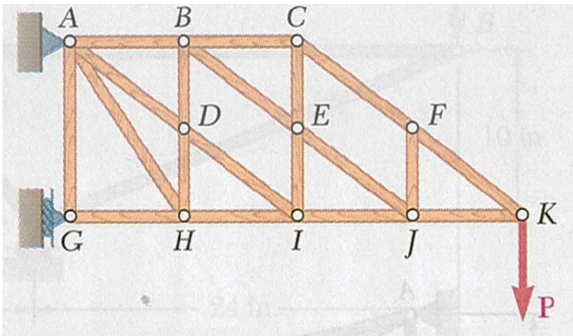
Introduction to Solid Mechanics
ME C85/CE C30

Midterm Exam 1

Fall, 2013

1. Do not open the exam until you are told to begin.
2. Put your name and SID on **every** page of your answer book.
3. You may not use a calculator, but you may use a straightedge to help you draw figures.
4. You may use one 8-1/2 x 11 sheet of notes, but not your book or any other notes.
5. Store everything else out of sight.
6. Turn off cell phones.
7. There will be no questions during the exam. Write your concerns or alternative interpretations in exam margins.
8. Write all answers in the answer book provided with this exam.
9. Be concise and write clearly. Identify your answer to a question by putting a box around it.
10. Use only the front sides of the answer sheets for your answers. You may use the backs of pages for "scratch" paper, but if there is work that we should see, be sure to point that out in the main body of the exam.
11. Time will be strictly enforced. At 9:00, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

Problem 1. (10 Points) Identify all zero force members in the truss shown below. This truss is supported by a pin at joint A and a roller at joint G.

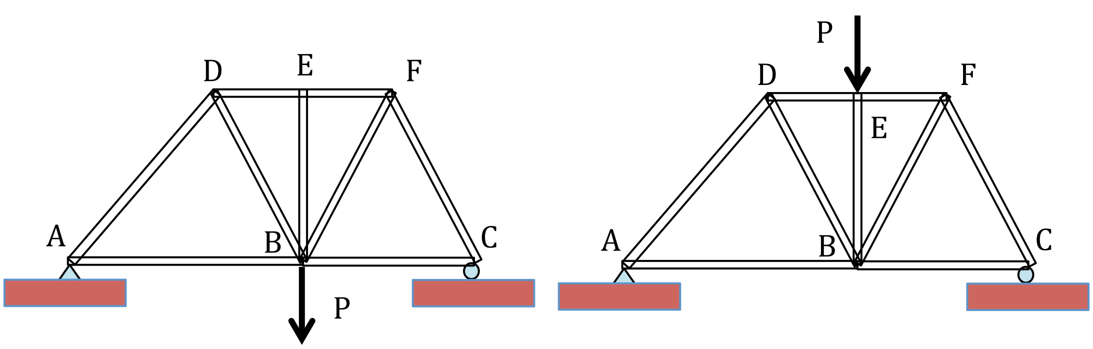


Problem 2. (25 points) The principle of transmissibility states that two forces acting on a rigid body are statically equivalent if they have the same magnitude and direction, and are collinear (i.e., they act on the body along the same line of action). You will consider the implications of this principle in the context of forces acting on a truss. Shown below is a truss loaded by a vertical force that acts at either joint B or joint E. While not explicitly shown, all members are assumed to be connected by ideal pins at the joints.

(a) In the context of determining the reaction forces at points A and C, are the two loads statically equivalent? Justify your answer.

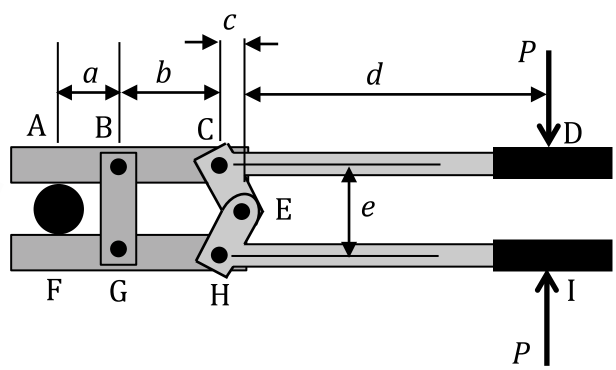
(b) Determine the force in member BE for each loading case, and from this conclude that the two cases are not necessarily statically equivalent in the context of the forces in the members of the truss.

