Prob. 1 Solution FOS MI1 lec

& grading scheme.

(✱) "NA" marked on exam for no answer given.

(a) ($\frac{1}{2}$ pt each)

(i) (T) (definition)

(ii) (F) (1 mole N_2 occupies 22 L at STP. So, 4L air will be $\approx \frac{1}{6}$ mol)

(iii) (F) ($\dot{Q} \propto T^4$. If $T \to 1.01T$; $\dot{Q}_{new} \to (1.01)^4 T^4 = 1.04 T^4 = 1.04 \dot{Q}_{old}$)
So, 4%

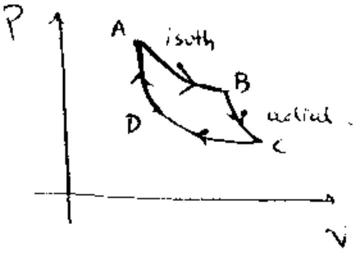
(iv) (T) (E_{int} is a state function → ΔE is path independent)

(v) (T) (If material expands; work is being done by it.
Some part of the heat input (at const. Pressure) is used to do this work
At const. volume; All the heat goes into raising T.
∴ For same ΔT, C_p will be $> C_v$.

(vi) (F) ($C_v = \frac{d}{2}R$ & $C_p = (\frac{d}{2}+1)R$. Both are greater for polyatomic

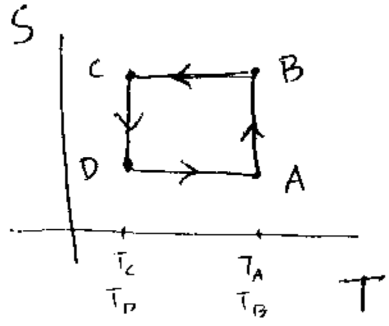
(vii) (T) (well, it is!)

(viii) (T) (



For AB & CD;
T = const (each point)
& ΔS = +ve (AB)
& ΔS = −ve (CD)
For BC & AD;
$Q = 0$ at each point
∴ S = const ($\Delta S = 0$)

$\longrightarrow$



(ix) (F)

(x) (T) (that's what the theorem states!)

(b) (1pt each)

(i) A B C D (all → (E)) Partial credit ($\frac{1}{2}$ pt) if at least 2 were circled

(ii) (C)

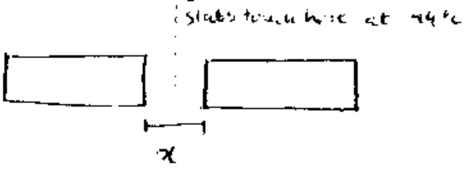
(iii) [D and E] → full credit [just D] → full credit
[just E] → $\frac{1}{2}$ pt (not AS important as D)

(iv) (E) ($\Delta l = l_0 \alpha \Delta T = (2737\text{ m})(12 \times 10^{-6}\ \frac{1}{^\circ C})(50^\circ C) \simeq 1.6\text{m}$)
= 160cm

(v) (C) (No change → density will change)

(C) (1pt each)

(i) D.



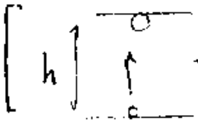
Slab expansion: $\Delta \ell = \ell_0 \alpha (T_{max} - 15°C)$

& $\frac{\Delta \ell}{2} = \frac{x}{2} \Rightarrow x = \Delta \ell$

$= (15)(12 \times 10^{-6})(29)$

$\approx 5.2 \times 10^{-3}$ m

(ii) D.



$P_2 = P_{atm} \sim 10^5 Pa$, V_2, T_2

$P_1 = P_{atm} + h\rho g = 3.8 \times 10^5 + 10^5 \sim 4.8 \times 10^5 Pa$, V_1, T_1

$T_1 = 5.4°C = 278.6 K$

$T_2 = 18.5°C = 291.7 K$

$V_1 = 1.16 cm^3$

moles in bubble = const.

$n = \frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$

$V_2 = V_1 \times \frac{T_2}{T_1} \times \frac{P_1}{P_2} = 5.8 cm^3$

(iii) C. When Temp. rises, Hg expands more than Fe ($\beta_{Hg} > \beta_{Fe}$). Since masses remain the same, density of Hg goes down MORE than that of Fe. So, Fe sinks lower.

(iv) A. (Expansion for normal materials is isotropic).

(v) F. $\Delta \ell = \ell_0 \alpha \Delta T = (160 m)(14.2 \times 10^{-6} (°C)^{-1})(-150°C - 100°C) = -0.568 m$ (decrease).

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d) (1pt each)

Partial credit given in EXCEPTIONAL cases if work was shown neatly and only an unimportant part was wrong (this was RARE)!

