1/1100 + 1/40)\Omega = 57.895V$ $I_{patient} = V/R_{patient} = 57.895V/1100\Omega = 5.2632 \times 10^{-2} = 52.632mA$

(d) [3 points] Does the person goes into heart disrupting fibrillation in case (b)? in case (c)? (Ventricular fibrillation is a very fast irregular heart rhythm in the lower chambers of the heart. During ventricular fibrillation the heart quivers, and provides little or no blood flow to the body. If not treated immediately ventricular fibrillation will cause sudden cardiac arrest.)

Case (b) yes, the person is likely to have disrupting fibrillation as the current is at the most lethal 100 mA.

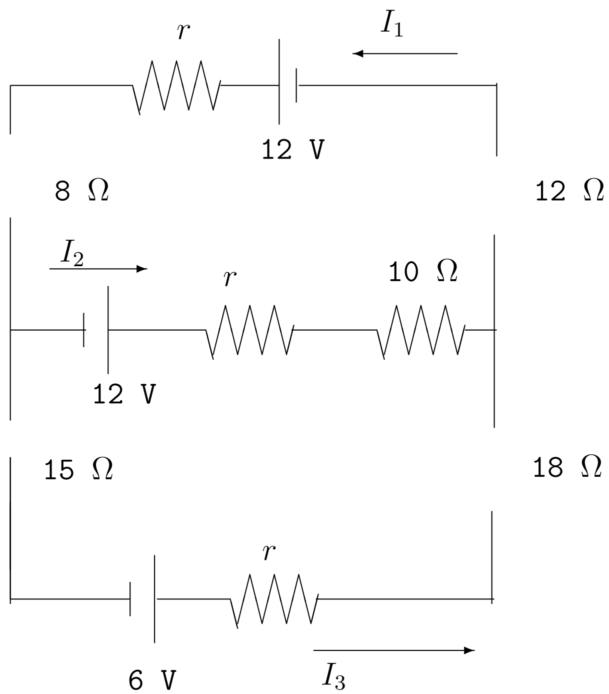
Case (c) Not unless the patient has a pre-existng condition. It would be painful but not likely to lead to death. It generally takes 100 mA for that to happen.

(e) [3 points] If a defibrillator is applied to reset and restart the heart of a person suffering ventricular fibrillation? It requires a short pulse of about one ampere for a short time. The defibrillator must deliver between 100 and 360 Joules (maximum) of energy to the person. A capacitor is charged with the appropriate amount of energy, say 200 Joules, the doctor shouts clear, and a switch connects the capacitor to the patient to make a circuit. In order to obtain 1 amp current, what voltage and what capacitance is needed?

Answer: If the body resistance remains $\Omega = 1100\Omega$ (It is actually reduced by conducting gels placed on the skin.) Then the necessary voltage is $V = IR = 1A \times 1100\Omega = 1100V$

The energy is $E = 0.5CV^2 = 200J$ so that $C = 2 \times 200J/(1100V)^2 = 3.3058 \times 10^{-4}F 330.58\mu F$

6. [21 points] For the circuit in the accompanying schematic assume that all three batteries have internal resistance $r = 1\Omega$.
(a) [5 points] Draw a circuit with the resistances reduced to an equivalent minimum number and show their values.




Answer: This is particularly simple as $r = 1$ and only reduction straight forward is series where $R_{eq} = R_1 + R_2$. One can reduce them further for purposes of solutions by recognizing some interchanges do not affect the other loops. E.g. the top loop where one can add the $12\Omega$ resistor to the series mix by a simple interchange. $R_{top} = 12 + 1 + 8 = 21\Omega$ $R_{middle} = 1 + 10 = 11\Omega$ and the bottom loop $R = 18 + 15 + 1 = 34\Omega$ In the end one

(b) [5 points] Write the appropriate equations and solve for the currents $I_1$, $I_2$, and $I_3$.

Answer: This is simple application of Kirchoff's laws;

$V_{12V} - I_1(R_{12} + R_1 + R_8) + V_{12V} + I_2(R_1 + R_{10}) = 0 = 24V - 21\Omega I_1 - 11\Omega I_2$ or $24V = 21\Omega I_1 + 11\Omega I_2$

$V_{12V} - I_2(R_1 + R_{10}) + I_3(R_{18} + R_1 + R_{15}) - V_{6V} = 0 = 6V - 11\Omega I_2 + 34\Omega I_3$ or $6V = 11\Omega I_2 - 34\Omega I_3$.

