$\boxed{1}$ Final Exam.

(a) Temperature is what's the same when 2 bodies are in contact for a long time.

(b) The spontaneous flow of energy from one body to another due to temperature differential.

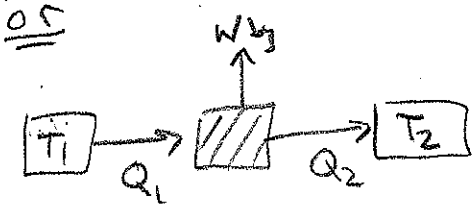
(c) $\Delta U = W_{on} + Q$; energy is conserved when you've accounted for all work on a body and heat.

(d) You can't take heat and turn it all into work with no other change.

or

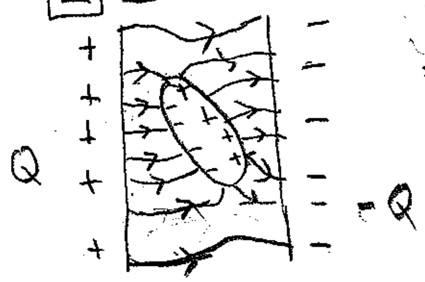
You can't take heat from cold ~~body and~~ give to hot body with no other change.

or



$Q_2 \neq 0$!

2 (a)

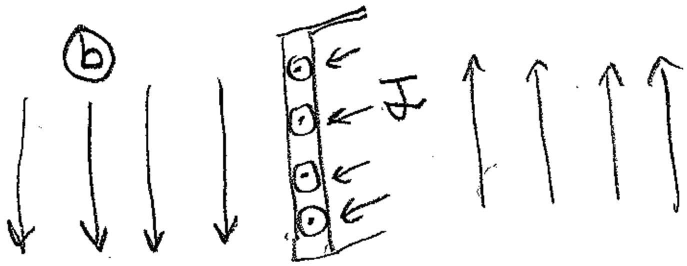


$\vec{E}=0$ in conductor.

$\vec{E} \perp$ to surface of conductors.

$\Delta \vec{E}_\perp = \frac{\sigma}{\epsilon_0}\hat{n}$ discontinuity in $\vec{E}_\perp$ due to presense of surface charge.

(b)

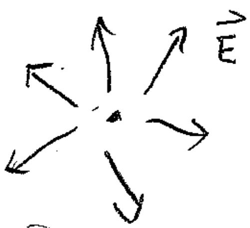


$\vec{B}_\parallel$ to surface of current sheet.

$\Delta \vec{B}_\parallel = \mu_0 J$

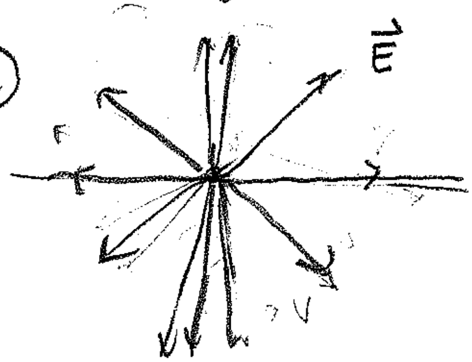
discontinuity in $\vec{B}_\parallel$ due to current sheet.

(ci)



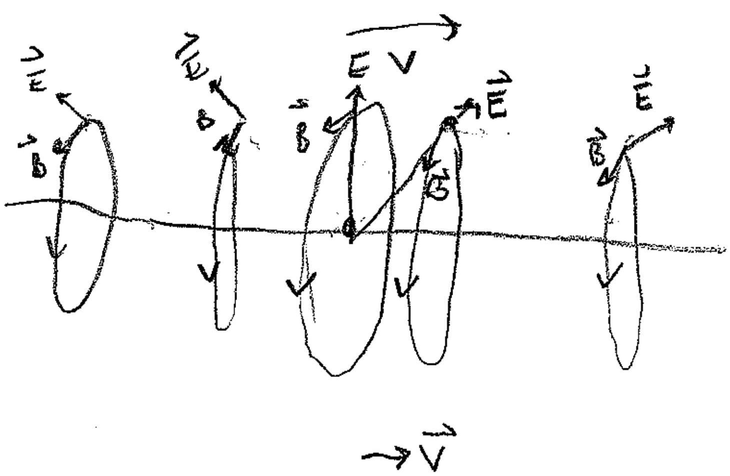
$\vec{B}$ is zero.

