

University of California, Berkeley
Department of Mechanical Engineering
ME 104, Fall 2013

Midterm Exam 2 (13 November 2013)

1. (30 points) Consider a particle X , of mass m , moving along a plane curve \mathcal{C} , which is parametrized by its arc length s . The unit tangent vector \mathbf{e}_t to \mathcal{C} is defined by

$$\mathbf{e}_t = \frac{d\mathbf{r}}{ds}, \quad (1)$$

and the principal unit normal vector to \mathcal{C} is

$$\mathbf{e}_n = \rho \frac{d\mathbf{e}_t}{ds}, \quad (2)$$

where $\rho (> 0)$ is the radius of curvature of \mathcal{C} .

(a) Show that the velocity and acceleration vectors of X may be expressed as

$$\mathbf{v} = \dot{s} \mathbf{e}_t = v \mathbf{e}_t, \quad \mathbf{a} = \dot{v} \mathbf{e}_t + \frac{v^2}{\rho} \mathbf{e}_n. \quad (3)$$

(b) Suppose that a car weighing 3000 lbf is being tested on a circular skidpad of radius 100 ft. If the maximum sustainable constant speed of the car is 37 mph, calculate the acceleration of the car in g 's. Also, calculate the total frictional force that the asphalt surface then exerts on the tires.

(c) Suppose that a projectile is launched with speed v_0 at an angle 45° from the horizontal plane. Neglect air resistance. Draw the free-body diagram of the projectile at the highest point A of its trajectory. Find the velocity vector $\mathbf{v}(A)$ and acceleration vector $\mathbf{a}(A)$ of the projectile, and indicate them on your sketch. What are the normal and tangential components of $\mathbf{a}(A)$? Express the radius of curvature $\rho(A)$ in terms of v_0 and g .

2. (35 points) Recall that the linear impulse of a force \mathbf{F} is defined by

$$\mathcal{I} = \int_{t_1}^{t_2} \mathbf{F} dt. \quad (4)$$

(a) If \mathbf{G} is the linear momentum of a particle of mass m , prove that

$$\mathcal{I} = \Delta \mathbf{G} = \mathbf{G}_2 - \mathbf{G}_1. \quad (5)$$

(b) A simplified model of a ballistic pendulum comprises: (i) a simple pendulum consisting of an inextensible string of length ℓ and a bob of mass M , which is initially at rest in its stable equilibrium position; and (ii) a bullet of mass m travelling with speed v on a collision course with the bob. Treat the collision as impulsive, and calculate the speed v^+ of the bob and the embedded bullet immediately after impact.

(c) Show that the maximum angle θ_m reached by the pendulum after impact is given by

$$1 - \cos \theta_m = \frac{(v^+)^2}{2g\ell}. \quad (6)$$

(d) If

$$\ell = 5 \text{ m}, \quad m = 0.2 \text{ kg}, \quad M = 20 \text{ kg}, \quad \theta_m = 10^\circ,$$

calculate the incoming speed v of the bullet, and express it as a Mach number, using $c = 343$ m/s for the speed of sound in air at a temperature of 20°C .

3. (35 points) Consider a planet of mass m orbiting the sun (mass M), regarded as fixed in an inertial frame of reference. Use polar coordinates, and recall that the velocity of the planet is given by

$$\mathbf{v} = \dot{r} \mathbf{e}_r + r\dot{\theta} \mathbf{e}_\theta. \quad (7)$$

(a) Calculate the angular momentum \mathbf{H}_O of the planet about the center of the sun, and deduce that

$$r^2\dot{\theta} = \text{const.} = h. \quad (8)$$

(b) Recall that the gravitational potential energy of the planet is $V = -\frac{GMm}{r}$, where $G = 6.6734 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$, and the mass of the sun is $1.990 \times 10^{30} \text{ kg}$. Deduce that

$$\frac{1}{2}v^2 - \frac{GM}{r} = \text{const.} = \frac{E}{m}. \quad (9)$$

(c) For the planet Mercury, the distances to perihelion and aphelion are

$$r_p = 46 \times 10^6 \text{ km}, \quad r_a = 69.82 \times 10^6 \text{ km}. \quad (10)$$

If the speed of Mercury at aphelion is $38.86 \times 10^3 \text{ m/s}$, calculate its speed at perihelion.

(d) Using aphelion data, calculate the value of specific orbital energy of Mercury.

(e) Where in its orbit does the maximum acceleration of Mercury occur? Calculate its value.