a) clockwise (right hand rule)

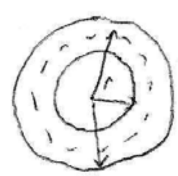
b) Use Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$ since the field lines must be circles

$I_{enc} = NI$ The loop encloses all turns

$B(2\pi r) = \mu_0 NI$

$\Rightarrow \boxed{B(r) = \dfrac{\mu_0 NI}{2\pi r}}$



c) Find flux through 1 loop.

$\Phi_0 = \int \vec{B}\cdot d\vec{A} = \int B\,dA$ $\quad dA = a\,dr$

$= \int \frac{\mu_0 NI}{2\pi r} a\,dr$

There are several choices for the bounds. If R is the inner radius, then go from R to $R+a$. If R is the outer radius, go from $R-a$ to R. If R is the radius to the center of the square cross section, go from $R-a/2$ to $R+a/2$. I'll do the latter.

$$\Phi_0 = \int_{R-a/2}^{R+a/2} \frac{\mu_0 NI}{2\pi r} a\,dr = \frac{\mu_0 NIa}{2\pi} \ln\left(\frac{R+a/2}{R-a/2}\right)$$

The flux through all the loops is

$$\Phi = N\Phi_0 = \frac{\mu_0 N^2 Ia}{2\pi} \ln\left(\frac{R+a/2}{R-a/2}\right)$$

Using $\Phi = LI$,

$$\boxed{L = \frac{\mu_0 N^2 a}{2\pi} \ln\left(\frac{R + a/2}{R - a/2}\right)}$$

If $a \ll R$, a simpler method works since B is approximately uniform across the square cross section. In this case, the flux through a loop is

$$\Phi_0 = B a^2 = \frac{\mu_0 N I a^2}{2\pi R}$$

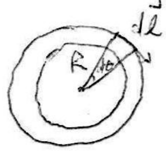
$$\Rightarrow \Phi = N \Phi_0 = \frac{\mu_0 N^2 I a^2}{2\pi R} \Rightarrow \boxed{L = \frac{\mu_0 N^2 a^2}{2\pi R}}$$

However, you were not told this approximation was valid for part c).

d) There is a net current of I that must flow clockwise around the loop. This produces a field

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

pointing downward. If you did not have this formula, it can be derived from the Biot-Savart Law.



$$d\vec{l} = -R\,d\theta\,\hat{\theta} \qquad r = R$$

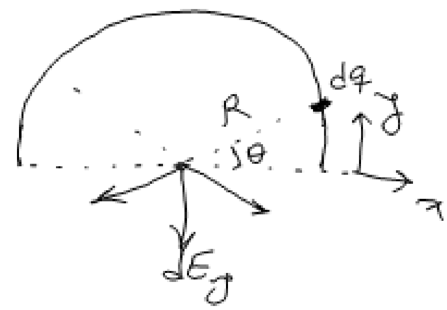
$$d\vec{l} \times \hat{r} = -R\,d\theta\,\hat{z}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{l} \times \hat{r}}{r^2} = -\frac{\mu_0}{4\pi} \frac{I R\,d\theta}{R^2} \hat{z}$$

$$\vec{B} = \int_0^{2\pi} -\frac{\mu_0 I\,d\theta}{4\pi R} \hat{z} = -\frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\theta\,\hat{z} = -\frac{\mu_0 I}{4\pi R}(2\pi)\hat{z}$$

$$\boxed{\vec{B} = -\frac{\mu_0 I}{2R} \hat{z}}$$ the same as above.

Lec 001 / Final / Problem 2



by symmetry $E_x = 0$

$$E_y = \int dE_y$$

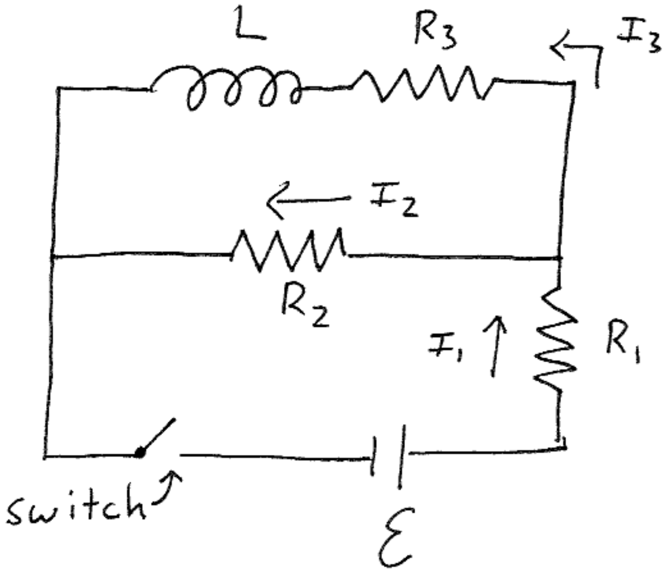
$$= \frac{1}{4\pi\epsilon_0}\frac{1}{R^2}\int dq \sin\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{R^2}\int \lambda R\, d\theta \sin\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda}{R}\underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2}$$

$$\boxed{E_y = \frac{\lambda}{2\pi\epsilon_0 R}} \downarrow$$

3.)



Physics 7B
Spring 2012
Professor Huang
Final Exam
#3 Solution

	I_1	I_2	I_3
a.) Just after switch is closed	$\frac{\mathcal{E}}{R_1+R_2}$	$\frac{\mathcal{E}}{R_1+R_2}$	0
b.) Long after switch is closed.	$\frac{\mathcal{E}}{R_1+\frac{1}{\frac{1}{R_2}+\frac{1}{R_3}}}$	$I_1 \cdot \frac{R_3}{R_2+R_3}$	$I_1 \cdot \frac{R_2}{R_2+R_3}$
c.) Just after switch is re-opened	0	Negative I_3 $\longrightarrow$	Same as in part (b)
d.) Long after switch is re-opened	0	0	0

• 1 pt. each $\Rightarrow$ 12 pts. total.