(i) E. $v_{rms} = \sqrt{\frac{3kT}{m}}$ (given on front of exam)

m = mass of particle $= m_{N_2} = (14 \text{ amu}) \times 2$

$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293.15}{28 \times 1.6605 \times 10^{-27}}}$

1 amu $= 1.6605 \times 10^{-27}$ kg (given on pg. 3. prob 1(a)(i))

≈ 502 m/s $\simeq 500$ m/s (2 sig figs) ← doesn't matter so much here.

OR: $v_{rms} = \sqrt{\frac{3RT}{M}}$ (M = molar mass = 28 gms/mol = 28×10^{-3} kg/mol. $R = 8.314$ J/mol.K).

(same answer)

(ii) B. $\ell_M = \frac{1}{\sqrt{2}\, 4\pi r^2 \frac{N}{V}}$, r = radius!! $\frac{N}{V} = \frac{P}{kT}$ (ideal gas law)

$\Rightarrow r^2 = \frac{kT}{\sqrt{2}\, 4\pi P \ell_M}$

$\therefore r = \sqrt{\left[\frac{1.38 \times 10^{-23} \times 273.15}{\sqrt{2} \times 4\pi \times 1.01 \times 10^5 \times 25 \times 10^{-3}}\right]} = \sqrt{0.8 \times 10^{-20}} \simeq 9.2 \times 10^{-11} m$

$\therefore \boxed{d = 2r \approx 1.8 \times 10^{-10} m}$

(d) Contd.

(iii) Ⓑ. $W_{\text{isothermal}} = \int_{v_1}^{v_2} P\,dV = \int_{v_1}^{v_2} \frac{nRT}{V}\,dV = nRT \ln\frac{V_2}{V_1} = nRT\ln 2$ $(V_2 = V_1)$

isothermal T = const.

(iv) Ⓒ Isothermal, reversible $\Rightarrow \Delta S = \frac{Q}{T} = \frac{\overset{0}{\cancel{\Delta E}} + W}{T} = \frac{nRT\ln 2}{T} = \underline{\underline{nR\ln 2}}$.

(isothermal ΔE = 0)

(v) Ⓔ or Ⓕ. (or both)

$\Delta E = nC_v\Delta T$ (any process)

$= 0$ (isothermal $\rightarrow \Delta T = 0$)

(e) 1pt each.

(i) Ⓐ In thermal equilibrium, the temperatures are the same.

↳ measure of [translational K·E]$_{\text{average}}$

$$\boxed{\underset{(\text{translational})}{K\cdot E_{avg}} = \frac{3}{2}kT}$$ Any ideal gas.

↳ just x, y, z linear motion.

$K\cdot E_{\text{total}} = \frac{d}{2}kT$ ← THIS depends on degs. of freedom.

(ii) Ⓑ. 1st law → energy conservation (not violated)

3rd law → not relevant - He's not trying to reach absolute 0

(iii) Ⓔ. Temperature constant at 0°C = 273.15K.

$\Delta S = \frac{Q_{\text{added}}}{T} = \frac{mL}{T} = \frac{1\,\text{kg}\cdot 333\left(\frac{\text{kJ}}{\text{kg}}\right)}{273.15\,\text{K}} = 1219\ \frac{\text{J}}{\text{K}}$

(iv) Ⓙ All correct (Google them to check).

(v) Ⓔ. 1 kcal = 4.18×10^3 J (front of exam) ←

∴ 1 cal = 4.18 J → Temp = 300K (large volume ⇒ this doesn't change temp. by much)

$\Delta S = \frac{4.18\,\text{J}}{300\,\text{K}} = k_B \ln\left(\frac{\Omega_F}{\Omega_i}\right)$

$\Rightarrow \frac{\Omega_F}{\Omega_i} = \exp\left[\frac{4.18}{1.38\times10^{-23}\times300}\right] = \exp\left[\frac{0.01}{10^{-23}}\right] = \exp\left[10^{21}\right] = e^{10^{21}}$

END

END

2)

a. By allowing the blocks to come to a thermal equilibrium through heat transfer alone, we can set up a calorimetry problem.

$$\sum_i Q_i = 0$$

$$Q_A + Q_B = 0$$

$$m_A c_A (T_F - T_A) + m_B c_B (T_F - T_B) = 0$$

$m_A = m = m_B$

$c_A = c = c_B$

$$2T_F - (T_A + T_B) = 0 \rightarrow \boxed{T_F = \frac{T_A + T_B}{2}} = \frac{100°C + 0°C}{2} = 50°C = \boxed{323 K = T_F}$$

b. Now we allow the blocks to come to thermal equilibrium, but by extracting work in the form of a reversible engine. The fact that the engine operates reversibly is critical to this solution.