(cii)



$\vec{E}$ points from instantaneous position of particle.

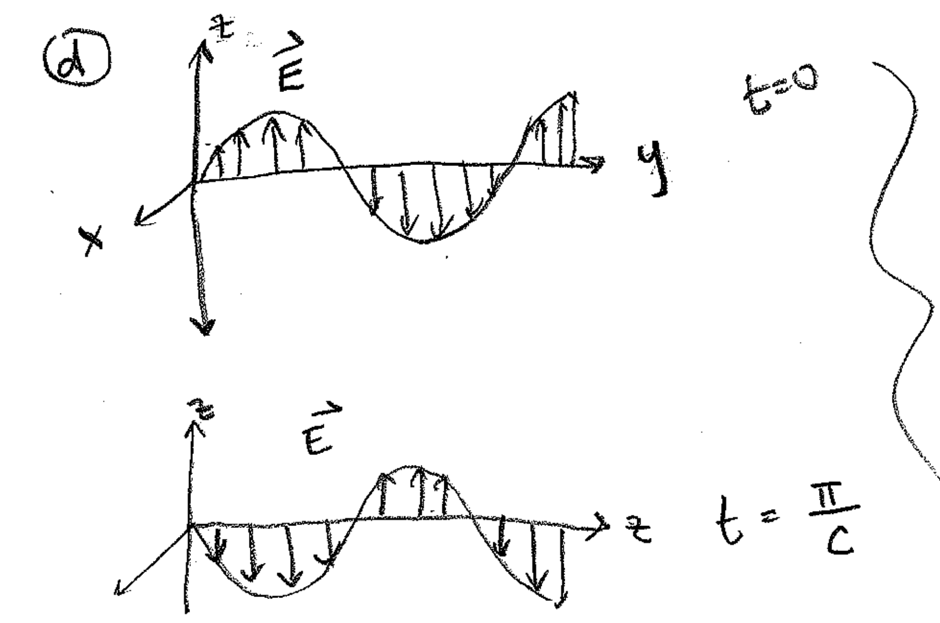
$\vec{E}$ is strongest in the plane $\perp$ to $\vec{v}$.



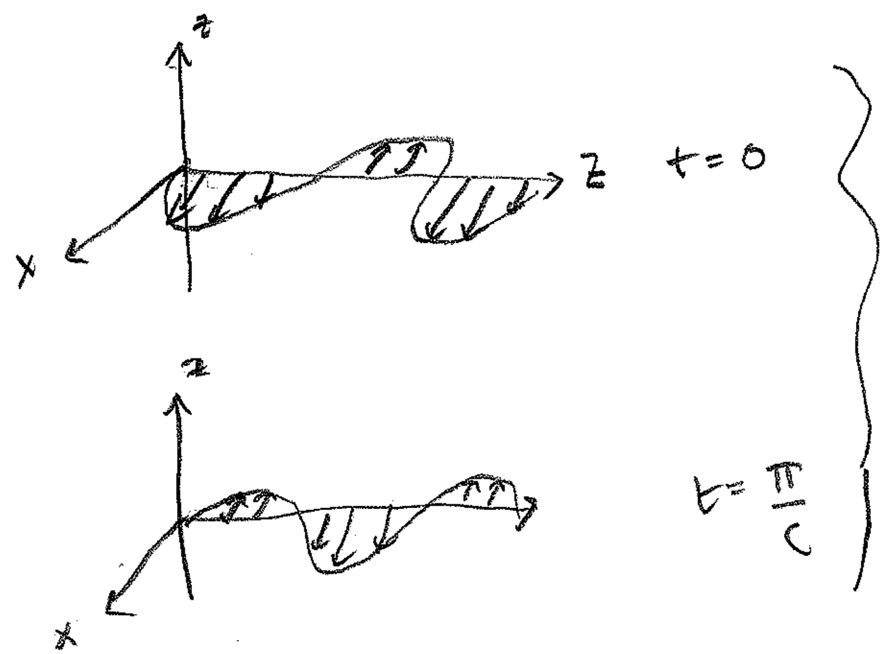
$$\vec{B} = \frac{\vec{v}}{c^2}\times\vec{E}$$

particle velocity

(d)



$\vec{E}$ same everywhere x-z plane,



$\vec{B}$ same everywhere in x-z plane.

