University of California, Berkeley
Department of Mechanical Engineering
ME 104, Fall 2013

Midterm Exam 1 (2 October 2013)

1. (20 points) **(a)** For a particle undergoing a rectilinear motion, the position, velocity, and acceleration vectors are given by

$$\mathbf{r} = x\,\mathbf{i}, \qquad \mathbf{v} = \dot{x}\,\mathbf{i} = v\,\mathbf{i}, \qquad \mathbf{a} = \ddot{x}\,\mathbf{i} = a\,\mathbf{i}. \tag{1}$$

Show that

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right). \tag{2}$$

(b) Consider a particle P of mass m travelling in a horizontal plane. Let its velocity vector be

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j} = v_x\,\mathbf{i} + v_y\,\mathbf{j}. \tag{3}$$

Suppose that v_x^2 and v_y^2 are specified by

$$v_x^2 = C_1 - \frac{1}{2}kx^2 \geq 0, \qquad v_y^2 = C_2 - \frac{1}{2}ky^2 \geq 0, \tag{4}$$

where k is a positive constant, and C_1 and C_2 are constants. Calculate a_x and a_y.

(c) Show that the force acting on P is centripetal.

(d) Obtain the differential equations for the x and y coordinates of P, and provide a simple solution of either one of them.

2. (30 points) Let $OABC$ be a rigid plate which is rotating at constant angular velocity $\boldsymbol{\omega} = \omega\,\mathbf{k}$ around a vertical axis OZ ($\omega = \dot{\theta} = const.$). Introduce a corotational basis $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ on the plate, as indicated in Fig. 1. Suppose that a rigid rod OAD is welded to the plate and that MN is a rigid guide (OMN is a right angle.). Let P be a pin of mass m that can slide on the rod and inside the guide.

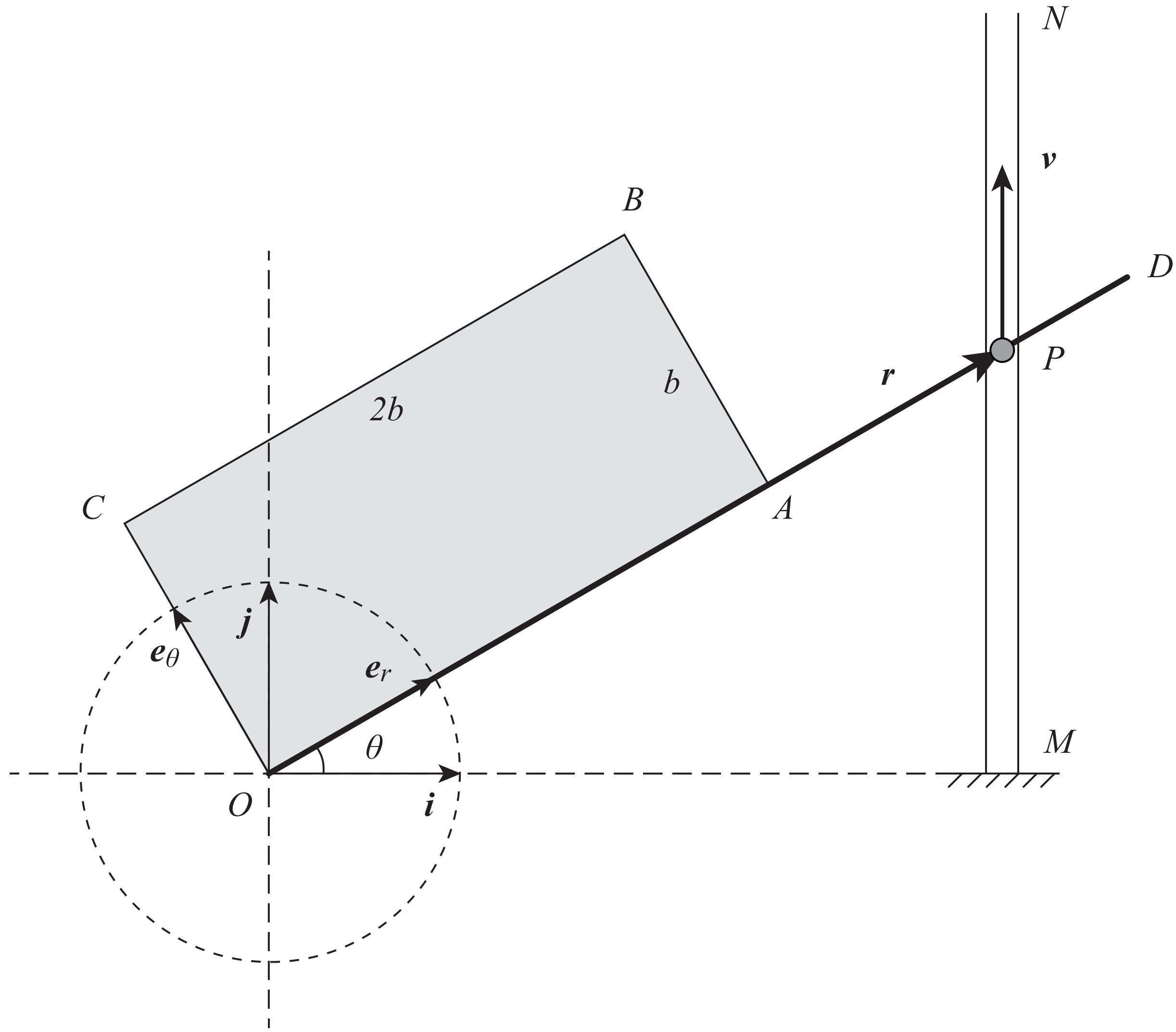

Figure 1: Problem 2

Figure 1.

(a) Write down the relationship between the corotational basis vectors $\mathbf{e}_r, \mathbf{e}_\theta$ and the fixed basis vectors $\mathbf{i}, \mathbf{j}$.

(b) Show that

$$\dot{\mathbf{e}}_r = \dot{\theta}\,\mathbf{e}_\theta = \boldsymbol{\omega} \times \mathbf{e}_r, \qquad \dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r = \boldsymbol{\omega} \times \mathbf{e}_\theta. \tag{5}$$

(c) Express the velocity and acceleration of P on both bases, given that $\dot{\theta} = const.$ Hence deduce that

$$\dot{r} = \dot{y}\,\mathbf{j}\cdot\mathbf{e}_r, \qquad r\dot{\theta} = \dot{y}\,\mathbf{j}\cdot\mathbf{e}_\theta. \tag{6}$$

(d) Suppose that at $\theta = 30°$,

$$r = 0.05 \text{ m}, \qquad \dot{r} = 0.2 \text{ m/s}, \qquad \ddot{r} = -0.025 \text{ m/s}^2. \tag{7}$$

(i) Calculate $\dot{\theta}$ and $\dot{y}$, and check that $\dot{y}^2 = \dot{r}^2 + r^2\dot{\theta}^2$.
(ii) Find $\ddot{y}$.
(iii) Calculate the magnitude of the total force $\mathbf{F}$ acting on P if the mass of P is $m = 0.5$ kg.

(e) Let η represent the distance along the diagonal OB of the plate. Calculate the velocity distribution $\mathbf{v}(\eta)$ along OB and sketch it.

(f) Also calculate the acceleration along the diagonal OB and indicate its variation on a sketch ($\dot{\theta} = const.$).