We know that since entropy is a state function $\Delta S^{cycle} = 0$ of the working substance, and this means that $|\Delta S_{in}| = |\Delta S_{out}|$, otherwise there would be a net increase/decrease in entropy over the course of a cycle.

Since the engine operates reversibly, we can use the relation

$$dS = \frac{\text{đ}Q}{T}$$

We also find $\Delta S = \int dS$, so

$$\Delta S = \int \frac{\text{đ}Q}{T}$$

Using $Q = mc\Delta T$, we then see $\text{đ}Q = mc\,dT$

Putting these together, we find

$$|\Delta S_{in}| = |\Delta S_{out}|$$

(if you don't like the absolute value, then use $\Delta S_{in} + \Delta S_{out} = 0$ to obtain the same result)

$$\left| \int \frac{\text{đ}Q_H}{T_H} \right| = \left| \int \frac{\text{đ}Q_L}{T_L} \right|$$

$$\left| \cancel{mc} \int_{T_A}^{T_F} \frac{dT_H}{T_H} \right| = \left| \cancel{mc} \int_{T_B}^{T_F} \frac{dT_L}{T_L} \right|$$

$$\left| \ln\left(\frac{T_F}{T_A}\right) \right| = \left| \ln\left(\frac{T_F}{T_B}\right) \right|$$

$$\ln\left(\frac{T_A}{T_F}\right) = \ln\left(\frac{T_F}{T_B}\right)$$

exponentiate both sides

$$\frac{T_A}{T_F} = \frac{T_F}{T_B} \rightarrow T_F^2 = T_A T_B$$

$$\boxed{T_F = \sqrt{T_A T_B}} = \sqrt{(373 K)(273 K)} \approx \boxed{319.1 K = T_F}$$

Common mistakes:

- many students said $T_F = 50°C$, but this assumes no work is done. The point of a heat engine is to do positive work!
- many students also assumed the Carnot efficiency, and some referenced Carnot's theorem. The point of part (2) is to show how this differs from a Carnot engine. I will transcribe here Carnot's theorem from Chapter 20 of Giancoli:

2) b. continued...

Quote: "All reversible engines operating between the same two constant temperatures T_H and T_L have the same efficiency. Any irreversible engine operating betw the same two fixed temperatures will have an efficiency less than this."

Clearly, in this situation we do not have two constant temperatures, and thus this is not a carnot engine. A Reversible engine is not necessarily a Carnot engine, but it is a requirement to produce the maximum efficiency possible.

C. We know our input heat goes into producing ~~heat~~ work and waste heat, so

$$Q_H = W_{net} + Q_L$$

$$\hookrightarrow W_{net} = Q_H - Q_L \quad , \text{ where } \quad Q_H = |mc\Delta T_H| \quad Q_L = |mc\Delta T_L|$$

$$= mc\left[(T_A - T_F) - (T_F - T_B)\right]$$

$$\boxed{W_{net} = mc\,(T_A + T_B - 2T_F)}$$

$$= (1000\,\text{kg})\left(\frac{1000\,\text{J}}{\text{kg}^\circ\text{C}}\right)(373 + 273 - 2[319.1])$$

$$\boxed{W_{net} \simeq 7.8 \times 10^6\,\text{J}}$$

Common mistakes:

Several students had the incorrect sign for Q_H. Q_H must be a positive quantity because it is positive heat delivered to the working substance. Q_L is also defined with absolute value above because of how we arrive at $W_{net} = Q_H - Q_L$:

From the first law: $\Delta E_{int} = Q_{in} - W_{by}$, so over the course of a cycle

$$\Delta E_{int}^{net} = Q_{in}^{net} - W_{by}^{net} \quad , \quad \Delta E_{int}^{net} = 0 \text{ over a cycle (since } E_{int} \text{ is a State function)}$$

$$Q_{in}^{net} = W_{by}^{net}$$

And we define $Q_{in}^{net} \equiv Q_H - Q_L$, so that $W^{net} = Q_H - Q_L$

d. The net thermal efficiency for this engine is

$$e_{actual} = \frac{W_{net}}{Q_H} = \frac{mc\,(T_A + T_B - 2T_F)}{mc\,(T_A - T_F)} \text{ — from part (c)}$$

$$\boxed{e_{actual} = \frac{T_A + T_B - 2T_F}{T_A - T_F}} \simeq \boxed{0.15}$$

The efficiency of a carnot engine is

$$\boxed{e_{carnot} = 1 - \frac{T_B}{T_A} \simeq 0.27}$$

We see that $e_{carnot} > e_{actual}$, as should be the case.