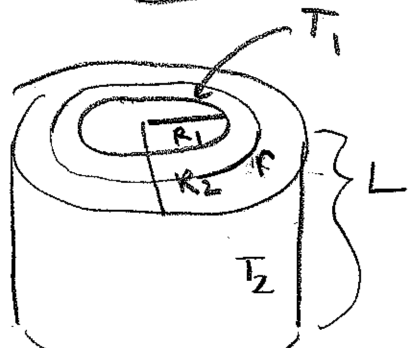
- Field pattern travels w/ speed c.
- $|\vec{E}| = c|\vec{B}|$ at every point in time in space.
- $\vec{E} \perp \vec{B} \perp$ direction of propagation.

3

(a) The analog of $\vec{h}$ is $\vec{E}$. The analog of s is ρ. The analog of T is ϕ. The electro-magnetic analog is electrostatics. We have:

$\vec{\nabla}\cdot\vec{h} = s$	$\vec{\nabla}\cdot\vec{E} = \left(\frac{\rho}{\epsilon_0}\right)$
$\vec{\nabla}\times\vec{h} = \vec{\nabla}\times(-k\vec{\nabla}T) = 0$	$\vec{\nabla}\times\vec{E} = 0$ (electro-statics)
$\vec{h} = -k\vec{\nabla}T$	$\vec{E} = -\vec{\nabla}\phi$ (electrostatics)

(b)



By gauss's theorem,

$$G = \int_V \vec{\nabla}\cdot\vec{h}\, dV = \oint_{S=\partial V} \vec{h}\cdot d\vec{a}$$

$$G = h\int \hat{r}\cdot d\vec{a} = h\, 2\pi r L$$

$$\vec{h} = -k\vec{\nabla}T$$

$$\vec{h} = -k\frac{dT}{dr}\hat{r}$$

$$G = -k\left(\frac{dT}{dr}\right) 2\pi r L$$

$$G\int_{R_1}^{R_2}\frac{dr}{r} = -2\pi K L\int_{T_1}^{T_2} dT$$

$$\boxed{G = \frac{2\pi K L\,(T_1 - T_2)}{\ln(R_2/R_1)}}$$

(c) G is the analog of charge Q, and we've also just solved a capacitor problem.

$\boxed{4}$

$PV = \frac{1}{3}U$ (5)

(a) for adiabatic processes, $Q=0$, so the first law gives

$$\Delta U = Q + W_{on} = W_{on} = -P\Delta V$$

$$dU = -P\,dV \quad (*)$$

From (5) we have

$$dU = 3P\,dV + 3V\,dP \quad (**)$$

Combining (*) and (**),

$$3P\,dV + 3V\,dP = -P\,dV$$

$$\int_{P_1}^{P_2} \frac{dP}{P} = -\frac{4}{3}\int_{V_1}^{V_2} \frac{dV}{V}$$

$$\ln\left(\frac{P_2}{P_1}\right) = -\frac{4}{3}\ln\left(\frac{V_2}{V_1}\right)$$

$$\boxed{PV^{4/3} = \text{Const}} \qquad \boxed{c = 4/3}$$

(b) $\left.\frac{dU}{dV}\right|_P = 3P$

(The general identity is $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$)

$$\left.\frac{dU}{dV}\right|_P = T\frac{dP}{dT} - P = 3P$$

$$4P = T\frac{dP}{dT}$$

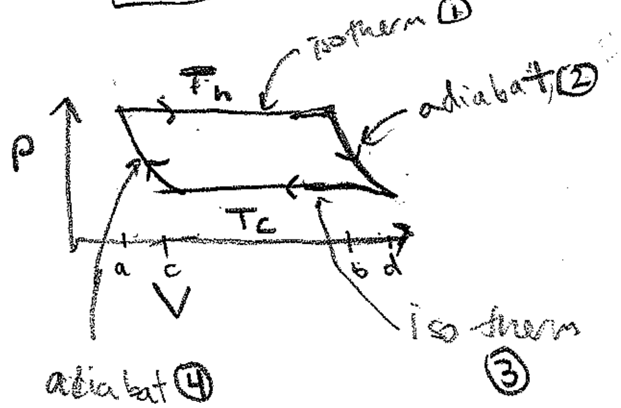
$$4\int_{T_1}^{T_2}\frac{dT}{T} = \int_{P_1}^{P_2}\frac{dP}{P}$$

