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UNIVERSITY OF CALIFORNIA
Department of Materials Science & Engineering

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MSE 113
Mechanical Behavior of Materials

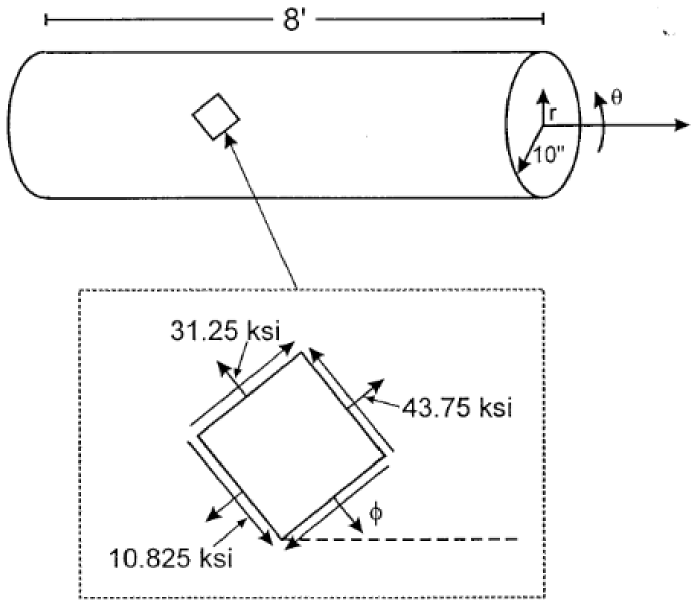
Midterm Exam #1

Name: ____

SID #: ____

Problem	Total	Score
1	35	
2	30	
3	35	

Problem 1 *35 points*



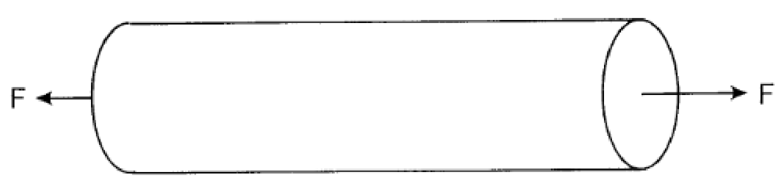
Consider a thin walled, cylindrical tank made of 1080 steel under internal pressure with inside radius, r = 10 in, wall thickness, t = 0.05 in, and a length of 8 feet. Given the stresses acting on an element in the θ-z plane of the cylinder wall oriented at an angle φ from the coordinate axes,

a) Find the angle φ that this element is oriented relative to the coordinate axes.

b) Find $\sigma_{\theta\theta}, \sigma_{zz}, and\ \sigma_{rr}$.

c) Find the internal pressure, P, in the cylinder.

d) Find the axial force, F, that needs to be applied to the cylinder to reach the onset of yielding given that 1080 steel has a tensile yield strength of 85 ksi and ultimate tensile strength of 140 ksi. (Use Tresca criteria.) Hint, this is a 3D problem.



Solution

a.

For the following equations consider: $\sigma_x = 43.75, \quad \sigma_y = 31.25, \quad \tau_{xy} = 10.825$

Center: $\frac{\sigma_x+\sigma_y}{2} = 37.5\ ksi$

Radius: $\sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = 12.5\ ksi$

$$\tan(2\psi) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.732$$

$\rightarrow \boldsymbol{\psi = 30°}$

b.

$\sigma_{max} = \boldsymbol{\sigma_{\theta\theta}} = 37.5 + 12.5 = \mathbf{50\ ksi}$

$\sigma_{min} = \boldsymbol{\sigma_{zz}} = 37.5 - 12.5 = \mathbf{25\ ksi}$

$\boldsymbol{\sigma_{rr} \approx 0}$

c.

$$\sigma_{\theta\theta} = \frac{pr}{t}$$

$$\boldsymbol{p} = \frac{\sigma_{\theta\theta} t}{r} = \frac{50{,}000\ psi * .05"}{10"} = \mathbf{250\ psi}$$

or

$$\sigma_{zz} = \frac{pr}{2t}$$

$$p = \frac{2\sigma_{\theta\theta} t}{r} = \frac{2 * 25{,}000\ psi * .05"}{10"} = 250\ psi$$

d.

