Final, Physics H7B

Dr. McCurdy, December 19, 2013

Please do all your work in a bluebook.

You will be graded on your solutions, and not just your answers. Show all work and thoroughly justify your solutions with figures, diagrams, equations, and words, as appropriate. Partial credit will be given to partially correct and/or partially complete solutions. No credit will be given to unjustified answers. Cross out any parts of solutions that you do not want to be graded.

Make sure that your answers to questions that ask for a vector quantity are given in the form of vectors (i.e. have vector components, or a magnitude and direction). Where appropriate, clearly label the choice of axes you are using and your choice of signs.

There are 7 problems and 150 possible points on the exam. Please read all 7 problems carefully at the beginning of the exam and attempt all problems to maximize your partial credit. You have 3 hours to complete the exam.

This is a closed-book exam. You may use three double-sided 3"×5" index card of notes. Calculators and electronic devices of any kind are not allowed.

On the front or your bluebook, write your:

- full name
- SID
- signature

Do not turn over the exam until you are told to do so. Good luck!!

$$
\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\
\vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \\
\vec{E}'_{||} &= \vec{E}_{||} \\
\vec{B}'_{||} &= \vec{B}_{||} \\
\vec{E}'_{\perp} &= \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B}) \\
\vec{B}'_{\perp} &= \gamma(\vec{B}_{\perp} - \frac{1}{c^2} \vec{v} \times \vec{E}) \\
x' &= \gamma(x - vt) \\
t' &= \gamma(t - \frac{v}{c^2} x) \\
\gamma &= (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \\
\frac{d^2 u}{dt^2} + 2A \frac{du}{dt} + B^2 u &= C sin(\omega t), \quad u(t) = u_c(t) + u_p(t), \quad A^2 < B^2 \\
u_c(t) &= c_1 e^{-At} sin(\sqrt{B^2 - A^2} + c_2), \quad A^2 < B^2 \\
u_p(t) &= \frac{C}{\sqrt{(B^2 - \omega^2)^2 + 4A^2\omega^2}} sin(\omega t - \phi) \\
and \quad tan\phi &= \frac{2A\omega}{B^2 - \omega^2} \\
\frac{\partial^2 u}{\partial t^2} &= v^2 \frac{\partial^2 u}{\partial x^2}, \quad u(x,t) = f(x + vt) + g(x - vt)
\end{aligned}
$$

1. **(8p)** Conceptual thermal physics questions. Answer in one sentence!

   (a) **(2p)** What is the fundamental definition of temperature? Partial credit if your answer requires kinetic theory.

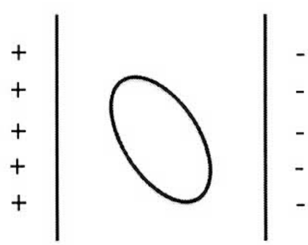
   (b) **(2p)** What is heat?

   (c) **(2p)** Give the statement of the first law of thermodynamics, and its physical interpretation. Define all terms.

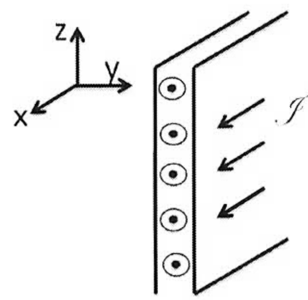
   (d) **(2p)** Give a statement of the second law of thermodynamics in terms of cyclic processes, heat, and work.

2. **(30p)** Drawing Electricity and Magnetism!

   (a) **(4p)** Draw electric field lines for a neutral oval conductor placed in a parallel plate capacitor of charge $\pm Q$. See Figure (1a). Briefly comment on boundary conditions.



(a) Problem (2a)



(b) Problem (2b)

Figure 1: Problem (2)

   (b) **(4p)** Draw the magnetic field on either side of an infinite steady current sheet with sheet current density $\mathcal{J}$ in the xz plane. See Figure (1b) Briefly comment on boundary conditions.

   (c) Draw the electric and magnetic field from a point charge $Q > 0$ in two frames. Briefly comment on any relationship between $\vec{E}$ and $\vec{B}$ in the following two frames.

      i. **(2p)** The rest frame of the point charge.

      ii. **(8p)** A frame where the point charge is moving with constant velocity $\vec{v} = v\hat{x}$, at one instant in time.

