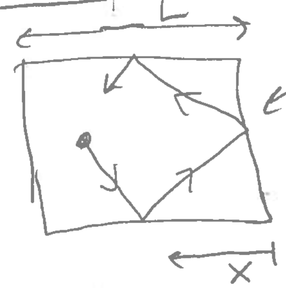
Midterm 1

1



area $A = L$ in 2D, volume $V = L^2$ in 2D
of wall, of box

$V_{\parallel} = V_x \qquad V_{\perp} = V_y$

$\hat{n} = \hat{x}$

ⓐ $\boxed{\Delta t = \frac{2L}{V_{\parallel}}}$

- time to travel the length of box and back.
- collisions are all elastic, symmetric, so changes in $V_{\perp}$ don't matter.

ⓑ $\Delta \vec{p} = \left[-mV_{\parallel} - (mV_{\parallel})\right]\hat{n} = \boxed{-2mV_{\parallel}\,\hat{n}}$ of particle

on wall is opposite particle

ⓒ $\bar{P} = \frac{F}{L} = -\frac{1}{L}\left(\frac{\Delta p}{\Delta t}\right) = \frac{2mV_{\parallel}}{L\left(\frac{2L}{V_{\parallel}}\right)} = \frac{m\overline{V_{\parallel}^2}}{L^2}$

2D, area = L

average $V_{\parallel}^2$

$$\overline{V^2} = \overline{V_{\parallel}^2 + V_{\perp}^2} = \overline{V_{\parallel}^2} + \overline{V_{\perp}^2}$$

$$\overline{V_{\parallel}^2} = \overline{V_{\perp}^2} \quad \text{isotropic}$$

$$\overline{V_{\parallel}^2} = \frac{1}{2}\overline{V^2}$$

V_{rms}^2 (test mistakenly said average)

$$\boxed{\bar{P} = \frac{1}{L^2}\left(\frac{m\overline{V^2}}{2}\right)}$$

ⓓ for N: $\boxed{\bar{P} = \frac{N}{L^2}\left(\frac{m\overline{V^2}}{2}\right) = Nu}$

$u = \frac{m\overline{V^2}}{2L^2}$

Units of u are $\boxed{\frac{\text{Joules}}{\text{meter}^2}}$

ⓔ 2 degrees of freedom

$U = kT$

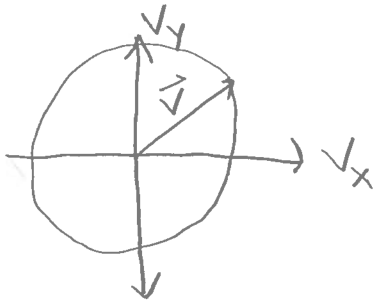
$kT = U = \frac{m\overline{V^2}}{2}$

$$\boxed{V_{rms} = \sqrt{\frac{2kT}{m}}}$$

$\boxed{2}$ 2D

(a) $\epsilon = \frac{1}{2} m v^2$

$$B(v) = e^{-\frac{1}{2}mv^2/kT}$$

(b) 

velocity space

$$A(v) = 2\pi v$$

(c) $$f(v) = \eta \, 2\pi v \, e^{-\frac{1}{2}mv^2/kT}$$

for 1 particle distribution (N particles normalized to N also accepted)

$$1 = \int f(v)\,dv = \eta \int_0^\infty 2\pi v \, e^{-\frac{1}{2}mv^2/kT}\,dv$$

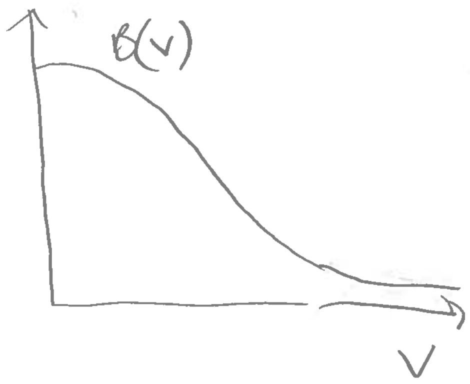
$$x = v \qquad \sigma_x = \sqrt{\frac{kT}{m}}$$

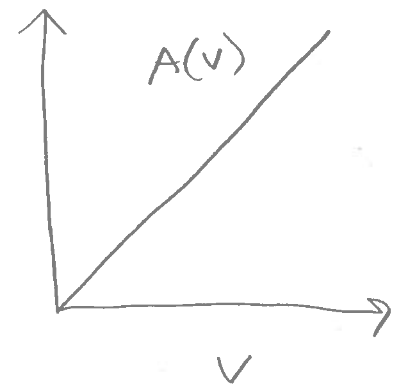
$$1 = 2\pi\eta \int_0^\infty x \, e^{-x^2/2\sigma_x}\,dx = 2\pi\sigma_x^2 = \frac{2\pi kT\eta}{m}$$

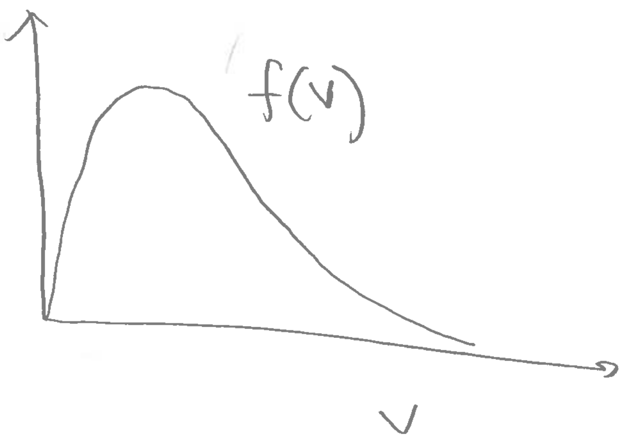
$$\boxed{\eta = \frac{m}{2\pi kT}}$$

$$\boxed{f(v) = \frac{mv}{kT} e^{-\frac{1}{2}mv^2/kT}}$$

(d)







