Midterm 2, Physics H7B

Dr. McCurdy, November 7, 2013

Please do all your work in a bluebook.

You will be graded on your solutions, and not just your answers. Show all work and thoroughly justify your solutions with figures, diagrams, equations, and words, as appropriate. Partial credit will be given to partially correct and/or partially complete solutions. No credit will be given to unjustified answers. Cross out any parts of solutions that you do not want to be graded.

Make sure that your answers to questions that ask for a vector quantity are given in the form of vectors (i.e. have vector components, or a magnitude and direction). Where appropriate, clearly label the choice of axes you are using and your choice of signs.

There are 4 problems and 100 possible points on the exam. Please read all 4 problems carefully at the beginning of the exam and attempt all problems to maximize your partial credit. You have 1.5 hours to complete the exam.

This is a closed-book exam. You may use one double-sided 3"$\times$5" index card of notes. Calculators and electronic devices of any kind are not allowed.

On the front or your bluebook, write your:

- full name
- SID
- signature

Do not turn over the exam until you are told to do so. Good luck!!

1. **(20p)** A non-uniform charge distribution.
A non-conducting sphere of radius R has a non-uniform charge distribution $\rho(r) = ar$.

   (a) **(5p)** Find the potential $\phi(r)$ everywhere.

   (b) **(5p)** Find $\vec{E}$ everywhere.

   (c) **(5p)** Where is $\nabla \cdot \vec{E}$ zero and non-zero, and why? (No calculation necessary.)

   (d) **(5p)** If instead the sphere was a conductor with the same amount of total charge Q_{tot}, how would the electric field change, and why?

2. **(20p)** Non-conducting shells.
For this problem, we'll look at non-conducting spherical shells and uniformly distributed total charge. Take two such nonconducting shells of uniformly distributed charge, both of radius R, but with total charges $Q, -Q$, respectively. The shells are touching. (Remember the shells are non-conducting, so the charges stay put.) See Figure (1).

   (a) **(5p)** Draw the electric field inside the positively charged shell. Can you create the electric field in this shell from a well-placed point charge? If so, how, and if not, why?

   (b) **(5p)** Draw the electric field inside the negatively charged shell. Can you create the electric field in this shell from a well-placed point charge? If so, how, and if not, why?

   (c) **(5p)** Draw the electric field outside both shells. Can you create the electric field outside the shells from well-placed point charges? If so, how, and if not, why?

   (d) **(5p)** What is the work required to separate the shells to infinity? Briefly explain your reasoning.

3. **(30p)** A simple RC circuit.
The circuit has a switch, a resistor with resistance R, a battery with electromotive force ϵ, and a parallel plate capacitor. At time $t < 0$, there is zero charge on the capacitor, and the circuit switch is open. For time $t \geq 0$, the switch is closed. See Figure (2).

   (a) **(5p)** Find the charge on the capacitor as a function of time, $Q(t)$, and plot $Q(t)$.

   (b) **(5p)** Find the current through the circuit as a function of time, $I(t)$, and plot $I(t)$.

   (c) Show that energy is conserved in the circuit.

      i. **(5p)** After a very long time $t = \infty$, what was the energy dissipated in the resister?

      ii. **(5p)** After a very long time $t = \infty$, what is the energy stored in the capacitor?

      iii. **(5p)** After a very long time $t = \infty$, what is the total work done by the battery?

      iv. **(5p)** Briefly explain energy conservation in the circuit.

4. **(30p)** Parallel plates and Special Relativity
You have two (approximately infinitely) parallel plates at angle $\theta = 45\,\text{deg}$ with uniform surface charge density $\pm\sigma$ (respectively) in the rest frame of the plates (frame F). There is an observer in frame F' moving with velocity $\vec{v} = -v\hat{x}$ as measured by an observer at rest in frame F. See Figure(3).

   (a) **(5p)** What is the electric field $\vec{E}$ (in cartesian coordinates) between the plates the frame F?

   (b) **(5p)** What is the electric field $\vec{E}'$ (in cartesian coordinates) between the plates the frame F'?

(c) **(5p)** What is the angle of the plates θ' in frame F'?

(d) **(5p)** Draw the electric field from a point charge element of the one of the plates at 3 points A, B, C in frame F. See Figure(4).

(e) **(5p)** Draw the electric field from a point charge element of the one of the plates at 3 points A', B', C' in frame F'. See Figure(4).

(f) **(5p)** Is the electric field perpendicular to the plates in frame F'? Give a qualitative argument using the results from parts (4b, 4c), and or (4d, 4e).

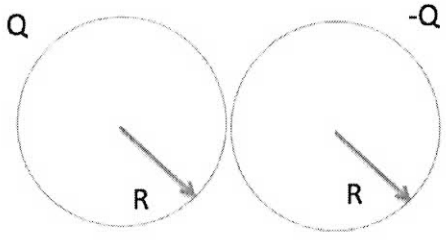

Figure 1. Problem (2).

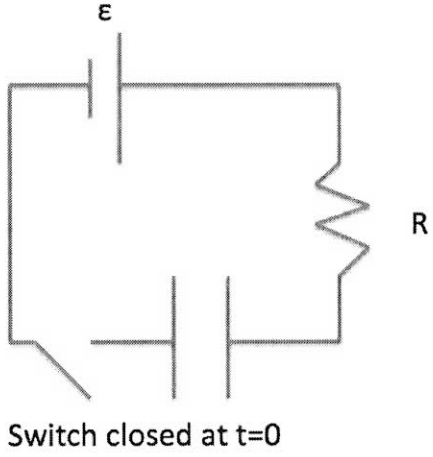

Figure 2. Problem (3).

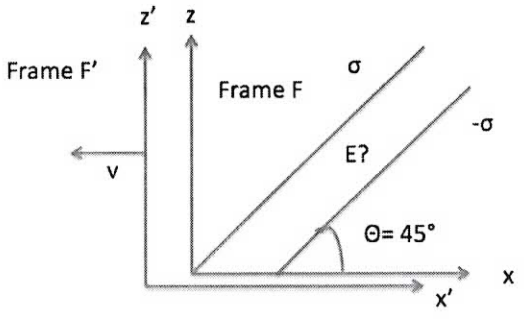

Figure 3. Problem (4).

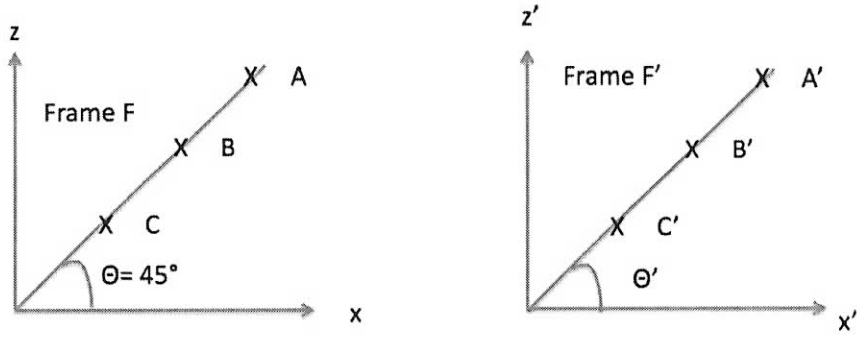

Figure 4. Problem (4), Part(4d, 4e)