Midterm 1, Physics H7B

Dr. McCurdy, October 3, 2013

Please do all your work in a bluebook.

You will be graded on your solutions, and not just your answers. Show all work and thoroughly justify your solutions with figures, diagrams, equations, and words, as appropriate. Partial credit will be given to partially correct and/or partially complete solutions. No credit will be given to unjustified answers. Cross out any parts of solutions that you do not want to be graded.

Make sure that your answers to questions that ask for a vector quantity are given in the form of vectors (i.e. have vector components, or a magnitude and direction). Where appropriate, clearly label the choice of axes you are using and your choice of signs.

There are 4 problems and 100 possible points on the exam. Please read all 4 problems carefully at the beginning of the exam and attempt all problems to maximize your partial credit. You have 1.5 hours to complete the exam.

This is a closed-book exam. You may use one double-sided 3"×5" index card of notes. Calculators and electronic devices of any kind are not allowed.

On the front or your bluebook, write your:

- full name
- SID
- signature

Do not turn over the exam until you are told to do so. Good luck!!

1. **(20p)** Kinetic Theory, $2D$
For this problem, pretend that the world has only 2 spatial dimensions. Use standard arguments from kinetic theory and the simplest model for an ideal gas. Take a single molecule of mass m to be bouncing around elastically with velocity $\vec{v}$ inside a volume L^2.

   (a) **(2p)** Single out one wall of the container, with normal $\hat{n}$. What is the average time per molecular collision with that wall, Δt, in terms of $v_{\|}$ (the component of molecule's speed parallel to $\hat{n}$) and L?

   (b) **(3p)** What is the average change in momentum of the molecule per collision $\Delta\vec{p}$, in terms of $v_{\|}$ and m?

   (c) **(5p)** What is the average pressure $\overline{P}$ on the wall, in terms of the average speed $\overline{v}$, L, and m? (Note that pressure in $2D$ has different units from pressure in $3D$.)

   (d) **(5p)** Generalizing now to N molecules, what is the pressure P as a function of of the energy density $u = \frac{U}{m^2}$?

   (e) **(5p)** What is v_{rms} in $2D$, in terms of T and constants?

2. **(25p)** Maxwell Distribution, $2D$
Find the Maxwell speed distribution $f(v)$ for a monotonic ideal gas of molecules of mass m in 2 spatial dimensions.

   (a) **(5p)** What is the Boltzmann factor $B(v)$ in terms of speed, v? (Recall the Boltzmann factor is $B(v) = e^{-\frac{\epsilon}{kT}}$, where ϵ is the energy of one particle.)

   (b) **(5p)** What is the area $A(v)$ of velocity v in velocity space?

   (c) **(5p)** The Maxwell distribution is the product of these two factors $f(v) = \eta A(v)B(v)$, where η is the normalization. Find η and give the distribution $f(v)$.

   (d) **(5p)** Qualitatively draw $B(v)$. Qualitatively draw $A(v)$. Qualitatively draw the 2−dimensional Maxwell distribution $f(v)$.

   (e) **(5p)** How would you calculate $\bar{v}$ and v_{rms}? Give integral expressions but do not integrate.

3. **(40p)** Engines and Thermodynamic Laws
An ideal gas with gas constant γ is taken through a quasi static cycle, shown in Figure 1.

   (a) **(25p)** We've drawn this process on a $P-V$ diagram, but processes can also be represented on a $T-S$ diagram.

      i. **(10p)** Derive $T(S)$ for each of the 3 legs on the $P-V$ diagram.

      ii. **(10p)** Draw this process on a $T-S$ diagram, clearly labeling all the points, line segments, and directions.

      iii. **(5p)** What is the interpretation of the area under the curve in this diagram?

   (b) **(10p)** Calculate the efficiency of this process. You can give your answers in terms of the volumes, pressures, and temperatures labeled on the diagram, and γ. Don't bother simplifying your answer.

   (c) **(5p)** Is this process reversible? Explain your reasoning.

4. **(15p)** Rubber band and Thermodynamic Laws
Imagine you have a mass m hanging on the end of a rubber band. The surface gravity of the earth is g. You've extended the rubber band-mass system beyond it's equilibrium length by ΔL, as in Figure 2.

   (a) **(5p)** Suddenly, you release the mass. After oscillating for a while, the rubber band and mass comes to rest at the equilibrium position. (The oscillations are not important for solving this problem.) This is not a quasistatic process, but it does happen quickly, so the process is adiabatic.

      i. **(3p)** What is the work done by the rubber band, W_{by}, in terms of ΔL and given constants?

      ii. **(2p)** What is the change in internal energy ΔU of the rubber band?

   (b) **(5p)** Now, allow the mass to come to it's equilibrium position slowly, so that as the mass rises, the temperature of the rubber band stays constant. With the appropriate caveats and constants, the rubber band equation of state is $\frac{F}{L} = -\kappa T$, where F is the force on the rubber band, L is the length (it's extension), and κ is the appropriate constant.

      i. **(3p)** What is the work done by the rubber band, W_{by}?

      ii. **(2p)** Draw this second process on the $F - L$ diagram.

   (c) **(5p)** What is the equivalent of free expansion for a rubber band?

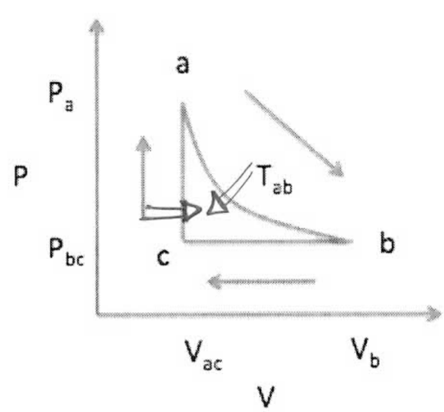

Figure 1. Problem (3).

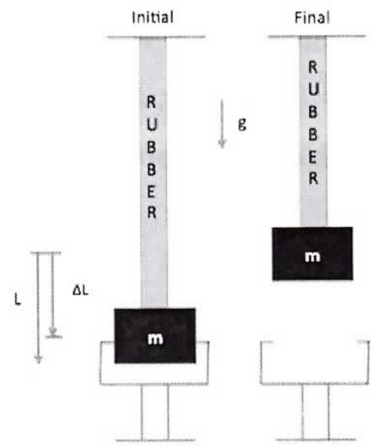

Figure 2. Problem (4).