$I_1 - I_2 - I_3 = 0$ or $I_1 = I_2 + I_3$

Solving this is fairly straightforward. Substituting the thirst equation into the first equation gives $24V = 21\Omega(I_2 + I_3) + 11\Omega I_2$. Regrouping gives two equations in two unknowns:

$24V = 32\Omega I_2 + 21\Omega I_3$.

$6V = 11\Omega I_2 - 34\Omega I_3$ Multiply by (21/34) yields

$3.7059V = 6.7941\Omega I_2 - 21I_3$

Adding the two equations together gives $27.7059V = 38.7941\Omega I_2$ or $I_2 = 0.7142A$

Multiply by (32/11) gives $17.4545V = 32\Omega I_2 - 98.9091 I_3$ subtracting gives $6.5455V = 119.9091 I_3$ or $I_3 = 0.0546A$

$I_1 = I_2 + I_3 = 0.7688A$

(c) [3 points] What is the terminal voltage on the 6-V battery?

Answer: $V = EMF - IR = 6V - (-0.4A)1\Omega = +6.4V$

(d) [4 points] Suppose the 6-V battery is replaced by an unknown emf $\xi$. If the current through the 10-$\Omega$ resistor is $I_2 = -0.30$ A (current to the left), what is $\xi$?

Answer: Set up our Kirchhoff equations again: $V_{12V} - I_1(R_{12} + R_1 + R_8) + V_{12V} + I_2(R_1 + R_{10}) = 0 = 24V - 21\Omega I_1 - 11\Omega I_2$ or $24V = 21\Omega I_1 + 11\Omega I_2$

$V_{12V} - I_2(R_1 + R_{10}) + I_3(R_{18} + R_{15}) + \xi = 0 = (12 + \xi)V - 11\Omega I_2 + 33\Omega I_3$ or $(12 + \xi)V = 11\Omega I_2 - 33\Omega I_3$.

$I_1 - I_2 - I_3 = 0$ or $I_1 = I_2 + I_3$ and $I_2 = 0.30A$

Solving this is fairly straightforward. Substituting the thirst equation into the first equation gives $24V = 21\Omega I_1 + 11\Omega 0.3A = 21\Omega I_1 - 3.3V$. Regrouping gives $I_1 = (24+3.3)V/21\Omega = 1.30A$ so that we now know $I_3$: $I_3 = I_1 - I_2 = 1.3A + 0.3A = 1.6A$

$(12 + \xi)V = 11\Omega I_2 - 33\Omega I_3 = -3.3V - 33\Omega 1.6A = -56.1V$ $\xi = -68.1V$ i.e. same sign as the 6 V battery. If we leave in the internal impedance $r = 1\Omega$, this becomes $\xi = -69.7V$.

(e) [4 points] What would be the current $I_1$, if the 18-$\Omega$ resistor were shorted out (replaced with a conductor)? (Use the original circuit as a starting place.)

Answer: We can get the same equations as before with the 18$\Omega$ resistor replaced with zero ohms:

$V_{12V} - I_1(R_{12} + R_1 + R_8) + V_{12V} + I_2(R_1 + R_{10}) = 0 = 24V - 21\Omega I_1 - 11\Omega I_2$ or $24V = 21\Omega I_1 + 11\Omega I_2$

$V_{12V} - I_2(R_1 + R_{10}) + I_3(R_1 + R_{15}) - V_{6V} = 0 = 6V - 11\Omega I_2 + 16\Omega I_3$ or $18V = 11\Omega I_2 - 16\Omega I_3$.

$I_1 - I_2 - I_3 = 0$ or $I_1 = I_2 + I_3$

Solving this is fairly straightforward. Substituting the thirst equation into the first equation gives $24V = 21\Omega(I_2 + I_3) + 11\Omega I_2$. Regrouping gives two equations in two unknowns:

$24V = 32\Omega I_2 + 21\Omega I_3$.

$6V = 11\Omega I_2 - 16\Omega I_3$ Multiply by (21/16) yields

$7.8750V = 14.4375\Omega I_2 - 21I_3$

Adding the two equations together gives $31.8750V = 46.4375\Omega I_2$ or $I_2 = 0.6864A$

Multiply by (32/11) gives $17.4545V = 32\Omega I_2 - 46.5455 I_3$ subtracting gives $6.5455V =$

$119.9091 I_3$ or $I_3 = 0.0546A$

$I_1 = I_2 + I_3 = 0.7410A = 0.783A?$

7. [15 points] Twelve resistors, each of resistance $R$ are connected as the edges of a cube. Determine the equivalent resistance

*Hint:* Apply an emf and determine currents; use symmetry at junctions.