3)

a. The maximum possible coefficient of performance for this refridgerator is when it runs on a Carnot-like Cycle where all the heat enters and leaves the system at constant temperatures, and the other steps are adiabatic.

In general, $COP = \frac{Q_L}{W_{net}} = \frac{Q_L}{Q_H - Q_L} = \frac{Q_L}{Q_L(\frac{Q_H}{Q_L} - 1)} = \frac{1}{\frac{Q_H}{Q_L} - 1} = COP$

Since entropy is a state function, we know that over the course of a cycle $\Delta S_{in} = \Delta S_{out}$, and since a carnot cycle is a reversible cycle,

$dS = \frac{dQ}{T} \rightarrow \Delta S = \int dS = \int \frac{dQ}{T}$

In a carnot cycle, all heat enters/leaves at a constant temperature, so

$\Delta S = \frac{1}{T} \int dQ = \frac{Q}{T}$, which means that

$\Delta S_{in} = \Delta S_{out}$ gives the relation

$\frac{Q_H}{T_H} = \frac{Q_L}{T_L} \rightarrow \frac{Q_H}{Q_L} = \frac{T_H}{T_L}$ (For a carnot Cycle)

Plugging this in above gives

$COP = \frac{1}{T_H/T_L - 1} = \boxed{\frac{T_L}{T_H - T_L} = COP_{max}} \approx \boxed{8.48}$

Common mistakes

Several students had the symbolic answer correct, but then used either temperatures in °C, or added T_H and T_L in the denominator (I suspect because they expected $COP < 1$)

b. First, a note on notation. Many students used Q_L or Q_H and W in this problem instead of $\dot{Q}_L$, $\dot{Q}_H$ and P. Notice that this problem deals with rates, and $\dot{Q}_L$, $\dot{Q}_H$, and P have units of $\frac{J}{s} = W$, whereas Q_L, Q_H, and W have units of J. The 'dot' typically signifies a quantity per unit time.

Here we want to find $\dot{Q}_L$, and we know the COP, so we use the relation

$COP = \frac{Q_L}{W} = \frac{\dot{Q}_L}{P}$, where $COP = F\left(\frac{T_L}{T_H - T_L}\right)$, so we write

$\boxed{\dot{Q}_L = P\,COP = PF\left(\frac{T_L}{T_H - T_L}\right)}$

$\dot{Q}_L = \left(\frac{3}{4}\text{hp}\right)\left(\frac{746\ J/s}{1\ \text{hp}}\right)(0.43)(8.48) \approx \boxed{2040\ J/s = \dot{Q}_L}$

3) c. Now we want to find $\dot{Q}_H$

$Q_H = W + Q_L$, so

$$\dot{Q}_H = P + \dot{Q}_L = \boxed{P\left(1 + F\left[\frac{T_L}{T_H - T_L}\right]\right) = \dot{Q}_H}$$

Another common solution was to use

$$COP = \frac{\dot{Q}_L}{P} \rightarrow P = \frac{\dot{Q}_L}{COP}$$, where $COP = F\left(\frac{T_L}{T_H - T_L}\right)$

$$\dot{Q}_H - \dot{Q}_L = \frac{\dot{Q}_L}{COP}$$

$$\downarrow$$

$$\boxed{\dot{Q}_H = \dot{Q}_L\left(1 + \frac{1}{COP}\right)}$$

$$= P\,COP\left(1 + \frac{1}{COP}\right)$$

$$\boxed{\dot{Q}_H = P(1 + COP)}$$

Any of these work, and they all give

$$\boxed{\dot{Q}_H \approx 2600\ J/s}$$

<u>Another Common mistake:</u>

Some students wrote

$\frac{a}{a-b} = 1 - \frac{a}{b}$, this is not true

$\frac{a-b}{a} = 1 - \frac{b}{a}$, so by inverting both sides

$\frac{a}{a-b} = \frac{1}{1 - b/a}$

d. We know from earlier, that

$\dot{Q}_L = PF\left(\frac{T_L}{T_H - T_L}\right)$, and from calorimetry that

$Q = m_w C_w \Delta T$ [illegible] $L_F m_w$, where $\Delta T < 0$

In terms of heat entering the ice, $\dot{Q}_L < 0$ as well, so our minus signs cancel, and the time required to freeze the ice (Δt) is

$$\boxed{\Delta t = \frac{m_w\left(C_w|\Delta T| + L_F\right)}{PF\left(\frac{T_L}{T_H - T_L}\right)} \approx 845\,s \approx 14 \text{ minutes} = \Delta t}$$

<u>common mistakes:</u>

Several students had the signs of the heat leaving the water for the cooling process and the freezing process opposite. In both instances, the heat flows in the same direction and so they must have the same sign.