   (d) **(12p)** Draw any time-varying solution to Maxwell's equations in free space at two snapshots in time. Briefly comment on why your solution is a solution.

3. **(20p)** Steady Heat Flow and Electricity and Magnetism
In class we argued that, for one-dimensional problems, the heat flow per unit time $\frac{dQ}{dt}$ was proportional to how the temperature changes with respect to distance,

$$\frac{dQ}{dt} = -KA\frac{dT}{dx} \tag{1}$$

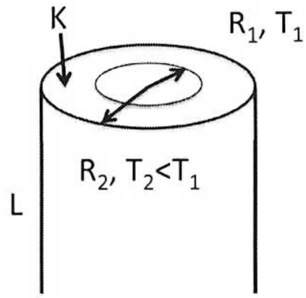


Figure 2: Problem (3b)

K is the conduction constant and is dependent on the material. More generally we can consider a vector field $\vec{h} \cdot \hat{n} = \frac{1}{A}\frac{dQ}{dt}$, where A is the area perpendicular $\hat{n}$, the normal unit vector. $\vec{h}$ is a vector field that represents the amount of heat flowing per unit time through unit area A. The generalization of (1) is:

$$\vec{h} = -K\vec{\nabla}T \tag{2}$$

The heat current $\vec{h}$ obeys the continuity equation

$$\vec{\nabla} \cdot \vec{h} = s \tag{3}$$

where s is the source of heat.

(a) **(6p)** There is an electromagnetic analog to $\vec{h}$, s, and T. Identify these analogs and say why the analogy holds.

Consider the case of a cylindrical pipe carrying steam of inner radius R_1. The at R_1 the temperature is T_1. The pipe is nested within another pipe with radius R_2. The outer pipe is cooler, at temperature $T_2 < T_1$. The rate of heat loss for length L of the pipe is a constant, $G = \frac{dQ}{dt}$. Between the pipes is a material with thermal conductivity K. You do not need part (3a) to do the parts below. In cyclindrical coordinates $\vec{\nabla}f = (\partial_r f)\hat{r} + \frac{1}{r}(\partial_\theta f)\hat{\theta} + (\partial_z f)\hat{z}$.

(b) **(10p)** Find G as a function of the quantities given in the problem and physical constants.

(c) **(4p)** What electromagnetic problem does this correspond to? What is the electromagnetic analog for G?

4. **(38p)** A photon gas
In class we used kinetic theory to show that in, three spatial dimensions, the pressure P and volume V relate to the mass m and velocity $\vec{v}$ of the particles in the following way:

$$PV = \frac{1}{3}N\langle mv^2\rangle \tag{4}$$

For a photon gas, this generalizes to the following equation of state ($\vec{p}$ is momentum and $\vec{v}$ is velocity and U is the energy):

$$PV = \frac{1}{3}N\langle \vec{p} \cdot \vec{v}\rangle = \frac{1}{3}U \tag{5}$$

(a) **(8p)** Find the adiabatic compressibility of a photon gas. This means find the exponent c_1 such that:

$$PV^{c_1} = \text{constant}. \tag{6}$$

(b) **(6p)** Use Equation (5) and the following identity Equation (7) to derive how the energy per unit volume $\frac{U}{V}$ of a photon gas depends on T. (Aside: this is not the most general form of the thermodynamic identity.)

$$\left(\frac{\partial U}{\partial V}\right)_{\text{P is constant}} = T\frac{dP}{dT} - P \tag{7}$$

Call your undetermined constant A. To solve this problem, you need to find the constant c_2 such that,

$$\frac{U}{V} = AT^{c_2} \tag{8}$$

(c) **(8p)** Draw two isotherms and two adiabats for a photon gas on a PV diagram, such that they make a cycle. How do these compare to adiabats and isotherms for a monatomic ideal gas?

(d) **(12p)** Calculate the efficiency of a photon gas traversing the cycle you've drawn. (Hint: it is not recommended to calculate the work, heat, and change in internal energy along each leg.)

