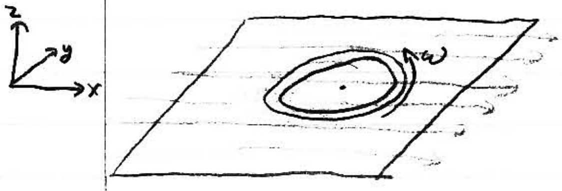
Packard - Fall '04

Section 2
Prob 1

Lecture 2

Solutions - Problem 1



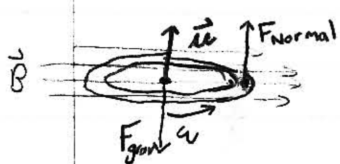
Mass M, charge Q, angular velocity ω, radius R

- Current: $I = \rho_e v = \frac{Q}{2\pi R}\cdot \omega R \Rightarrow I = \frac{Q\omega}{2\pi}$

- Magnetic Dipole: $\vec{\mu} = I \cdot A \hat{n}$ $\hat{n}$ is the normal vector

$$= \left(\frac{Q\omega}{2\pi}\right)(\pi R^2)\,\hat{z} = \frac{Q\omega R^2}{2}\hat{z}$$

Let's say that ω is such that there is only 1 point feeling a normal force, but the ring is still parallel to the plane.



Net force on loop is 0 by symmetry.
We also want net tourque to be 0.

Go Go Gadget 7A Knowledge! $\vec{\tau} = \sum \vec{r} \times \vec{F}$

Force: $F_{normal}\hat{z} + F_{grav}(-\hat{z}) = 0$ $\boxed{F_{normal} = Mg}$

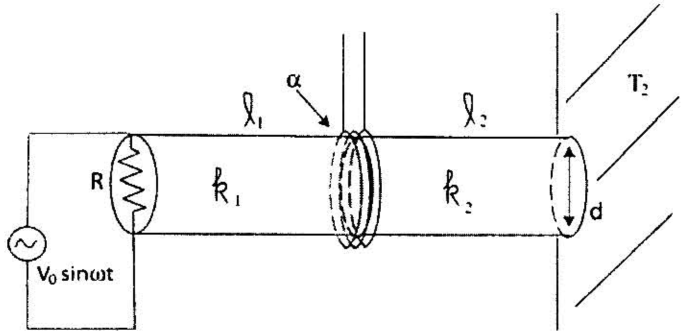
Tourques about Center of the ring:

- Due to dipole-B-field: $\vec{\mu}\times\vec{B} = \frac{Q\omega R^2}{2}\cdot B\,(\hat{z}\times\hat{x}) = \frac{QR^2\omega B}{2}\hat{y}$
- Due to gravity: $\vec{r}=0 \Rightarrow \tau_{grav} = 0$
- Due to normal force: $\tau_N = (R\hat{x})\times(Mg\hat{z}) = -RMg\,\hat{y}$

$$\Rightarrow \tau_N = \left(\frac{QR^2\omega B}{2} - RMg\right)\hat{y} = 0 \Rightarrow \frac{QR^2\omega B}{2} = RMg$$

$$\therefore \boxed{\omega = \frac{2Mg}{QRB}}$$

2. (50 points) A loop of wire is wrapped around the connection of two solid cylinders, one of length ℓ_1 and thermal conductivity k_1 and the second of length ℓ_2 and thermal conductivity k_2. Both cylinders have diameter d. An electric heater element of resistance R is pasted against end 1 so that all the power dissipated in it must flow down the cylinders. The heater is connected across an oscillating voltage $V_o \sin\omega t$. The temperature of end 2 is thermally fixed at temperature T_2. The wire loop at the cylinder's joint has a linear temperature coefficient of resistance α. Assume initially that when the heater is off, the entire bar is at temperature T_2. (a) find the temperature of the end where the heater is attached. (b) Find the fractional change in the wire loop's resistance when the heater is turned on and all temperatures stabilize.