(c) Now consider the forces in members AB, BD and DE, which could be determined using the method of sections. Draw an appropriate free body diagram for each loading case, and use them to argue that these member forces (F_{AB}, F_{BD} and F_{DE}) must be the same for the two loading cases. You should not attempt to solve for these forces, but it may be useful to show which equations would be used if the details of the geometry had been given.

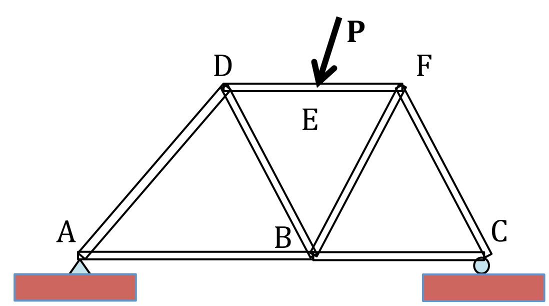


Problem 3. (25 Points) Shown below are a photograph of a bolt cutter and a simplified diagram of this machine showing the five rigid members (ABC, DCE, BG, FGH and IHE) which are joined with (ideal) pins at points B, C, E, G and H. These pins can support forces in the horizontal and vertical directions, but offer no resistance to rotation. The cutter works by placing a bolt between points A and F, and exerting vertical forces of magnitude *P* on the handles at points D and I.

Determine the magnitude of the forces acting on the bolt at A and F in terms of the applied force *P* and the specified lengths *a, b, c, d* and *e*. For full credit, you must draw free body diagrams for relevant members (you need not draw a FBD for every member) and clearly show which equilibrium equations are used for which member.



Problem 4. (20 Points) The structure shown is loaded by a force **P** at the midpoint E of member DF. Because the loading is not at a joint, this structure cannot be treated as a truss.

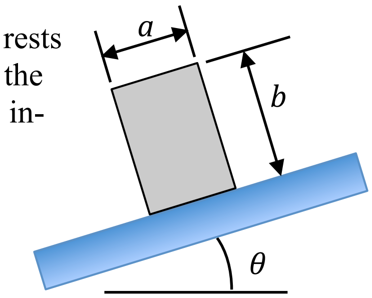


(a) Explain why use of the truss analysis is not appropriate for member DF, but that all other members can be treated as if they were part of a truss.

(b) Show that the forces in members BD and BF are of equal magnitude, but opposite signs (so that one is in tension and one is in compression).

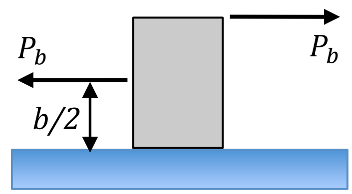
(c) Draw free body diagrams for member DF and for joint D.

Problem 5. (20 Points) A block of weight W, width a and height b rests on an inclined flat surface. The coefficient of static friction between the block and the surface is μ_s. It is found that as the inclination angle θ increases, the block tips over before it slips.

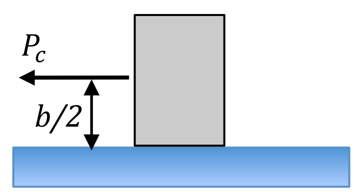


(a) Show that for this "tip before slip" to occur, $\mu_s > a/b$.