(e) $\bar{v} = \int v\, f(v)\, dv$

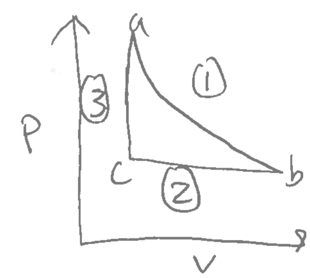
$v_{rms} = \sqrt{\int v^2 f(v)\, dv}$

not the same! in fact, the variance of v is given by

$$\boxed{\mathrm{var}(v) = v_{rms}^2 - (\bar{v})^2}$$

$\boxed{3}$

a i



1 isothermal $T(S) = T_{ab}$

2 isobaric $dS = \frac{dQ}{T} = \frac{n\,C_p\,dT}{T}$

$$S - S_0 = n\,C_p \ln\left(\frac{T}{T_0}\right)$$

$$T(S) = T_0\, e^{(S - S_0)n/C_p} = T_0\, e^{(S-S_0)n/\gamma C_v}$$

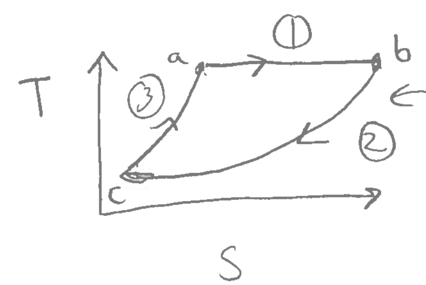
$C_p = \gamma C_v$

$\gamma > 1$

3 isochoric $dS = \frac{dQ}{T} = \frac{n\,C_v\,dT}{T}$

$$T(S) = T_0\, e^{(S-S_0)n/C_v}$$

2



less steep, $\gamma^{-1} < 1$

3 Area $= \oint T\,dS = Q_{net}$

Complete cycle + first law

$$\cancel{\Delta E} = Q_{net} - W_{by}$$

0

$$Q_{net} = W_{by}$$

(c) Not reversible because Q transfer occurs at finite ΔT in legs (2) & (3). Even though $\Delta S_{gas} = 0$, $\Delta S_{universe} > 0$.

(b) $\boxed{e = \frac{W_{net}}{\sum Q_{in}}}$

(1) $W_{by} = \int_a^b P\,dV = \int_{V_{ac}}^{V_b} \frac{NkT_{ab}}{V}\,dV = NkT_{ab} \ln\left(\frac{V_b}{V_{ac}}\right)$

$W_{by} > 0$

$\Delta E = 0 = Q - W_{by}$ (isothermal)

$Q = W_{by} = NkT_{ab} \ln\left(\frac{V_b}{V_{ac}}\right) > 0$ Q enters

(2) $W_{by} = \int_b^c P\,dV = P_{bc}(V_{ac} - V_b)$

$\Delta E = \frac{f}{2} Nk\Delta T = \frac{1}{\gamma-1} Nk\Delta T = \frac{1}{\gamma-1} P_{bc}(V_{ac} - V_b)$

Use ideal for each T

$Q = \Delta E + W_{by}$

$Q = \underbrace{\frac{\gamma}{\gamma-1}}_{0 >} \underbrace{P_{bc}(V_{ac} - V_b)}_{< 0} < 0$, Q leaves

(3) $W_{by} = 0$ $(dV = 0)$

$\Delta E = \frac{1}{\gamma-1} Nk\Delta T$

ideal

$Q = \Delta E + \cancel{W_{by}}^{0} = \frac{1}{\gamma-1} Nk\Delta T = \underbrace{\frac{1}{\gamma-1}}_{>0} V_{ac} \underbrace{(P_a - P_{bc})}_{>0} > 0$

Q enters

$$e = \frac{W_{net}}{\sum Q_{in}} = \frac{NkT_{ab}\ln\left(\frac{V_b}{V_{ac}}\right) + P_{bc}(V_{ac} - V_b)}{NkT_{ab}\ln\left(\frac{V_b}{V_{ac}}\right) + \frac{1}{\gamma-1} V_{ac}(P_a - P_{bc})}$$

4

(a i) $W_{by} = -\int \vec{F}_{on}\cdot d\vec{L} = -mg\int_{\underbrace{L_i}_{\Delta L}}^{\overbrace{L_f}^{0}} dL = mg(L_i - L_f)$

$W_{by} = mg\Delta L > 0$

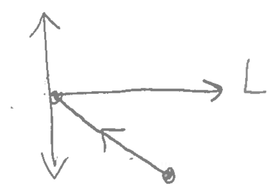
positive wirk done by

(a ii) $\Delta U = \underbrace{\cancel{Q}}_{0,\ \text{adiabatic}} - W_{by} = -mg\Delta L < 0$

rubber band looses internal eregy

(b i) $W_{by} = -\int \vec{F}_{on}\cdot d\vec{L} = -\int_{\Delta L}^{0} (-LkT)\, dL = \underbrace{-\frac{1}{2}kT(\Delta L)^2}_{<0}$

(b ii)



probably because of the typo, many found the sign confusing; both signs accepted.

Note: There was a typo on the exam, and the real-life eqn of state for (b) should have read $\boxed{\frac{F}{L} = kT}$

then (i) $W_{by} > 0$ and (ii)

(c) For gas free expansion, $\Delta U = Q = W_{by} = 0$. For a rubber band, this means letting it contract w/ out constraint.