Power delivered by resistor

$$P = I_{rms}^2 R = \frac{1}{2} I_0^2 R = \frac{1}{2}\frac{V_0^2}{R}$$

Rate of Heat flow through cylinders

$$\dot{Q} = \frac{KA}{\ell}\Delta T = \frac{1}{r}\Delta T \qquad r = \frac{\ell}{KA} \text{ thermal resistance} \qquad A = \pi\left(\frac{d}{2}\right)^2$$

2 resistors in series $\quad r_{Total} = r_1 + r_2 = \frac{\ell_1}{K_1 A} + \frac{\ell_2}{K_2 A} = \frac{4}{\pi d^2}\left(\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2}\right)$

$$\dot{Q} = \frac{\pi d^2}{4}\left(\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2}\right)^{-1}(T_1 - T_2) \qquad T_1 = \text{temperature of left end}$$

Power delivered by resistor = Rate of heat flow through cylinders

$$\frac{1}{2}\frac{V_0^2}{R} = \frac{\pi d^2}{4}\left(\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2}\right)^{-1}(T_1 - T_2)$$

a) $$\boxed{T_1 = \frac{2V_0^2}{\pi d^2 R}\left(\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2}\right) + T_2}$$

b) Consider heat flow through first cylinder

$$\frac{1}{2}\frac{V_0^2}{R} = \frac{K_1 \pi d^2}{4\ell_1}(T_1 - T_m) \qquad T_m = -\frac{2V_0^2 \ell_1}{\pi d^2 R K_1} + T_1 \qquad T_m = \text{temperature at middle}$$

$$\frac{\Delta\rho}{\rho} = \alpha\Delta T = \alpha(T_m - T_2) \xrightarrow{\text{algebra}} \boxed{\frac{\Delta\rho}{\rho} = \frac{2V_0^2 \alpha \ell_2}{\pi d^2 R K_2}} = \frac{\Delta R_{wire}}{R_{wire}}$$

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#3

(a) For the oscillation amplitude to drop to $\frac{1}{2}$ of its initial value,

$$e^{-\frac{Rt}{2L}} = \frac{1}{2}$$

as $Q = \underbrace{Q_0 e^{-Rt/2L}}_{\text{amplitude}} \cos(\omega' t + \phi)$

$$\Rightarrow t = \frac{2L}{R} \ln(2)$$

(15) points

(b) Initial Energy $= \frac{1}{2} \frac{Q_0^2}{C}$

Final Energy $= \frac{1}{2} \frac{(Q_0/2)^2}{C} = \frac{1}{8} \frac{Q_0^2}{C}$

$\therefore$ Energy lost through resistor $= \frac{1}{2} \frac{Q_0^2}{C} - \frac{1}{8} \frac{Q_0^2}{C}$

$$= \frac{3}{8} \frac{Q_0^2}{C}$$

(10) points

$$mc\,(T_f - T_i) = \frac{3}{8} \frac{Q_0^2}{C}$$

$$\Rightarrow T_f = \frac{3}{8 \rho V c} \cdot \frac{Q_0^2}{C} + T_i$$

(10) points

Entropy change, $dS = \frac{dQ}{T}$

$$\Delta S = \int dS = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln\left(\frac{T_f}{T_i}\right)$$

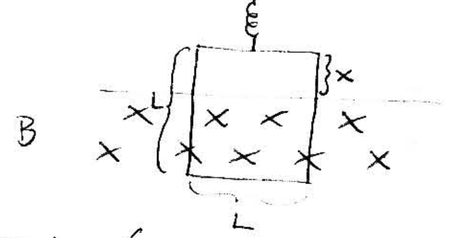
(10) points

$$= \rho V c \ln\left[1 + \frac{3}{8 \rho V c T_i} \frac{Q_0^2}{C}\right]$$

CORRECT ALGEBRA & CORRECT ANSWER (5) points

(15) + (10) + (10) + (10) + (5) = (50)

4. Damped oscillations mean that something is dissipating energy in this system. The only thing that can do that here is the resistance of the wire loop, so we'll need the force exerted on the current. This is given by the Lorentz force law, $\vec{F} = I\,\vec{\ell} \times \vec{B}$.



The Lorentz force acting on the two vertical portions of the loop will cancel, so we're only concerned about the bottom part:

$|F| = ILB$ since the wire is $\perp$ to $\vec{B}$.

