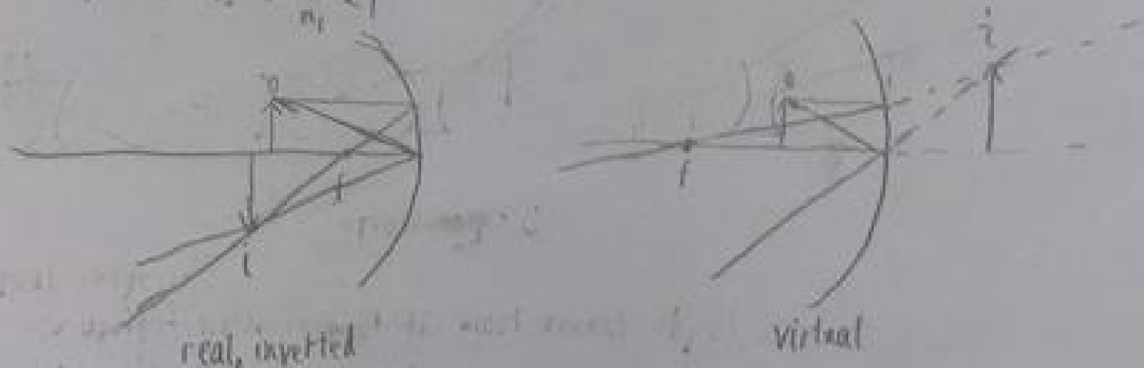
1a no, total internal reflection requires $n_2 \leq n_1$

$n_1 \sin\theta = n_2 \to \frac{n_2}{n_1} < 1$

1b



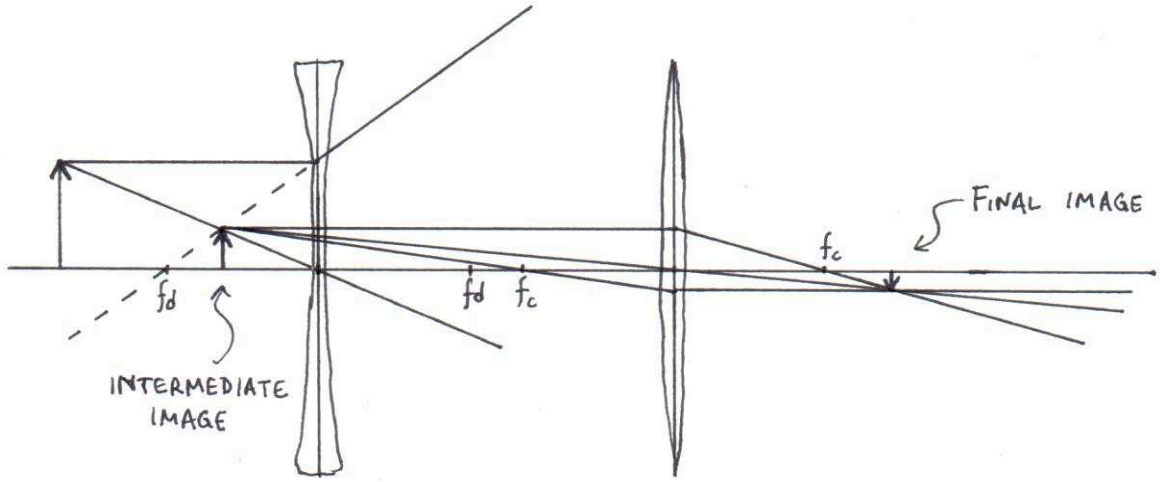
(allow upright if in conjunction w/ virtual object

1c holding $kx - \omega t$ constant, increasing t by δt increases x by $\frac{\omega}{k}\delta x$
(constant position on wavefront)

therefore $v = +\frac{\omega}{k}\hat{x}$

1d yes, the two curvitures enter symmetrically

2) a)



b) $\dfrac{1}{f_d} = \dfrac{1}{d_{o,d}} + \dfrac{1}{d_{i,d}} \Rightarrow \dfrac{1}{f_d} = \dfrac{d_{o,d} + d_{i,d}}{d_{o,d} \cdot d_{i,d}}$

$$\Rightarrow f_d = \frac{d_{o,d} \cdot d_{i,d}}{d_{o,d} + d_{i,d}} = \frac{d_1 d_{i,d}}{d_1 + d_{i,d}} \qquad [1]$$