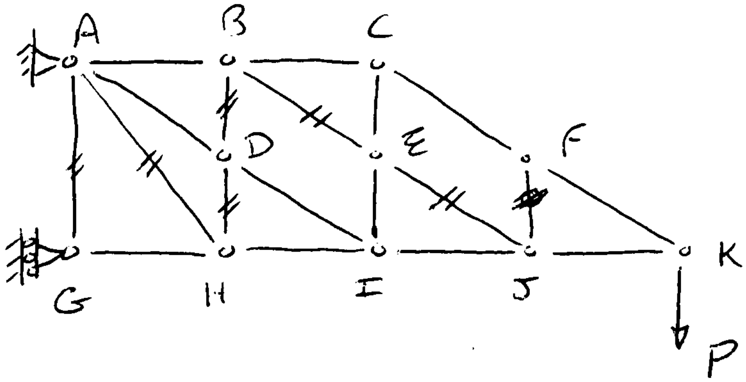
(b) Now suppose that the surface is returned to horizontal and that the coefficient of friction is $\mu_s = a/b$. Let the block be acted upon by two horizontal forces as shown. At what magnitude of force P_b will the block be subject to impending motion? At this point, is the block about to slip or about to tip? Express your answer in terms of a, b and W.



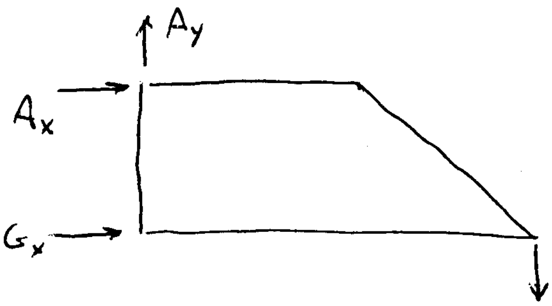
(c) Finally, let the block be acted upon by a single horizontal force P_c. Assuming that the coefficient of friction is the same as in part (b) of this problem, determine the magnitude of force P_c that will cause the block be subject to impending motion? At this point, is the block about to slip or about to tip?



1. ZERO FORCE MEMBERS:

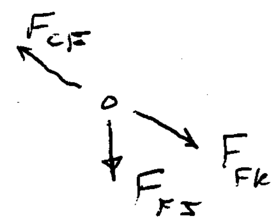


FBD OF ENTIRE TRUSS



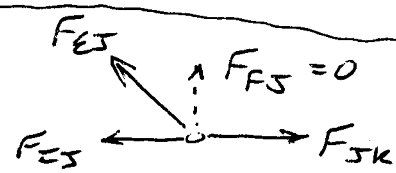
FOR ZERO FORCE MEMBERS:

BEGIN AT JOINT F -



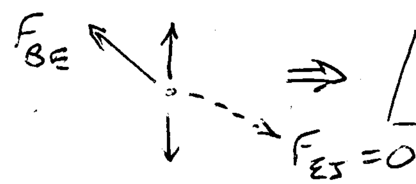
F_{FJ} IS THE ONLY FORCE $\perp$ CFK $\therefore$ $\boxed{F_{FJ} = 0}$

NOW MOVE TO JOINT J:



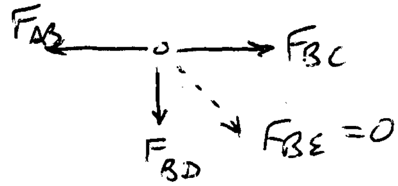
$\Rightarrow \boxed{F_{EJ} = 0}$

JOINT E:



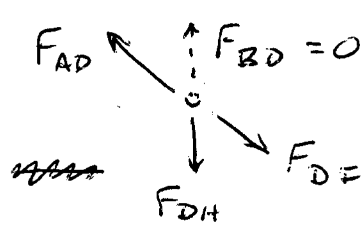
$\Rightarrow \boxed{F_{BE} = 0}$

JOINT B:



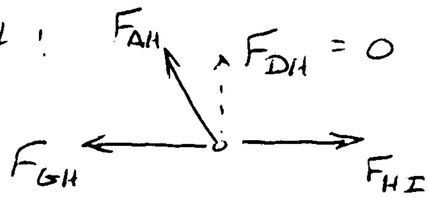
$\Rightarrow \boxed{F_{BD} = 0}$

JOINT D:



$\Rightarrow \boxed{F_{DH} = 0}$

JOINT H :



$\Rightarrow \boxed{F_{AH} = 0}$

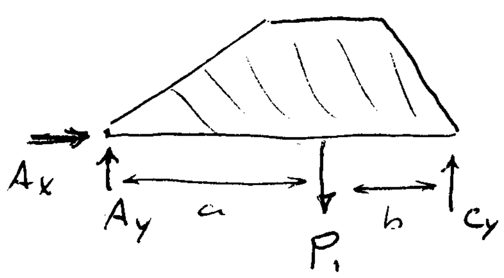
JOINT G :