We get I from Faraday's law:

$$I = \frac{1}{R}|\mathcal{E}|$$
$$= \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right|$$
$$= \frac{1}{R}\left|\frac{d}{dt}\left[BL(L-x)\right]\right|$$
$$= \frac{BL}{R}|\dot{x}| \leftarrow \text{ so } |F| = \frac{(BL)^2}{R}|\dot{x}|,$$ the term we wanted.

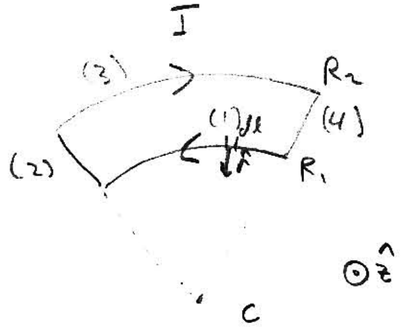
Finally, we need to calculate R:

$$R = \rho \cdot \frac{4L}{a}$$

so $|F| = \frac{a}{4L\rho}(BL)^2|\dot{x}|$, and if we write $F = -b\dot{x}$ we must identify

$$b = \frac{B^2 a L}{4\rho}$$

Problem 5



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\hat{\ell} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \sum_{i=1}^{4} \int_{(i)} \frac{d\hat{\ell} \times \hat{r}}{r^2}$$

$d\hat{\ell} \perp \hat{r}$

$$\int_{(1)} \frac{d\hat{\ell} \times \hat{r}}{r^2} = \int_{(1)} \frac{\hat{z}\, d\ell}{R_1^2}$$

$$= \int_{(1)} \frac{R_1\, d\theta}{R_1^2}$$

$$= \frac{1}{R_1} \int_0^{\theta} d\theta$$

$$= \frac{\theta}{R_1} \hat{z}$$

$$\Rightarrow \int_{(4)} \frac{d\hat{\ell} \times \hat{r}}{r^2} = \int_{(4)} \frac{-\hat{z}\, d\ell}{R_2^2}$$

$$= -\frac{\theta}{R_2} \hat{z}$$

$$\int_{(2)} \frac{d\hat{\ell} \times \hat{r}}{r^2} = \int_{(4)} \frac{d\hat{\ell} \times \hat{r}}{r^2} = 0 \quad (\text{b/c } d\hat{\ell} \perp \hat{r})$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \sum_{i=1}^{4} \int_{(i)} = \frac{\mu_0 I \hat{z}}{4\pi} \left(\frac{\theta}{R_1} + 0 - \frac{\theta}{R_2} + 0 \right)$$

$$= \frac{\mu_0 I \theta \hat{z}}{4\pi} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \boxed{\frac{\mu_0 I \theta (R_2 - R_1)}{4\pi R_1 R_2} \hat{z}}$$

6. 50 points.

a) For a wire of length l in a constant B field, the EMF is traveling at speed v.

$$\mathcal{E} = Blv$$

So for a wire of infinitesimal length,

$$\frac{d\mathcal{E}}{dl} = Bv \Rightarrow d\mathcal{E} = Bv\,dl$$

· For this system, B is a function of position:

$$B(x) = \frac{\mu_0 I}{2\pi x}$$

$$\Rightarrow d\mathcal{E} = v\frac{\mu_0 I}{2\pi}\frac{dx}{x}$$

$$\mathcal{E} = \int d\mathcal{E} = \int_{x=b}^{b+a} \frac{v\mu_0 I}{2\pi}\frac{dx}{x} = \frac{v\mu_0 I}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

This EMF must also be the EMF across the capacitor:

$$\mathcal{E} = V_{cap}$$

$$E_{cap} = \frac{V_{cap}}{d} = \boxed{\frac{v\mu_0 I}{2\pi d}\ln\left(\frac{b+a}{b}\right)}$$

b) $Q = CV = ?$

$C_0 = \frac{A\epsilon_0}{d}$ if the capacitor was filled with nothingness, i.e, a vacuum.

$$C = KC_0 = \frac{A\epsilon_0 k}{d}$$

$$Q = \frac{A\epsilon_0 k}{d}\,\frac{v\mu_0 I}{2\pi}\ln\left(\frac{b+a}{b}\right)$$