MATH 53 — MIDTERM #2

Each problem counts 20 points.

Problem #1. Calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

for the vector field $\mathbf{F} =< -y, x >$ and the half circle C given parametrically by $\mathbf{r}(t) =< \cos t, \sin t >$ for $0 \leq t \leq \pi$.

Problem #2. Find all solutions (x, y, z, λ) of the equations given by the Lagrange multiplier method for the problem of determining the points on the surface $z^2 = xy + 4$ closest to the origin.

Of these solutions, which give the points closest to the origin?

Problem #3. For the vector field $\mathbf{F} =< yz+x^2, xz+y, xy+z >$, compute the value of

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is any curve connecting the point $(1, 2, 0)$ to $(2, 0, 3)$.

Problem #4. Use the transformation

$$x = 2u,\ y = 4u + v$$

to evaluate the integral

$$\iint_R (2x(y - 2x))^{\frac{1}{2}}\ dA,$$

where R is the parallelogram with vertices $(0, 0)$, $(0, 1)$, $(2, 4)$, $(2, 5)$.

Problem #5. Find the volume of the solid lying within the sphere $\rho = 4\cos\phi$ and below the cone $\phi = \frac{\pi}{4}$.