$\Rightarrow \boxed{F_{AG} = 0}$

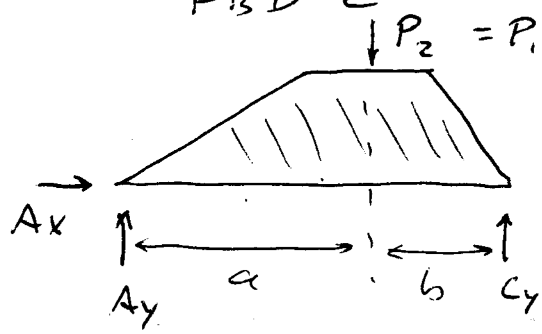
2. (a) IN THE CONTEXT OF DETERMINING THE REACTION FORCES, THE TWO FORCES ARE STATICALLY EQUIVALENT

JUSTIFICATION: TREAT THE ENTIRE STRUCTURE AS A RIGID BODY.

FBD 1



FBD 2



THIS SITUATION OF LOADS OF EQUAL MAGNITUDE ACTING ON A RIGID BODY ALONG THE SAME LINE OF ACTION SATISFIES THE PRINCIPLE OF TRANSMISSIBILITY.

WE CAN ALSO SHOW THAT IN BOTH CASES, THE REACTION FORCES ARE THE SAME:

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

$$\Sigma M_c = 0 \Rightarrow A_y = P_1\left(\frac{b}{a+b}\right) = P_2\left(\frac{b}{a+b}\right)$$

P_1

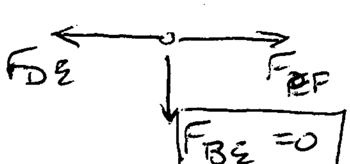
$$\Sigma M_A = 0 \Rightarrow C_y = P_1\left(\frac{a}{a+b}\right) = P_2\left(\frac{a}{a+b}\right)$$

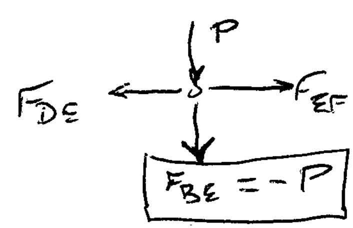
NOTE: a + b WERE NOT GIVEN, BUT WERE INTRODUCED HERE TO FACILITATE THE SOLUTION.

(b) CASE 1: FBD OF E

BE IS A ZERO FORCE MEMBER

CASE 2: FBD OF E



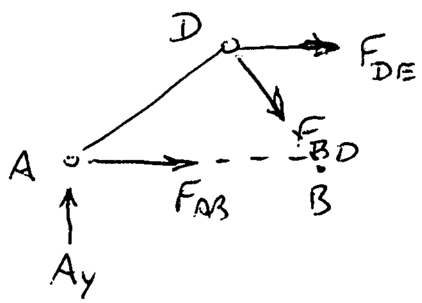


IN CASE 1, $F_{BE} = 0$

IN CASE 2, $F_{BE} = -P$ (FORCE MAGNUDE OF P IN COMPRESSION)

∴ THE APPLIED FORCES ARE <u>NOT</u> STATICALLY EQUIVALENT IN TERMS OF MEMBER FORCES.

(c) SINCE WE ARE CONSIDERING ONLY MEMBERS AB, BD & DE WE CAN USE THE METHOD OF SECTIONS. IN EITHER CASE SECTIONING THROUGH THE MEMBERS OF INTEREST + DRAWING THE FBD OF THE PORTION TO THE LEFT OF THE CUT ⇒



AS SHOWN IN PART (a), A_y IS THE SAME FOR BOTH LOADING CASES. ∴ THE MEMBER FORCES MUST ALSO BE THE SAME.