$$4\ln\left(\frac{T_2}{T_1}\right) = \ln\frac{P_2}{P_1}$$

$$P = (\text{const})\,T^4 = \frac{1}{3}AT^4$$

$$\boxed{\frac{U}{V} = (\text{const})\,T^4 = A\,T^4}$$

(c)



Adiabats –
Photon gas adiabats have $P(V) = \frac{\text{const}}{V^{4/3}}$ while ideal gas adiabats have $P(V) = \frac{\text{const}}{V^{5/3}}$

Isotherms
for photon gas, Because $P \propto T^4$ is independent of V, isotherms are horizontal,
for ideal gas, isotherms have $P(V) = \frac{\text{const}}{V}$.

(d)

$$e = \frac{W_{by,TOT}}{Q_{in}}$$

since ΔU is a state variable, for the complete cycle,

$$\Delta U = Q_{TOT} - W_{by,TOT}$$

$$W_{by,TOT} = Q_{TOT},$$

$$Q_{TOT} = Q_{in}\ (\text{along } ①) + Q_{out}\ (\text{along } ③)$$

$$Q_{in} = \Delta U + P\Delta V = (A)(\Delta V)T_h^4 + (\tfrac{1}{3}A)(\Delta V)T_h^4$$

$$Q_{in} = (\tfrac{4}{3}A)\ T_h^4\ (V_b - V_a)$$

$$Q_{out} = \tfrac{4}{3}A\ T_c^4 (V_c - V_d)$$

Using $PV^{4/3} = \text{const}$ and $P = \text{const}\ T^4$

We also have $T^3V = \text{const}$ for adiabatic processes. This means that

$$T_h^3 V_b = T_c^3 V_d \qquad T_h^3 V_a = T_c^3 V_c$$

We can rewrite $Q_{out} = \tfrac{4}{3}A)\ T_c (T_h^3 V_a - T_h^3 V_b)$

$$W_{by,TOT} = \tfrac{4}{3}A\ T_h^4 (V_b - V_a)\left[1 - \frac{T_h}{T_c}\right]$$

the efficiency

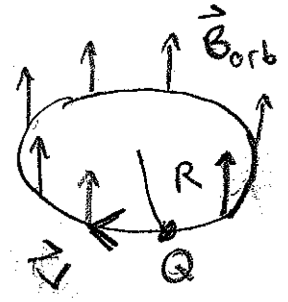
$$e = \frac{\frac{4}{3}A \cancel{T_h^4 (V_b - V_a)} \left[1 - \frac{T_h}{T_c}\right]}{\frac{4}{3}A \cdot \cancel{T_h^4} (V_b - V_a)} = 1 - \frac{T_h}{T_c}$$

This is the same as an ideal gas Carnot between T_h & T_c,

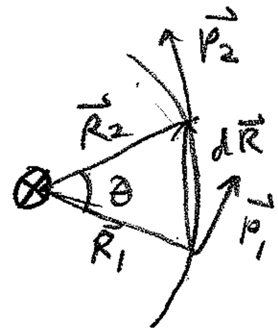
This is as you expect for a Carnot Cycle of any working substance.

(e) This process is reversible because all heat transfer occurs at constant T. equivalently, it has the maximum efficiency.