e. Find ΔS_{air}, $\Delta S_{ice/water}$, ΔS^{min}_{uni}

ΔS_{air}: We assume the heat exhausted into the air is not enough to raise its temperature, so

$$\Delta S_{air} = \frac{Q_H}{T_H}, \text{ where } Q_H = \overset{\text{from(c)}}{\dot{Q}_H}\,\overset{\text{from(d)}}{\Delta t} \rightarrow \boxed{\Delta S_{air} = \frac{\dot{Q}_H \Delta t}{T_H}} \approx \boxed{7472\,\frac{J}{K} = \Delta S_{air}}$$

$$\Delta S_{ice/water} = \frac{Q_{Freeze}}{T_{Freeze}} + \int_{T_i}^{T_f} \frac{m_w C_w\, dT}{T} = \boxed{-\frac{L_F m_w}{273K} + m_w C_w \ln\left(\frac{273K}{T_w}\right) = \Delta S_{air/water}}$$

$T_w = 18°C = 291K$

$$\Delta S_{ice/water} = \left[-\frac{\left(333\frac{kJ}{kg}\right)}{273K} - \frac{4.18\,kJ}{kg \cdot K}\ln\left(\frac{291K}{273K}\right)\right](4.2\,kg) \approx \boxed{-6244\ J/K \approx \Delta S_{air/water}}$$

3) e. continued...

$$\boxed{\Delta S_{\text{uni}}^{\text{min}} = \Delta S_{\text{air}} + \Delta S_{\text{ice/water}}}$$

$$\boxed{\Delta S_{\text{uni}}^{\text{min}} = 1228\ \text{J/K}}$$

Common mistakes:

Again, people used opposite signs for the entropy change of the cooling and freezing processes of the ice/water. When water freezes, it becomes less entropic, and the same is true when water cools.

f. In order for the refrigerator to work at its maximum possible coefficient of performance, it must operate on a reversible cycle.

Therefore, $\boxed{\Delta S_{\text{univ}}^{\text{cycle}} = 0}$

To determine $\frac{t_{\text{ideal}}}{t_{\text{actual}}}$, we realize

$$COP_{\text{max}} = \frac{\dot{Q}_L^{\text{ideal}}}{P} = \frac{T_L}{T_H - T_L}$$

$$\dot{Q}_L^{\text{ideal}} = P\left(\frac{T_L}{T_H - T_L}\right)$$

$$\left.\begin{aligned} t_{\text{ideal}} &= \frac{Q}{\dot{Q}_L^{\text{ideal}}} \\ t_{\text{actual}} &= \frac{Q}{\dot{Q}_L^{\text{actual}}} \end{aligned}\right\} \frac{t_{\text{ideal}}}{t_{\text{actual}}} = \frac{\dot{Q}_L^{\text{actual}}}{\dot{Q}_L^{\text{ideal}}} = \frac{F\cancel{P}\cancel{\left(\frac{T_L}{T_H - T_L}\right)}}{\cancel{P}\cancel{\left(\frac{T_L}{T_H - T_L}\right)}} = \boxed{F = \frac{t_{\text{ideal}}}{t_{\text{actual}}}} = \boxed{0.43}$$

from (b)

So that $t_{\text{ideal}} \doteq (0.43)(14\ \text{min})$
$\doteq 6$ minutes

Common mistakes:

- Some students found $\frac{t_{\text{ideal}}}{t_{\text{actual}}} = \frac{1}{F}$. Remember, if the refrigerator operates more efficiently, it will draw more heat out for the same work put in, so the ideal time will be shorter.
- $\Delta S_{\text{univ}}^{\text{cycle}} = 0$ because the engine operates on a cycle, and entropy is a state function. $\Delta S^{\text{cycle}} = 0$ is always true for the working substance, but $\Delta S_{\text{univ}} = \Delta S_{\substack{\text{working}\\\text{substance}}} + \Delta S_{\text{environme}}$ and $\Delta S_{\text{environment}} = -\Delta S_{\substack{\text{working}\\\text{substance}}}$ is only true for reversible processes, thus $\Delta S_{\text{univ}} = 0$ only for reversible processes.

