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UNIVERSITY OF CALIFORNIA
Department of Materials Science & Engineering

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MSE 113
Mechanical Behavior of Materials

Midterm Exam #2

Name: ______________________________

SID #: ______________________________

Problem	Total	Score
1	35	
2	40	
3	25	

1) Deformation (35 points)

You are given a uniaxial tensile specimen, with in a uniform cross-sectional area, of a ductile polycrystalline material that behaves according to the following constitutive law:

$$\sigma = k\varepsilon^n$$

where σ and ε are, respectively, the normal true stress and strain, *n* is the strain hardening coefficient and *k* is a scaling constant.

a) On the uniaxial tensile stress-strain diagram below, draw and label the following:
   i. engineering stress vs. engineering strain
   ii. true stress vs. true strain
   iii. where necking occurs
   iv. the ultimate tensile strength, S_u

b) Briefly explain (3 sentences or less) the difference between true stress-strain and engineering stress-strain diagrams.

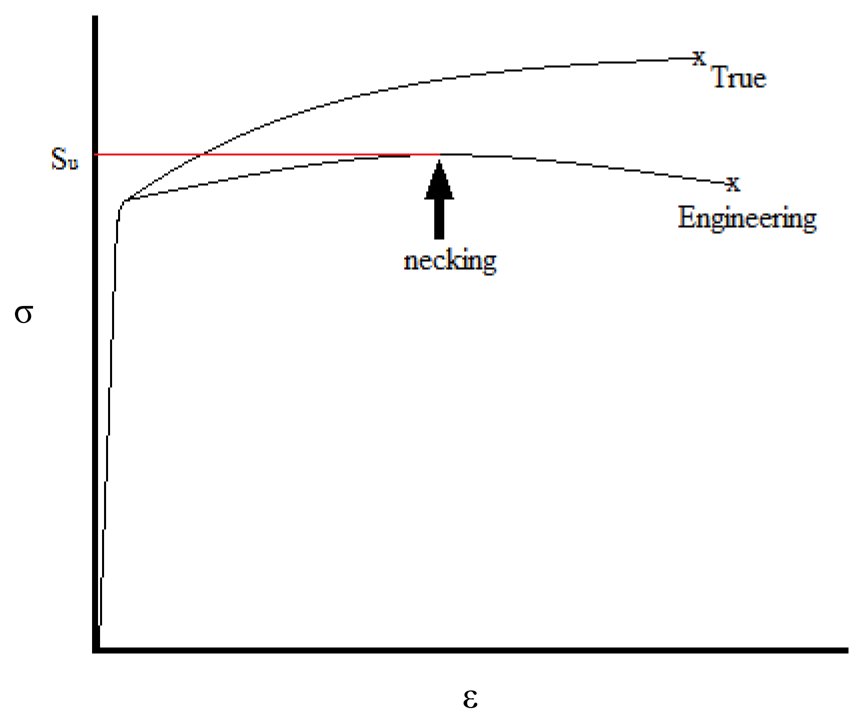
c) Briefly describe (3 sentences or less) what occurs at the microscopic and macroscopic scales at the onset of:
   i. plastic deformation
   ii. necking

d) Derive
   i. an expression relating true stress to engineering stress and engineering strain only
   ii. an expression relating true strain to engineering strain only
   iii. expressions for the true *and* engineering strains at necking

Clearly state the assumptions that you make in the derivations

<u>Solution</u>

1a)



1b)
True stress-strain compensates for changing cross sectional area, while engineering stress-strain uses the original area.

1c)
i. Microscopically, plastic deformation results from dislocation motion and bond breaking. On a macroscopic scale, this results in permanent deformation when the load is removed.

ii. On a microscopic scale, necking results from geometric softening which results in a locally reduced cross sectional area of the test specimen, on a macroscopic scale.

1d)
i. Assume a constant volume:

$$A_o l_o = Al$$
$$\frac{A_o}{A} = \frac{l}{l_o}$$

and

$$\varepsilon_{eng} = \frac{l - l_o}{l_o} = \frac{l}{l_o} - 1$$
$$\frac{l}{l_o} = 1 + \varepsilon_{eng}$$
$$\rightarrow \frac{A_o}{A} = 1 + \varepsilon_{eng}$$

Finally,

$$\sigma_{eng} = \frac{P}{A_o}, \quad \sigma_{true} = \frac{P}{A}$$
$$\sigma_{true} = \sigma_{eng} \frac{A_o}{A}$$
$$\rightarrow \boldsymbol{\sigma_{true} = \sigma_{eng}(1 + \varepsilon_{eng})}$$

ii.

$$d\varepsilon_{true} = \frac{dl}{l}$$
$$\varepsilon_{true} = \int_{l_o}^{l} \frac{dl}{l} = \ln\left(\frac{l}{l_o}\right)$$
$$\rightarrow \boldsymbol{\varepsilon_{true} = ln(1 + \varepsilon_{eng})}$$

iii. At necking: dP = 0

$$P = \sigma_{true} A$$
$$dP = 0 = A d\sigma_{true} + \sigma_{true} dA$$
$$\frac{\sigma_{true}}{d\sigma_{true}} = -\frac{A}{dA}$$