GIVEN THE DETAILS OF THE GEOMETRY, THESE FORCES COULD BE OBTAINED AS FOLLOWS:

$\Sigma F_y = 0 \Rightarrow F_{BD}$

$\Sigma M_D = 0 \Rightarrow F_{AB}$

$\Sigma M_B = 0 \Rightarrow F_{DE}$

NOTE THAT P ITSELF DOES NOT APPEAR IN THE FBD. HOWEVER, IT HAS BEEN USED ALREADY IN DETERMINING A_y.

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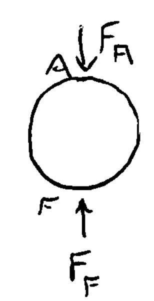
3. THERE ARE A NUMBER OF POSSIBLE FBDs THAT CAN PROVIDE USEFUL INFORMATION.

① AF :

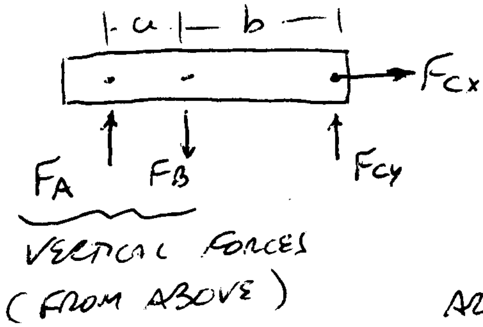


THE BOLT AF IS A 2-FORCE MEMBER SO F_A + F_F ARE VERTICAL.

② BG :

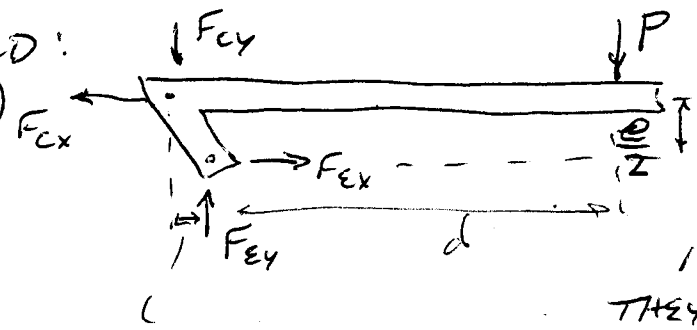
BG IS A 2-FORCE MEMBER SO F_B & F_G ARE VERTICAL

ABC : ③



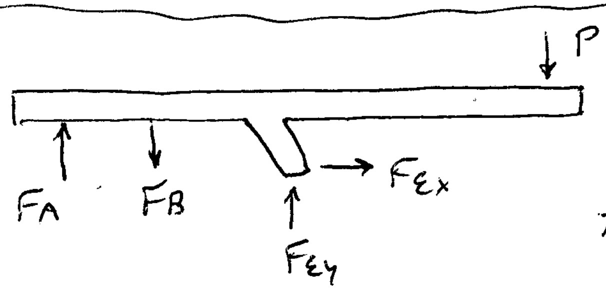
AT THIS POINT, WE DO NOT KNOW ANYTHING ABOUT F_{CX} & F_{CY}. THE DIRECTIONS OF F_A + F_B ON THIS FBD ARE DETERMINED FROM FBD 1 + 2.

ECD : ④



NOTE THAT ONCE THE DIRECTIONS OF F_{CX} + F_{CY} ARE GIVEN IN THE FBD FOR ABC, THEY MUST BE IN THE OPPOSITE DIRECTIONS FOR ECD.

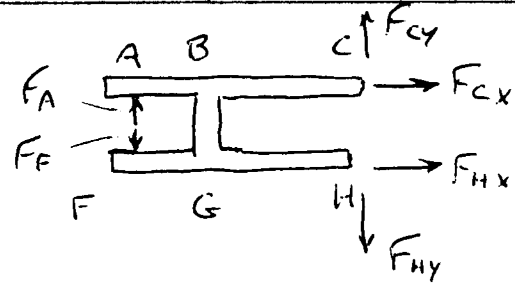
ABCDE ⑤



NOTE THAT THE FORCES AT PIN C ARE INTERNAL TO THIS BODY.