5



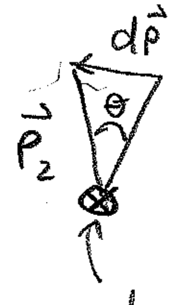
$\vec{B}_{orb} \perp \vec{v}$



Lorentz

(a) $\vec{F}_B = Q\vec{v} \times \vec{B} = -Q v B_{orb}\,\hat{r}$

$\frac{d\vec{p}}{dt} = \vec{F}$



$dp = F\,dt = Q v B_{orb}\,dt$

$\frac{|dp|}{|p|} = \frac{|dR|}{|R|}$

because $\vec{p} \perp \vec{r}$, θ is same,

$dp = \frac{p\,dR}{R} = \frac{p\,v\,dt}{R}$

$p\frac{v}{R}dt = Q v B_{orb}\,dt$

$$\boxed{R = \frac{p}{Q B_{orb}}}$$

Faraday

(b) $\varepsilon = \int_C \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} = \frac{d}{dt}\left(\int_{S,\,\partial S = C} \vec{B}\cdot d\vec{a}\right) = \frac{d}{dt}\left(B_{av}\cdot \pi R^2\right)$

$2\pi R E_{tan} = \pi R^2 \frac{dB_{AV}}{dt}$

$\vec{E} = \frac{R}{2}\frac{dB_{av}}{dt}\hat{\theta}$

$\vec{F} = Q\vec{E} = \frac{QR}{2}\frac{dB_{av}}{dt}\hat{\theta} = \frac{d\vec{p}}{dt}$

$$\boxed{\Delta p = \frac{QR}{2}\,\Delta B_{av}}$$ in time Δt,

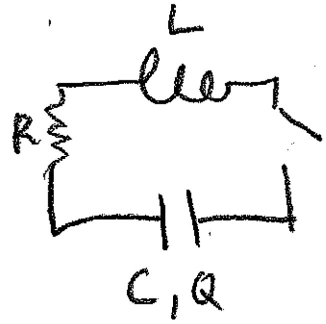
ⓒ Since for acircular orbit we must have, (part (a))

$$p = RQ\,B_{orb}$$

$$\Delta p = RQ\,\Delta B_{orb}$$

$$\boxed{\Delta B_{av} = 2\,\Delta B_{orb}}$$

6



underdamped

$$\frac{R^2}{4L^2} < \frac{1}{LC}$$

(a) $IR + L\frac{dI}{dt} + \frac{Q}{C} = 0$ ← no forcing

no particular sol$\overline{n}$.

$$\frac{dQ}{dt} = I$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$$

@ $t = 0$, $Q = Q_0 = c_1$

$c_2 = -\frac{\pi}{2}$

$A = \frac{R}{2L}$

$B^2 = \frac{1}{LC}$

$A^2 < B^2$ ☑

$$Q(t) = Q_0 e^{-At}\cos\left(\sqrt{B^2 - A^2}\, t\right)$$

$$\left.\begin{array}{l} Q(t=0) = Q_0 \\ Q(t=\infty) = 0 \end{array}\right\} ☑$$

(b) $\mathcal{U} = \mathcal{U}_E + \mathcal{U}_B$

$$\mathcal{U}_E = \frac{1}{2} C V^2 = \frac{1}{2C} Q^2(t)$$

$$\mathcal{U}_B = \frac{1}{2} L I^2(t)$$

(c) $\frac{d\mathcal{U}}{dt} = \frac{d\mathcal{U}_E}{dt} + \frac{d\mathcal{U}_B}{dt}$

$$\frac{d\mathcal{U}_E}{dt} = \frac{1}{C} Q \frac{dQ}{dt} = \frac{1}{C} Q I$$

$$\frac{d\mathcal{U}_B}{dt} = L I \frac{dI}{dt}$$

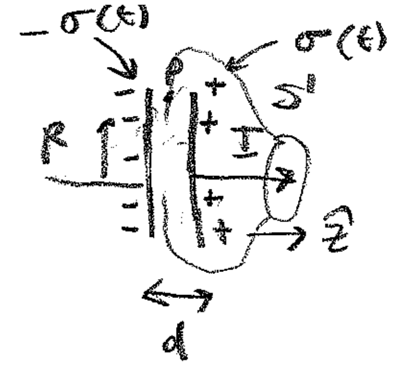
$$L \frac{dI}{dt} = -IR - \frac{Q}{C}$$ ← diff eqn

$$\frac{d\mathcal{U}}{dt} = \cancel{\frac{1}{C} Q I} + I\left(-IR - \cancel{\frac{Q}{C}}\right)$$

$$\boxed{\frac{d\mathcal{U}}{dt} = -I^2(t) R}$$

Good! your change in energy in your circuit is equal to the power dissipated across the resistor.