$d_{o,c} = d_2 - d_{i,d}$

$$\Rightarrow \frac{1}{f_c} = \frac{1}{d_{o,c}} + \frac{1}{d_{i,c}} = \frac{1}{d_2 - d_{i,d}} + \frac{1}{d_3}$$

$$\Rightarrow \frac{1}{d_2 - d_{i,d}} = \frac{1}{f_c} - \frac{1}{d_3} = \frac{d_3 - f_c}{d_3 f_c}$$

$$\Rightarrow d_2 - d_{i,d} = \frac{d_3 f_c}{d_3 - f_c}$$

$$\Rightarrow d_{i,d} = d_2 - \frac{d_3 f_c}{d_3 - f_c}$$

Plug into [1] above:

$$f_d = \frac{d_1\left(d_2 - \frac{d_3 f_c}{d_3 - f_c}\right)}{d_1 + d_2 - \frac{d_3 f_c}{d_3 - f_c}} = \frac{d_1\left[d_2(d_3 - f_c) - d_3 f_c\right]}{d_1(d_3 - f_c) + d_2(d_3 - f_c) - d_3 f_c}$$

$$= \frac{d_1(d_2 d_3 - d_2 f_c - d_3 f_c)}{d_3(d_1 + d_2) - f_c(d_1 + d_2 + d_3)} \qquad \blacksquare$$

NOTE: Due to sign ambiguity for f_d, <u>overall</u> sign differences were allowed in final result. Individual terms needed the same <u>relative</u> sign as shown above.

2) c) $M_d = \frac{-d_{i,d}}{d_{o,d}} = \frac{-d_{i,d}}{d_1}$

$$M_c = \frac{-d_{i,c}}{d_{o,c}} = \frac{-d_3}{d_2 - d_{i,d}}$$

$$M_{sys} = M_d M_c = \frac{d_3(d_{i,d})}{d_1(d_2 - d_{i,d})}$$

$$\frac{1}{f_d} = \frac{1}{d_{o,d}} + \frac{1}{d_{i,d}} \Rightarrow \frac{1}{d_{i,d}} = \frac{1}{f_d} - \frac{1}{d_{o,d}} = \frac{d_1 - f_d}{d_1 f_d}$$

$$\Rightarrow d_{i,d} = \frac{d_1 f_d}{d_1 - f_d}$$

$$\Rightarrow M_{sys} = \frac{d_3\left(\frac{d_1 f_d}{d_1 - f_d}\right)}{d_1\left(d_2 - \left(\frac{d_1 f_d}{d_1 - f_d}\right)\right)}$$