Physics 7B Fall 2008: Lecture 3 Midterm 1, Prob 4

Data

$$
\begin{gathered}
A = 25cm^2 \quad T_h = 100^\circ C \quad T_c = 0^\circ C \\
L_{Cu} = 26.0cm \quad L_{Al} = 33.0cm \\
k_{Cu} = 400\frac{W}{m\cdot^\circ C} \quad k_{Al} = 240\frac{W}{m\cdot^\circ C}
\end{gathered}
$$

(a) Interface temperature

The heat flows through each section must be equal to each other and to the overall heat flow through the entire composite bar at steady state. If they aren't equal, there would be a net heat dump or removal at the interface, which would raise or lower the temperature there (which wouldn't be steady state). Let the temperature at the interface be T. Then, the equations for heat conduction become:

$$
\begin{aligned}
\frac{dQ}{dt}\|_{Cu} &= \frac{dQ}{dt}\|_{Al} \qquad \text{at steady-state.} \\
\frac{k_{Cu}A(T_h - T)}{L_{Cu}} &= \frac{k_{Al}A(T - T_c)}{L_{Al}} \qquad (1) \\
\frac{k_{Cu}}{L_{Cu}}T_h + \frac{k_{Al}}{L_{Al}}T_c &= T\left(\frac{k_{Cu}}{L_{Cu}} + \frac{k_{Al}}{L_{Al}}\right)
\end{aligned}
$$

Finally,

$$\boxed{T = \frac{\frac{k_{Cu}}{L_{Cu}}T_h + \frac{k_{Al}}{L_{Al}}T_c}{\left(\frac{k_{Cu}}{L_{Cu}} + \frac{k_{Al}}{L_{Al}}\right)}} \qquad (2)$$

Numerically,

$$T = \frac{\frac{400W/m\cdot^\circ C}{0.26m}100^\circ C + \frac{240W/m\cdot^\circ C}{0.33m}0^\circ C}{\left(\frac{400W/m\cdot^\circ C}{0.26m} + \frac{240W/m\cdot^\circ C}{0.33m}\right)}$$

$$\boxed{T = 68^\circ C = 3.4 \times 10^2 K} \qquad (3)$$

(2 sig figs)

Grading scheme:
1pt: Showing that you know WHY the two heat flows are equal (steady state), SOMEWHERE in the writeup.
1pt: Correct order of the 2 temperature differences
1pt: final symbolic answer
2pt: final numeric answer. Work not necessary (but useful for partial credit here).

(b) For $T_{interface} = 50°C$

We can directly rearrange eq. 1 to get L_{Al}:

$$\boxed{L_{Al} = \frac{k_{Al}}{k_{Cu}} \frac{(T - T_c)}{(T_h - T)} L_{Cu}} \tag{4}$$

Numerically, for the interface temperature $T = 50°C$:

$$\boxed{L_{Al} = \frac{240W/m \cdot^{\circ} C}{400W/m \cdot^{\circ} C} \frac{(50°C - 0°C)}{(100°C - 50°C)} 26cm = 15.6cm \approx 16cm} \tag{5}$$

(2 sig figs)

Grading scheme:
1pt: Correct order of the 2 temperature differences
2pt: final symbolic answer. Partial credit given depending on how much numerical work was shown.
2pt: final numeric answer. Work not necessary (but useful for partial credit here).

(c) Heat flow along the original rods

Again, we can use the original eq. 1 directly (any one of the 2 expressions is fine):

$$\boxed{\frac{dQ}{dt}\|_{Al} = \frac{dQ}{dt}\|_{Cu} = \frac{k_{Cu} A (T_h - T)}{L_{Cu}}} \tag{6}$$

Numerically,

$$\boxed{\frac{dQ}{dt}\|_{Al} = \frac{dQ}{dt}\|_{Cu} = \frac{(400W/m \cdot^{\circ} C)(25 \times 10^{-4} m^2)(100°C - 67.9°C)}{0.26m} = 123.5W \approx 0.12kW} \tag{7}$$

(2 sig figs)

Grading scheme:
3pt: final symbolic answer (any one expression, with proper quantities - NOT generic T's, L's, etc.)
Again, a LOT of explicit numeric plugging-in gave you some partial credit here.
2pt: final numeric answer. Work not necessary (but useful for partial credit here). Missing/incorrect units are penalized.

(d) Rate of entropy production in the process

At steady state, the rod itself is merely acting as a conduit for the heat flow and absorbs no net heat. The heat flow calculated in (c) is being lost ($\dot{Q}_h$) by the hot reservoir (maintained at constant temperature T_h) and is gained ($\dot{Q}_c$) by the cold reservoir maintained at constant temperature (T_c). So, $\dot{Q}_h = -|\dot{Q}|$ and $\dot{Q}_c = |\dot{Q}|$, where $|\dot{Q}|$ is the absolute value of the heat flow from (c).