7



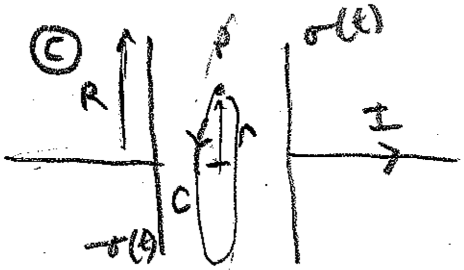
(a) $\vec{E} = -\frac{\sigma(t)}{\epsilon_0}\hat{z}$

(b) $\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = -\frac{\partial \sigma(t)}{\partial t}\hat{z}$ $\quad \left(\frac{\partial \sigma}{\partial t} < 0;\ \vec{J}_d \text{ points to right}\right)$

$$\Phi = \int_{S'} \vec{J}_d \cdot d\vec{A} = J_d A = \frac{\partial(\overbrace{\sigma A}^{Q})}{\partial t} = \frac{dQ}{dt} = I(t)$$

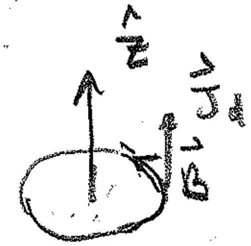
$\vec{J}_d$ only non-zero between plates

(c)



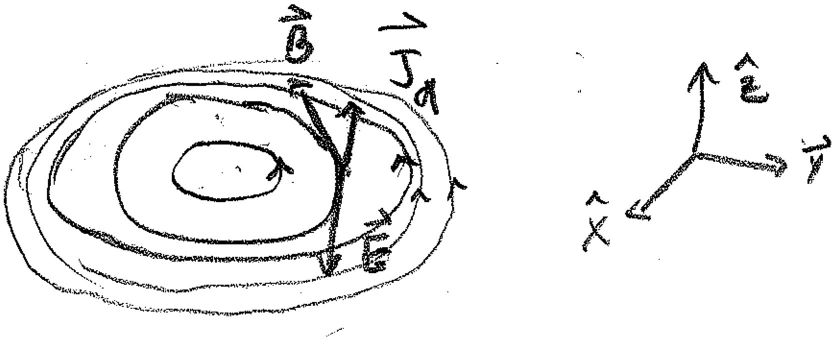
$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 \int_S \left(\vec{J}_d + \cancelto{0}{\vec{J}}\right) \cdot d\vec{a}$$

$$2\pi r B = \mu_0 \left(\frac{I(t)}{2\pi R^2}\right) 2\pi r^2$$



$$\boxed{\vec{B}(t, r) = \frac{\mu_0 I(t)\, r}{2\pi R^2}\hat{\theta}}$$

(d)



$\xrightarrow{\quad}$ $|\vec{B}|$ stronger w/ r,

(e) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ · Direction of Poynting gives direction of energy flow.

$\vec{S} \parallel \hat{r}$ since $-\hat{z}\times\hat{\theta} = \hat{r}$

(f) $S = \frac{1}{\cancel{\mu_0}}\frac{\sigma(t)}{\epsilon_0}\left(\frac{\cancel{\mu_0} I(t)}{2\pi R}\right)$ Poynting times area normal to it (flux of S) gives power, rate of change of stored energy.

$P = S\cdot 2\pi \cancel{R} d = \frac{\sigma(t) I(t) d}{\epsilon_0}$

$P = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{\epsilon_0}{2}(\text{Volume})\,E^2\right)$ ← rate of change of stored energy

$= \cancel{2}\frac{\epsilon_0}{\cancel{2}} A d\, E\cdot\frac{\partial E}{\partial t}$

$P = \cancel{\epsilon_0} A d\left(\frac{\sigma}{\epsilon_0}\right)\frac{\partial}{\partial t}\left(\frac{\sigma}{\cancel{\epsilon_0}}\right) = \frac{d}{\epsilon_0}\sigma(t)\overbrace{\frac{\partial(A\sigma)}{\partial t}}^{I}$

(e) They match!