$$= \frac{d_1 d_3 f_d}{d_1\left(d_2(d_1 - f_d) - d_1 f_d\right)}$$

$$= \frac{\pm d_3 f_d}{d_1 d_2 \mp f_d(d_1 + d_2)} \quad \blacksquare$$

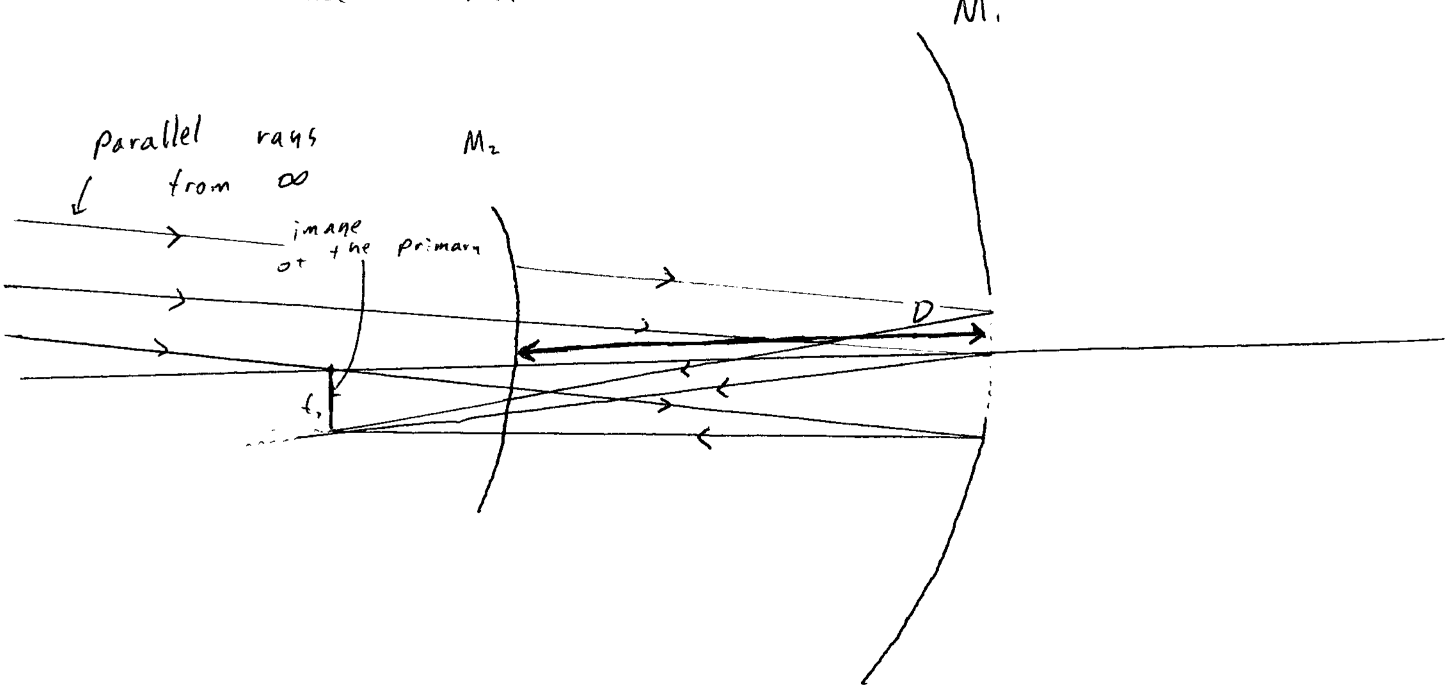
NOTE: Again, due to sign ambiguity for f_d, internal sign differences were not marked incorrect, provided it was consistent with prior work.

* Other solutions involving f_c, using a similar solving process as above, were possible.

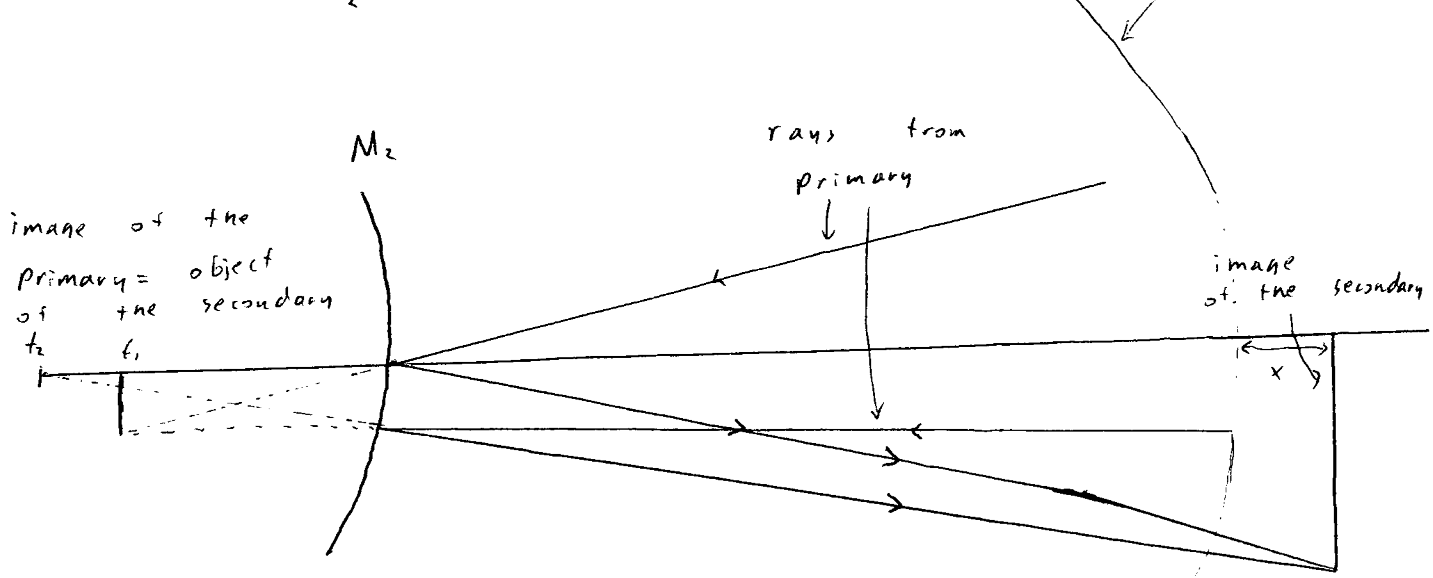
The radii of curvature are given - translate this to a focal length:

$$f_1 = \frac{R}{2} \qquad f_2 = -\frac{r}{2}$$

a) First trace M_1



Now trace M_2



From this: Image of the primary: real

Object of the secondary: virtual

[Image of the secondary is also real]

6) - First locate the image of the primary

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_1} \rightarrow \frac{1}{+\infty} + \frac{1}{d_i} = \frac{1}{f_1} = \frac{2}{R_1}$$

$$\rightarrow f_1 = \frac{R_1}{2}$$

- Next, figure out the object for the secondary

$$|d_o| = \frac{R_1}{2} - D$$

This is a virtual object so $d_o < 0$ ->

$$d_o = D - \frac{R_1}{2} = \frac{2D - R_1}{2}$$

f_2 is negative!

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_2} \rightarrow \frac{2}{2D - R_1} + \frac{1}{d_i} = -\frac{2}{r}$$

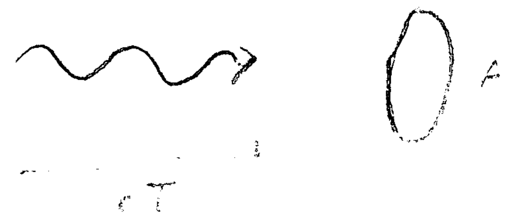
$$\frac{1}{d_i} = \left(-\frac{2}{r} - \frac{2}{2D - R_1} \right)$$

$$d_i = \left(\frac{2}{R_1 - 2D} - \frac{2}{r} \right)^{-1}$$

The distance from the primary to the final image is given by

$$x = d_i - D = \left(\frac{2}{R_1 - 2D} - \frac{2}{r} \right)^{-1} - D$$

4a)



In a time T, a volume of AcT of the wave will hit the sail. The energy is related to the energy density by a factor of the volume.

$$U_{abs} = AcT \langle u \rangle$$
$$= AcT \left[\tfrac{1}{2} \epsilon_0 E_0^2 \right]$$

$$E_0 = \sqrt{\frac{2 \, U_{abs}}{\epsilon_0 \, AcT}}$$

4b) The Poynting vector is the power per unit area. At a distance D, the Poynting vector has constant magnitude on a sphere of surface area $4\pi D^2$.

$$P = 4\pi D^2 \langle S \rangle$$
$$= 4\pi D^2 \left[\frac{1}{2\mu_0 c} E_0^2 \right]$$
$$= 4\pi D^2 \left[\frac{1}{2\mu_0 c} \frac{2 U_{abs}}{\epsilon_0 AcT} \right]$$
$$= \frac{4\pi D^2}{A} \frac{U_{abs}}{T}$$

4c)
$$-\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E}$$
$$= k E_0 \cos(kx - \omega t) \hat{z}$$
$$\vec{B} = \frac{k}{\omega} E_0 \sin(kx - \omega t) \hat{z}$$
$$= \frac{E_0}{c} \sin(kx - \omega t) \hat{z}$$

4d) $\vec{E}$ (and $\vec{B}$) must be perpendicular to the direction of propagation, which in this case is the $\hat{x}$ direction. Therefore, $\vec{E}$ can point anywhere in the y-z plane.

4e) No. To satisfy the wave equation, a plane wave must be a function of $(\vec{k} \cdot \vec{r} - \omega t)$; otherwise it would change [illegible] propagated