(e) **(2p)** How does the efficiency of part (4d) compare to the efficiency of an ideal gas working through a Carnot cycle between the same two temperatures? Explain what you observe in one sentence.

(f) **(2p)** Explain why or why not this process is reversible in one sentence.

5. **(18p)** The betatron.
A particle of charge $Q > 0$ and rest mass m is moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. In the plane of the particle's motion, $\vec{B} \parallel \hat{z}$, $\vec{B} \perp \vec{v}$. $\vec{B}$ is not uniform, but it is axially symmetric. See Figure (3a).

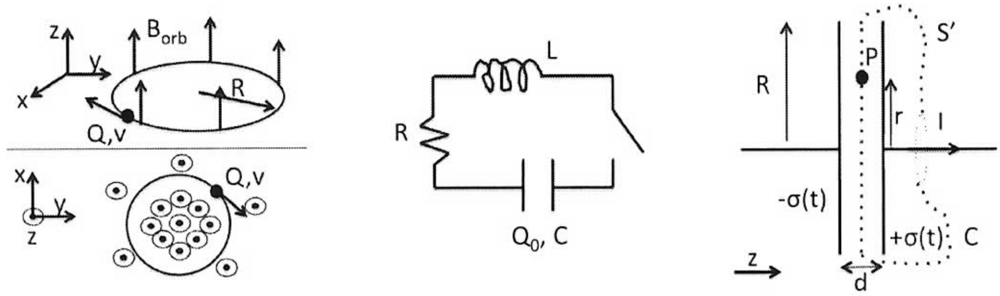
(a) **(8p)** Show that the particle's path is a circle of radius R, and solve for R in terms of the momentum $\vec{p}$ of the particle, B_{orb}, Q, and constants. B_{orb} is the magnitude of magnetic field at the orbit of the particle, and it is the same at every point along the orbit.

Now we allow the magnetic field $\vec{B}$ to vary with respect to time, but we require that our particle continue moving in a circle with radius R.

(b) **(6p)** Find the change in the magnitude of the momentum Δp due to the average magnetic field enclosed by the particle's orbit, ΔB_{av}, in some time Δt.

(c) **(2p)** What is the condition on ΔB_{av} and ΔB_{orb} so that the particle's orbit stays circular?

Think about what you just discovered how to do!



(a) Problem (5)   (b) Problem (6)   (c) Problem (7)

Figure 3

6. **(30p)** Series LCR circuit
   Consider a series LCR circuit with no driving force, where initial conditions have been set up so that the charge across the capacitor before the switch is closed at time $t = 0$ is Q_0. This circuit is underdamped. See Figure (3b).

   (a) **(6p)** Use Kirchoff's rules to write a differential equation for the charge $Q(t)$ in the circuit.

   (b) **(8p)** Guess the solution to your differential equation, and fit the constants.

   (c) **(6p)** Write an expression for the energy $U(t)$ in the circuit. You do not need to plug in your answer from part (6b).

   (d) **(10p)** Relate your expression for $U(t)$ to the rate of energy dissipated across the resistor. It is possible to do this without using part (6b).

7. **(36p)** A circular plate capacitor.
   Consider a discharging circular plate capacitor. The capacitor has capacitance C and charge density $\pm\sigma(t)$. The distance d between the parallel plates is very small compared to the radius R of the plates.

   (a) **(2p)** What is the electric field $\vec{E}(t)$ between the plates as a function of the charge density $\sigma(t)$? No need to derive, and you can neglect edge effects.

   (b) **(8p)** Without using Faraday's law, what is the flux of the displacement current $\vec{J}_d(t)$ through the surface S' in terms of the current $I(t)$? (Use Faraday's law for partial credit.)

   (c) **(6p)** Solve for the magnitude and direction of $\vec{B}(t)$ at point P equidistant between the plates in terms of the current $I(t)$, variables given, and physical constants. Point P is radius r from the center axis of the capacitor.

   (d) **(6p)** Draw $\vec{E}(t)$ and $\vec{B}(t)$ between the plates.

   (e) **(6p)** Solve for the direction of energy flow between the plates.

   (f) **(8p)** Solve for the total power leaving the capacitor, in two ways.

   (g) **(2p)** Comment in one sentence what you've found in part (7e).