Assume volume constancy

$$dV = 0$$
$$V = lA$$

$$dV = 0 = Adl + ldA$$
$$-\frac{A}{dA} = \frac{l}{dl}$$
$$\rightarrow \frac{\sigma_{true}}{d\sigma_{true}} = \frac{l}{dl}$$

Where,

$$d\varepsilon_{true} = \frac{dl}{l}$$
$$\frac{\sigma_{true}}{d\sigma_{true}} = \frac{1}{d\varepsilon_{true}}$$
$$\sigma_{true} = \frac{d\sigma_{true}}{d\varepsilon_{true}}$$
$$\rightarrow k\varepsilon_{11}^{n} = \frac{d(k\varepsilon_{11}^{n})}{d\varepsilon_{11}}$$
$$k\varepsilon_{11}^{n} = nk\varepsilon_{11}^{n-1}$$
$$\boldsymbol{\varepsilon_{true} = n}$$

Finally,

$$\varepsilon_{true} = ln\left(1 + \varepsilon_{eng}\right)$$
$$\boldsymbol{\varepsilon_{eng} = exp(n) - 1}$$

Note that the subscript 11 is used to note that we are looking at a uniaxial test specimen and that we are considering the strain along the loading or the 1 direction.

2) Fracture Mechanics (40 points)

a) What do you understand by the mechanical property of *toughness*? How does it differ from *strength* and *ductility*?

A processing error during manufacture resulted in a critical steel support rod for a nuclear reactor component being made with a circumferential crack in it. The rod is several feet in length and has a circular cross-section, 12 inches in diameter. It is designed to carry a load of 600 kips. The crack, which emanates from the surface, is estimated to be roughly half way through the cross-section. The material is made of carbon steel and has a plane strain toughness of 50 $ksi\sqrt{in}$. It has a yield strength of 50 *ksi*, elastic modulus *E* of 29,000 *ksi*.

b) Estimate the plastic zone size at the design load? Can we apply LEFM to study the fracture in this case? Will your answer be conservative?

c) Assuming that LEFM can be applied, what is the critical crack depth given:
   i. an operating load of 600 kips, assuming a variable notch size?
   ii. a load enough to cause yielding, assuming the crack is exactly half of the rod thickness?

d) Given what you found above, is the rod about to fail assuming typical operating load?

e) If it is found that the steel has a ductile-brittle-transition-temperature of 50°F. On a cold chilly Bay Area winter morning, the toughness is measured to be 10 $ksi\sqrt{in}$. Is the rod safe given typical operating loads?

State all your assumptions.

<u>Solution</u>

2a) Strength is a measure of the maximum amount of stress a material can withstand, while ductility is a measure of the maximum amount of plastic strain a material can withstand. The combination of strength and ductility yields toughness and is defined as the area under the stress-strain curve. For engineering materials it is desirable to have a combination of strength and ductility, or toughness, in order to have a damage tolerant structure.

2b) Estimate the plastic zone size:

$$r_y = \frac{1}{2\pi}\left(\frac{K_1}{\sigma_y}\right)^2$$

$$r_y = \frac{1}{2\pi}\left(\frac{50\ ksi\sqrt{in}}{50\ ksi}\right)^2$$

$$\boldsymbol{r_y = .16\ in}$$

The current flaw size is:

$$a = \frac{(D-d)}{2} = \frac{(12-6)}{2} = 3\ in$$

LEFM validity criterion and conservative check:

$$a, b, B, w \ \geq 15r_y = 2.39\ in$$

The only relevant dimensions here are the diameters of the notched and un-notched portions of the rod, and the flaw size, which all satisfy the LEFM criterion. Therefore we **can apply LEFM** in this case. Our answer will **not be conservative** unless we specifically apply a Safety Factor in the calculation.

2c) From the stress intensity handout, we consider case 10 for a Circumferentially-notched rod:

$$K_1 = \frac{.932\ P\sqrt{D}}{\sqrt{\pi}\ d^2}$$

Given D = 12 in and d = D/2 = 6 in. The solution is only good for when 1.2 ≤ D/d ≤ 2.1, where our D/d = 2. Note that the critical flaw size in this case is actually not in the equation, we need to calculate it from the notched diameter.

i. Given P = 600 kips and K_{1C} = 50 $ksi\sqrt{in}$, we can solve for the notched diameter which will cause critical failure:

$$K_{1C} = K_1 = \frac{.932 * 600kips\ \sqrt{12\ in}}{\sqrt{\pi}\ (d_c)^2} = 50\ ksi\sqrt{in}$$

$$\rightarrow \boldsymbol{d_c = 4.68\ in}$$

Therefore the critical crack size is half the difference:

$$a_c = \frac{(D-d)}{2} = \frac{(12-4.68)}{2} = \boldsymbol{3.66\ in}$$

ii. Now the notched diameter is assumed to be d = 6 in. The load necessary to cause localized yielding is:

$$P_y = \frac{\pi}{4} d^2 \sigma_y$$

$$P_y = \frac{\pi}{4}(6\ in)^2(50\ ksi)$$

$$P_y = 1413.7\ kips$$

$$K_{1C} = K_1 = \frac{.932 * 1413.7\ kips\ \sqrt{12\ in}}{\sqrt{\pi}\ (d_c)^2} = 50\ ksi\sqrt{in}$$

$$\rightarrow \boldsymbol{d_c = 7.18\ in}$$

The new critical flaw size is:

$$a_c = \frac{(D-d)}{2} = \frac{(12-7.18)}{2} = \boldsymbol{2.41\ in}$$

2d) The current flaw size is:

$$a = \frac{(D-d)}{2} = \frac{(12-6)}{2} = 3\ in$$

Given the operating loading configuration, the critical flaw size, 3.66 in, is larger than the current flaw size. The rod is expected to survive.

2e) The new lower toughness value will reduce the critical notched diameter, recomputed:

$$K_{1C} = K_1 = \frac{.932 * 1413.7\ kips\ \sqrt{12\ in}}{\sqrt{\pi}\ (d_c)^2} = 10\ ksi\sqrt{in}$$
$$\rightarrow d_c = 10.45\ in$$

The critical notched diameter is 10.45 in which is larger than the current notched diameter of 6 inches. The material is expected to fail under the new conditions.

3) Dislocation and Plasticity (25 points)

In a commercially pure metal with a face-centered cubic crystal structure, a perfect dislocation, with Burger's vector $\boldsymbol{b} = {}^{a}\!/_{2}\,[110]$, can be dissociated into two Shockley partial dislocations, with Burger's vectors $\boldsymbol{b_1} = {}^{a}\!/_{6}\,[121]$ and $\boldsymbol{b_2} = {}^{a}\!/_{6}\,[21\bar{1}]$.

a) Is this process energetically favorable?

b) Once the dissociation occurs, what happens to the material between the two partial dislocations?

c) The material has a shear modulus G = 45 GPa, lattice constant $a = 3.68$ Å and a stacking fault energy of $\gamma_s = 14\ x\ 10^{-3}\ J/m$. The width of separation is given by:

$$w = \frac{Gb^2}{2\pi\gamma}$$

Does the value that you get seem realistic?

d) How does the plastic deformation behavior on the microscale of a material with high stacking fault energy compare to one with low stacking fault energy? Briefly discuss the relationship between dissociation of dislocations, dislocation mobility and deformation behavior.

<u>Solution</u>

3a) using Frank's rule, an energetically favorable dislocation disassociation follows:

$$\boldsymbol{b}^2 \geq \boldsymbol{b}_1^2 + \boldsymbol{b}_2^2$$

First check to see if disassociation even makes sense:

$$\boldsymbol{b_1} + \boldsymbol{b_2} = \frac{a}{6}[121] + \frac{a}{6}[21\bar{1}]$$
$$= \frac{a}{6}[330] = \frac{a}{2}[110] = \boldsymbol{b}$$

Now use Frank's rule:

$$\boldsymbol{b}^2 = \frac{a^2}{2^2}(1^2 + 1^2 + 0^2) = \frac{a^2}{2}$$

$$\boldsymbol{b}_1^2 + \boldsymbol{b}_2^2 = \frac{a^2}{6^2}(1^2 + 2^2 + 1^2) + \frac{a^2}{6^2}(2^2 + 1^2 + (-1)^2) = \frac{a^2}{6} + \frac{a^2}{6} = \frac{a^2}{3}$$

$$\boldsymbol{b}^2 \geq \boldsymbol{b}_1^2 + \boldsymbol{b}_2^2$$
$$\frac{a^2}{2} \geq \frac{a^2}{3}$$

Therefore, the disassociation is not only possible but **energetically favorable**.

3b) In between the two partial dislocations is a stacking flaw. This occurs when the stacking order between nearest atomic planes is disrupted. The perfect dislocation keeps the stacking order intact, however the first partial dislocation interrupts the order, but the second one restores the order. In between is material that has normal stacking order again.

3c) The burger's vector for the two partials are $\frac{a}{\sqrt{6}}$, so:

$$w = \frac{45 * 10^9 Pa * (3.68 * 10^{-10} m)^2}{6 * 2\pi(14 * 10^{-3}\ \mathrm{J/m})} = 1.15 * 10^{-8}\ m = \mathbf{11.5\ nm}$$

The width is 11.5 nm, which is about 30 times larger than the lattice spacing. This is an <u>acceptable</u> figure.

3d) In the formula width is inversely related to the stacking fault energy. If the energy is high, the stacking fault width is small. Conversely, low stacking fault energy means high stacking fault width. Stacking fault inhibits dislocation movement. Because when a dislocation has to cross-slip onto another plane, it must re-associate (i.e. remove the stacking fault area) and then glide followed by disassociation again. Therefore a wide stacking fault area, or low stacking fault energy will necessarily cause the material to have higher yielding strength. Conversely, a high stacking fault energy, which has low stacking fault width, will be easier for the dislocation to be mobile and therefore tends to be softer.