Physics 7B
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#4 Solution

4.) Energy density of photon gas:

$$\frac{U}{V} = A \cdot (k_B \cdot T)^4$$

$$\Rightarrow U = A \cdot V \cdot (k_B T)^4$$

10 pts: a.) $dU = dQ - \cancel{dW}$ //First Law

0, since process is isochoric

$dU = T \cdot dS$ //Using $dQ = T \cdot dS$

$A \cdot V \cdot k_B^4 \cdot 4 \cdot T^3 \cdot dT = T \cdot dS$ // By above expression for U

$$\Rightarrow \boxed{dS = 4 \cdot A \cdot V \cdot k_B^4 \cdot T^2 \cdot dT}$$

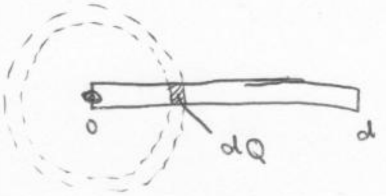
5 pts: b.) $S(T) = \int dS = \int_0^T 4 \cdot A \cdot V \cdot k_B^4 \cdot T'^2 \cdot dT'$

$$= 4 \cdot A \cdot V \cdot k_B^4 \cdot \int_0^T T'^2 \cdot dT'$$

$$\boxed{S(T) = \frac{4}{3} A \cdot V \cdot k_B^4 \cdot T^3}$$

MT001

#5



Tot $\frac{\text{Charge}}{\text{length}} = \frac{Q}{d}$

$$dQ = \left(\frac{Q}{d}\right) dr \quad ; \quad r : 0 \to d$$

current due to chunk dQ:

$$dQ = \left(\frac{Q}{d}\right) dr$$

$$dI = \frac{dQ}{dt} = dQ \left(\frac{\text{velocity}}{\text{distance}}\right)$$

linear velocity $V = \omega r$

$$= dQ \left(\frac{\omega r}{2\pi r}\right)$$

$$dI = dQ \frac{\omega}{2\pi} = \frac{Q}{d} \frac{\omega}{2\pi} dr$$

Area of loop : $A = \pi r^2$ at any r.

$d\mu = dI(A)$ ← moment due to current dI

$$= \frac{Q}{d} \frac{\omega}{2\pi} dr \left(\pi r^2\right)$$

$$= \frac{\omega Q}{2d} r^2 dr$$

Tot magnetic moment:

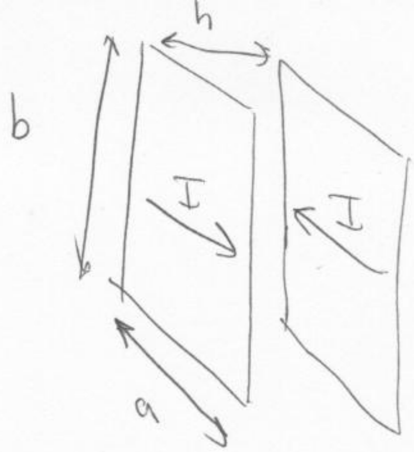
$$\mu = \int d\mu = \int_{r=0}^{d} \frac{\omega Q}{2d} r^2 dr$$

$$= \frac{\omega Q}{2d} \left(\frac{1}{3} r^3\right)\Big|_0^d$$

$$= \frac{\omega Q}{2 \cdot 3 d} d^3$$

$$\boxed{\mu = \frac{\omega Q}{6} d^2}$$

MT 001
#6



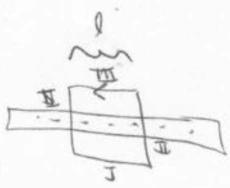
a) $\vec{B}$ direction + magnitude

$\vec{B}$ is up

(Lenz's Law)

Magnitude: use ampère law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



for one sheet use this loop

$$\oint \vec{B}\cdot d\vec{l} = \int_{I} B\,dl + \underbrace{\cancel{\int_{II} \vec{B}\cdot d\vec{l}}}_{0} + \int_{III} B\,dl + \underbrace{\cancel{\int_{IV} \vec{B}\cdot d\vec{l}}}_{0}$$

$$= 2Bl$$

$$2Bl = \mu_0 \underbrace{\left(\frac{I}{b}\right)}_{\text{current per length}} l$$

$$B_{sheet} = \frac{\mu_0 I}{2b}$$

for 2 sheets, B follows superposition

$$B = 2B_{sheet} = \boxed{\frac{\mu_0 I}{b}}$$

b) $u = \frac{B^2}{2\mu_0}$

$$U = u(\text{volume}) = u(bah) = \frac{1}{2\mu_0}\,\frac{\mu_0^2 I^2}{b^2}\,bah$$

$$\boxed{U = \frac{\mu_0}{2} I^2 \frac{ah}{b}}$$

c) $\Phi = LI$

$$\Phi = \int \vec{B}\cdot d\vec{A} = BA_{\text{cross section}} = Bah$$

cross section $\perp$ to $\vec{B}$

$$L = \frac{\Phi}{I} = \frac{\left(\frac{\mu_0 I}{b}\right)ah}{I} \quad ; \quad L = \boxed{\frac{\mu_0 ah}{b}}$$

d) $E = \frac{1}{2} L I^2 = \boxed{\frac{1}{2}\mu_0 \frac{ah}{b} I^2}$