ABCHGF

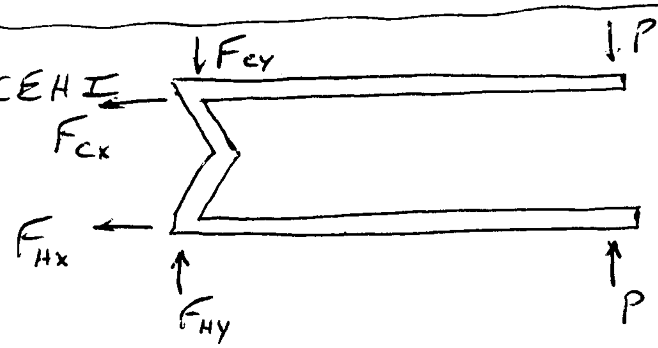
(6)



NOTE THAT THE FORCE AT B IS NOT SHOWN SINCE IT IS <u>INTERNAL</u> FOR THIS BODY.

DCEHI

(7)



NOTE THAT THE FORCES AT E ARE NOT SHOWN SINCE THEY ARE INTERNAL FOR THIS BODY

SOLUTION FOR F_A:

USING FBD 3 $\quad \Sigma F_x = 0 \Rightarrow F_{cx} = 0$

$$\Sigma M_B = 0 = F_{cy} b - F_A a$$

$$F_A = F_{cy}\left(\frac{b}{a}\right)$$

USING FBD 4 $\quad \Sigma F_x = 0 = F_{Ex}$

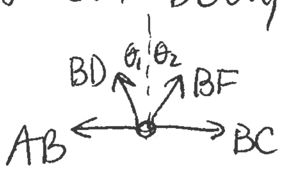
$$\Sigma M_E = F_{cy} c - Pd = 0$$

$$F_{cy} = P\left(\frac{d}{c}\right)$$

$$\Rightarrow \boxed{F_A = P\left(\frac{db}{ac}\right)}$$

4) a) Ideal ~~trusses~~ trusses are defined as structures composed of straight, massless members joined @ ideal pins (no friction) and loaded only at joints. Since member DF is loaded in the center, it violates the rules for an ideal truss. The center loading makes DF a 3-force member but since the rest of the members don't experience any external loading except that @ the pins, they are all 2-force members. Thus we cannot assume DF is experiencing force directed only along the beam and traditional truss analysis will not apply. The rest of the members are 2-force and ∴ do experience loading directed only along their length.

b) An appropriate FBD for this analysis is one involving BD & BF as "external loads". The easiest body to use is the pin @ B.



$\Sigma F_x = AB + BC - BD\sin\theta_1 + BF\sin\theta_2 = 0$

- too many unknowns (can't assume AB = BC yet)

$\Sigma F_y = BD\cos\theta_1 + BF\cos\theta_2 = 0$

$\rightarrow BD\cos\theta_1 = -BF\cos\theta_2$

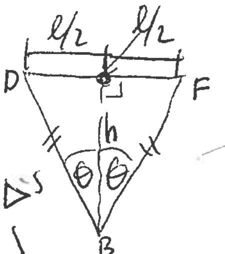
$\boxed{BD = -BF}$

given: E @ midpoint of DF

- identical Δs

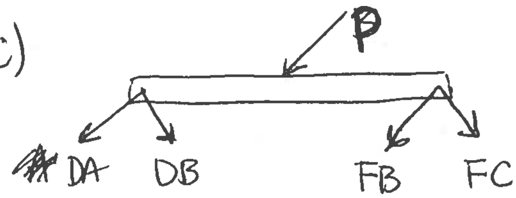
↓

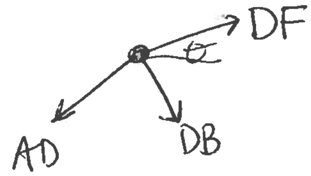
$\theta_1 = \theta_2$ (SAS thm)

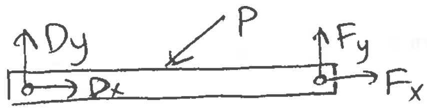


θ unknown!