(a) [5 points] between two ends of any resistor. (i.e. two corners of one edge).

Answer: We must make use of the symmetry to solve this easily and quickly. There is one path with resistance R and two with resistance 3 R all in parallel. The next resistance is $\frac{1}{r} = \frac{1}{R} + \frac{1}{3R} + \frac{1}{3R}$ So that $r = \frac{3}{5}R$ Doing it correctly with current and voltage loops $\frac{7}{12}R$ The other resistors span across identical by symmetry potentials so that they carry no current.

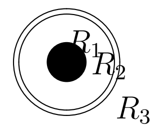
(b) [5 points] between any two opposite corners of the same face of the cube. (i.e. the diagonal corners on a face of the cube)

Answer: We must make use of the symmetry to solve this easily and quickly. There are two paths with two resistors in series and one with two resistors in series plus two - two in series in parallel. First the effective resistance of the third path $r_3 = R + R + R = 3R$ The parallel resistance is $\frac{1}{r} = \frac{1}{2R} + \frac{1}{3R} + \frac{1}{2R} = \frac{8}{6R}$ the result is then $r = \frac{3}{4}R$

(c) [5 points] between the opposite volume diagonal corners. (i.e. to get from one corner to the other one must go through at least 3 resistors.)

Answer: This is a very symmetric case. every path leaving one of the diagonal corners is equivalent. Thus the current divides in 3 and each current divides again to two resistors which combine back to three resistors. The first and last level have resistance R/3 and the middle resistance is R/6. The total resistance is 5 R / 6.

8. [20 points] A solid metal sphere of radius $R_1$ is surrounded by a concentric spherical shell of inner radius $R_2$ and outer radius $R_3$. Initially, there is no charge on either sphere. A charge $+Q$ is now placed on the inner sphere via a very fine insulated wire. (Assume that




the potential at infinity and grounded is zero.)

(a) [4 points] What is the potential of the inner sphere?

Answer: This is a simple application of Gauss's law and the fact that the electric field is zero in a conductor. We make a spherical Gaussian surface just into the metal (conductor) and the net flux is zero and thus the enclosed charge is zero. Thus the charge on the inner surface must be $-Q$ and the charge on the outer conductor must be $+Q$.

$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right]$

(b) [4 points] Suppose that the shell is momentarily grounded. Now what is the potential of the inner sphere?

Answer: All the other charge is bled off leaving only $-Q$ and $+Q$.

$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

(c) [4 points] Finally, after that momentary grounding, the inner sphere is momentarily grounded. What is now the potential of the shell?

Answer: Not all the charge runs out of inner sphere since grounded means zero potential.

$V_i = \frac{1}{4\pi\epsilon_0}\left[\frac{Q'}{R_1} - \frac{Q'}{R_2} - \frac{(Q-Q')}{R_3}\right] = 0$

$Q'(1/R_1 - 1/R_2 + 1/R_3) = Q/R_3$

$Q' = R_1R_2/(R_1R_2 - R_1R_3 + R_2R_3)Q$

$Q - Q' = R_3(R_2 - R_1)/(R_1R_2 - R_1R_3 + R_2R_3)Q$

$V = -(Q - Q')/4\pi\epsilon_0 R_3 = -Q/4\pi\epsilon_0(R_2 - R_1)/(R_1R_2 - R_1R_3 + R_2R_3)$

(d) [4 points] A dielectric material (water with dielectric constant K = 80) now fills the space between the sphere and shell. What is the capacitance of the system?

Answer: The capacitance is defined by the relationship $C = Q/V$. If this is part of a circuit then the inside sphere is one side of the capacitor and the outside of the shell is the other. That means we only need consider the effect of the inside charges. $V = \frac{Q}{4\pi\epsilon}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$C = \frac{4\pi\epsilon}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]} = 4\pi\epsilon\frac{R_1R_2}{R_1R_2} = K4\pi\epsilon_0\frac{R_2-R_1}{R_1R_2}$

(e) [4 points] The dielectric material has a bulk resistivity $\rho$. What is the resistance between the inner sphere and the outer shell?

Answer: The resistance of a differential shell has area $A = 4\pi r^2$ at radius $r$ and thus differential resistance of $dR = \rho dr/A = \rho dr/4\pi r^2$ Integrating from the inner sphere radius $R_1$ and inside of shell radius $R_2$ is $R = \frac{\rho}{4\pi}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\rho}{4\pi}\frac{R_2-R_1}{R_1R_2}$