The total rate of entropy change for the entire system is then:

$$\begin{aligned}\dot{S} &= \dot{S}_h + \dot{S_{rods}} + \dot{S}_c \\ &= \frac{\dot{Q}_h}{T_h} + 0 + \frac{\dot{Q}_c}{T_c}\end{aligned}$$

So,

$$\boxed{\dot{(S)} = |\dot{Q}| \left(\frac{1}{T_c} - \frac{1}{T_h} \right)} \tag{8}$$

Numerically,

$$\boxed{\dot{(S)} = 0.12 \times 10^3 J/s \left(\frac{1}{273.15K} - \frac{1}{373.15K} \right) \approx 0.12 \frac{J/K}{s}} \tag{9}$$

Grading scheme:
2pt: Saying SOMETHING about why the rods don't contribute to the entropy production and justifying the signs used for the heat flows.
1pt: final symbolic answer. Credit given only if symbols are defined/make sense.
2pt: final numeric answer. Work not necessary (but useful for partial credit here). Again, units are important.

(e) Scaling questions

If all linear dimensions of the rods are increased by a factor of 2, the cross-section area would go up by a factor of 4 and the length would go up by a factor of 2. There is an additional factor that must be considered (this can be done in two equally correct ways):

1. If you use the heat flow equation (eq. 6) derived in part (c) to analyse this question, you have to comment on why the interface temperature (T) doesn't change when you change ALL the linear dimensions. Just concluding this (by inspection) from your symbolic answer for part (a) (eq. 2) would be sufficient (notice that if both the Cu and Al lengths change by the SAME factor there, T is unchanged).
2. An equivalent way to solve the entire problem (but NO one did it this way) was to calculate the effective R-value of the composite rod and write the heat flow equation in terms of this effective R-value. Then of course, you would have had to show why this effective R-value didn't change with a scaling of all the dimensions.

With these 3 ingredients, we see that the heat flow would INCREASE by a factor of 2 overall.

Grading scheme:
1pt: Showing that the effective R-value or the interface temperature do NOT change (read above).
1pt: Knowing that $L \rightarrow 2L$
1pt: Seeing that $A \rightarrow 4A$
2pt: Final conclusion - heat flow increases by factor of 2.

Common mistakes

If you have any of these, chances are that I agonized over your grade to begin with so it most likely won't change. Also, any time you see grades crossed out/changed on your exam, I spent a LOT of time on those, so the chances that there was a grading mistake are pretty slim. Also, remember that where partial credit is concerned, I judged your understanding of the problem as a whole and rewarded good explanations.

(a) No justification for equating heat flows. Symbolic work not done/no symbolic answer. Incorrect numeric answer (bad units, wrong substitution, etc.)

(b) No symbolic answer given. Using numerical heat flow from part (a). Claiming that the total length stays the same (no reason for this, but a lot of people did this).

(c) Not writing symbolic answers. Forgetting to convert the area and/or length to SI units/converting incorrectly. Not writing units on your answers (general mistake for all parts).

(d) Very few got this correct. The most common mistake was writing a strange integral and obtaining a natural log of temperatures. This received no credit. Only a handful got full credit on this one.

(e) Only 1 person got full credit. Most common mistake was forgetting about $T_{interface}$ (described in the solution). Second most common mistake was forgetting that area would also change. Partial credit was given to wrong answers only if some explanation was offered.

(d) [5 points] Water vapor and CO_2 added to the atmosphere increase the emissivity. CO_2 increases the emissivity in the infrared (where the Earth's thermal radiation peaks) according to $\epsilon CO_2 = A + B * ln(C/Co)$, where C is the increased atmospheric density of CO_2 compared to the pre-driving density of CO_2 called Co. For this problem assume that $A = 0.05$, $B = 0.04$, and $Co = 300$ppmv (parts per million by volume) from pre-1960. It is 383 ppmv now. How much has the infrared emissivity of the atmosphere increased due to the CO_2 increase between 1960 and now? What would be the corresponding temperature rise for the Earth?

$\epsilon CO_2 = A + B * ln(C/Co) = 0.05 + 0.04 * ln(383/300)$

$\Delta\epsilon = 0.04 * ln(383/300) = 0.00977$ about 1%

The change in temperature $\epsilon_o T^4 = (\epsilon_0 - \Delta\epsilon)(T + \delta T)^4$

$\delta T \sim 0.75C$

(e) [2 points] The Greenhouse Effect is more difficult to calculate when including water vapor because there is positive feedback. A temperature rise of the ground, and especially of oceans and lakes causes water vapor in the atmosphere to increase. (So also is reflecting cloud cover.) For a 1°C rise from 14°C by how much would the water vapor fraction rise? The saturated vapor pressure of water at 14°C and 15°C are about 1.6×10^3Pa (= N/m^2) and 1.71×10^3Pa (= N/m^2) respectively and an average relative humidity for the earth is about 50%. What would be fractional content of the atmosphere in water vapor by volume?