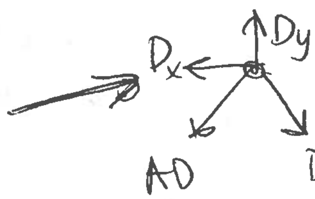
c)







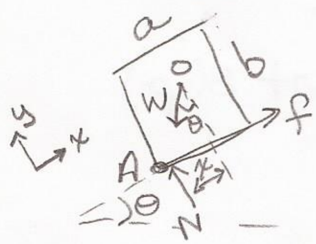
(Also acceptable)



sum of moments for DF will show that $D_y = F_y = P_y/2$ but no assumptions can be made about D_x & F_x without more information!!

Problem 5

a) FBD



$$\sum M_o = 0 = f(b/2) - N(x)$$

Assume simultaneos slip and tip

$$(f = \mu_s N \; ; \; x = a/2)$$

$$0 = \mu_s N(b/2) - N(a/2)$$

$$\rightarrow \mu_s = a/b$$

$\therefore$ for tip to occur before slip,

$$\mu_s > a/b$$

Alternatively:

① $\sum F_x = 0 = f - W\sin\theta \rightarrow f = W\sin\theta$

② $\sum F_y = 0 = N - W\cos\theta \rightarrow N = W\cos\theta$

③ $\sum M_A = W\sin\theta(b/2) - W\cos\theta(a/2) + N(\frac{a}{2} - x) = 0$

Assume simultaneous slip and tip

① + ② $\Rightarrow$ $\mu_s(W\cos\theta) = W\sin\theta$

④ $\mu_s = \tan\theta$

③ $\Rightarrow$ $W\sin\theta(b/2) = W\cos\theta(a/2)$

⑤ $\tan\theta = a/b$

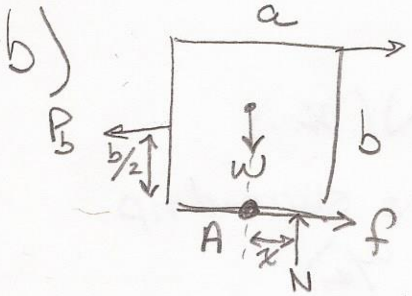
④ + ⑤ $\Rightarrow$ $\mu_s = a/b$

$\therefore$ for tip to occur before slip, $\mu_s > a/b$

Problem 5 (cont.)

FBD

b)



$$\Sigma F_x = 0 = P_b - P_b + f$$
$$\hookrightarrow f = 0 \text{ (cannot slip)}$$

$$\Sigma F_y = 0 = N - W$$
$$\hookrightarrow N = W$$

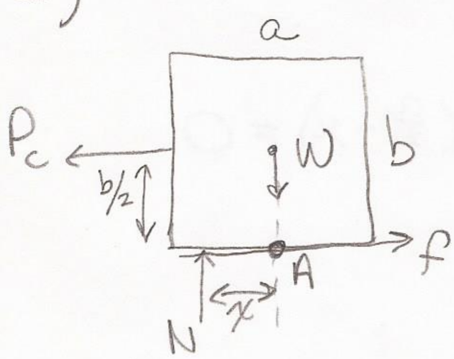
$$\Sigma M_A = 0 = N(x) + P_b(b/2) - P_b(b)$$

Assume tipping $\rightarrow x = a/2$

$$N(a/2) = P_b(b/2) \Rightarrow P_b = N\,a/b$$

$$\Rightarrow P_b = W\,a/b \text{ @ impending tip}$$

c) FBD



$$\Sigma F_x = 0 = f - P_c \Rightarrow P_c = f$$

$$\Sigma F_y = 0 = N - W \Rightarrow N = W$$

Assume impending slip: $f = \mu_s N$

$$\Rightarrow P_{cs} = \mu_s N = \mu_s W = \left(\frac{a}{b}\right) W$$

$$\Sigma M_A = 0 = P_c(b/2) - N(x)$$

Assume impending tip: $x = a/2$

$$P_c(b/2) = N(a/2) = W(a/2)$$

$$\Rightarrow P_{ct} = W\left(\frac{a}{b}\right)$$

Since the magnitude of P_c for impending slip is equal to that of impending tip, they happen simultaneously.