$0.5 \times 1.17 \times 10^3 / 1.013 \times 10^5 = 0.0058$

(f) [5 points] Why does the water vapor partial pressure rise increasingly rapidly with temperature? See attached figure and provide a rough physical argument/reason and an approximate formula.

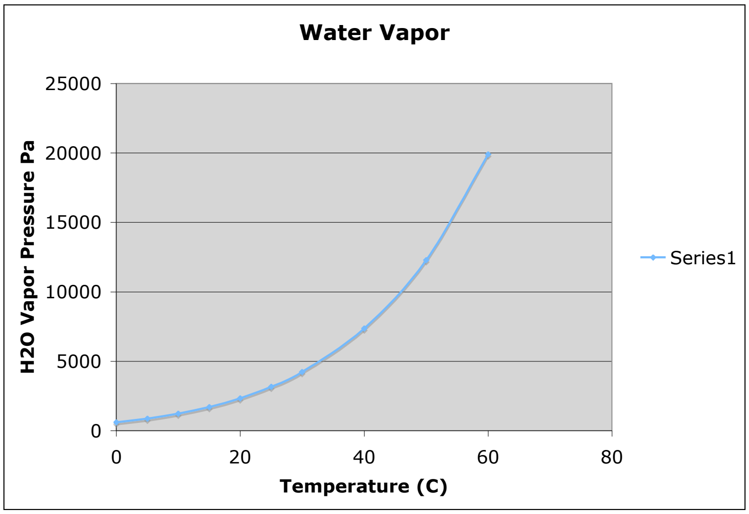

Figure 3: Water Vapor Partial Pressure Versus Temperature.

Boltzmann factor $e^{-\Delta E/kT}$ =153843000000*EXP(-5284/(A12+273.15)) fit by making a semilog. $f_{H_2O} = Aexp{-}B/T$ so that $ln(f_{H_2O}) = ln(A) - B/T$ pick two values and solve for ln(A) and B.

(g) [3 points] At the same temperature, a column of dry air will be denser or heavier than a column of air containing any water vapor. Thus, any volume of dry air will sink if placed in a larger volume of moist air. Also, a volume of moist air will rise or be buoyant if placed in a larger region of dry air. As the temperature rises the proportion water vapor in the air increases, its buoyancy will become larger. This increase in buoyancy can have a signicant atmospheric impact, giving rise to powerful, moisture rich, upward air currents when the air temperature and sea temperature reaches 25°C or above. This phenomenon provides a significant motivating force for cyclonic and anticyclonic weather systems (tornados and hurricanes).

What is the difference in density at 15°C (1.71×10^3Pa) between dry and saturated air? How about at 25°C (3170 Pa), what is the difference in density? What is the difference in pressure in the two cases? What is the expected wind speed?

Solution: Water vapor content is at 15°C is 1.71×10^3Pa and at 25°C is (3170 Pa). One standard atmosphere is 1.013×10^5 So the fractional differences is

$f(T = 15°C) = 1.71 \times 10^3/1.013 \times 10^5 = 1.688 \times 10^{-2}$ and

$f(T = 25°C) = 3.17 \times 10^3/1.013 \times 10^5 = 3.13 \times 10^{-2}$.

The mean molecular weight of air is $MW = 0.8 \times 28 + 0.2 \times 32 = 28.8$ The molecular weight of the water vapor is 18. Thus the displaced difference is 10.8 in molecular weight or fraction = 10.8/28.8 = 0.375.

The difference in density is

$\Delta\rho(T = 15°C) = f(T = 15°C) \times 0.375 \times \rho_0 = 1.688 \times 10^{-2} \times 0.375 \times 1kg/m^3 = 0.00633$

$\Delta\rho(T = 25°C) = f(T = 25°C) \times 0.375 \times \rho_0 = 3.13 \times 10^{-2} \times 0.375 \times 1kg/m^3 = 0.0117$

The difference in pressure is then

$\Delta P(T = 15°C) = 1710 \times 0.375$ Pa = 641 Pa

$\Delta P(T = 25°C) = 3170 \times 0.375$ Pa = 1190 Pa

Use Bernoulli equation (energy conservation changed to energy density conservation)

KE + PE = constant → KE/Volume + PE/Volume = constant

$\frac{1}{2}\rho v^2 + \Delta P + P_0 = P_0 = constant$

$v = \sqrt{2\Delta P/\rho} = \sqrt{2 \times 641 Pa/1kg/m^3}$ = 36 m/s = 128 km/hr = 77 mi/hr

$v = \sqrt{2\Delta P/\rho} = \sqrt{2 \times 1190 Pa/1kg/m^3}$ = 49 m/s = 176 km/hr = 106 mi/hr