Use superposition: Tension + Pressure

$$\tau_{max} = k = \frac{85 + 0}{2} = 42.5\ ksi$$

$\rightarrow \sigma_{tensile\ yield} = 2 * \tau_{max} = 85\ ksi$

For yielding: $\sigma_{zz} = 85\ ksi = \sigma_{zz}^{tension} + \sigma_{zz}^{pressure}$

$\rightarrow \sigma_{zz}^{tension} = 85 - 25 = 60\ ksi$

$$\sigma_{zz}^{tension} = \frac{F}{Area} = \frac{F}{2\pi rt}$$

$\rightarrow \boldsymbol{F} = \sigma_{zz}^{tension} * 2\pi rt = 60\ ksi * 2 * \pi * 10" * .05" = \mathbf{188.5\ kips}$

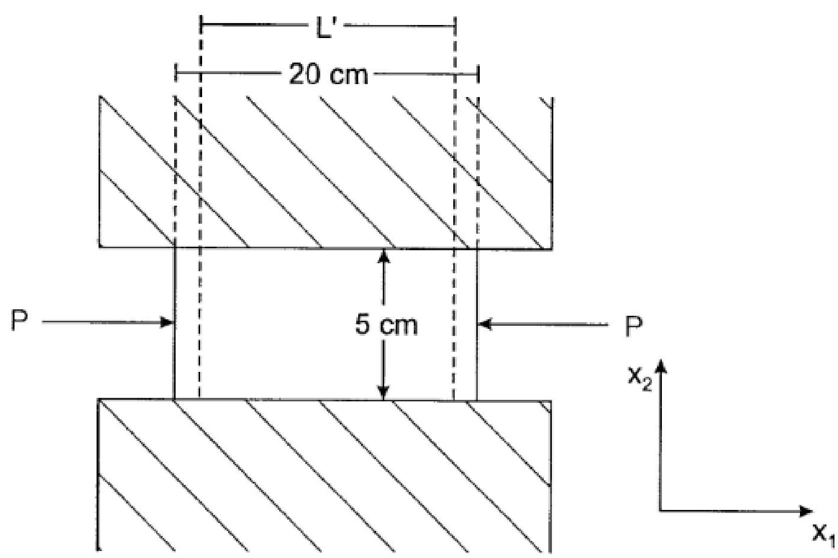
Problem 2 *30 points*

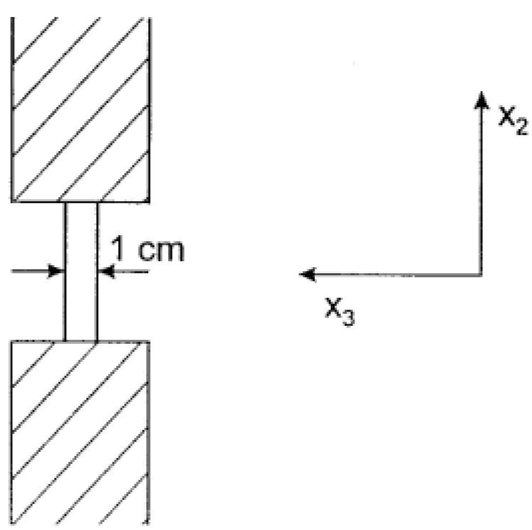
Suppose you have a long thin plane made of 6061-T4 aluminum between two rigid (non-deforming) walls acted on by a compressive force P of 10,000 N in the x_1 direction. The plate has original dimensions: L = 20 cm, b = 5 cm, and t = 1 cm.

Find σ_{22} , ε_{33}, and the new length L'.

Properties of 6061-T4 Aluminum

Young's Modulus (GPa)	Shear Modulus (GPa)	Poisson's Ratio	Tensile Yield Strength (MPa)	Ultimate Tensile Strength (MPa)
70	26.3	1/3	145	240





Solution

$\sigma_{11} = \dfrac{-P}{bt}$ $\quad$ *negative because it is a compressive force*

$\sigma_{11} = \dfrac{-10{,}000\ N}{0.05\ m * 0.01\ m}$

This will produce a stress in the 2 direction, σ_{22}, because of the constraint, $\varepsilon_{22} = 0$. There won't be a stress in the 3 direction, $\sigma_{33} = 0$, because there is no constraint. Instead the material will deform in this direction, ε_{33}.

Note $\sigma_{33} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0$

Setup strain equations:

$\varepsilon_{11} = \frac{1}{E}(\sigma_{11} - \upsilon\sigma_{22})$ $\quad$ $\varepsilon_{22} = \frac{1}{E}(\sigma_{22} - \upsilon\sigma_{11})$ $\quad$ $\varepsilon_{33} = \frac{1}{E}\big(-\upsilon(\sigma_{11} + \sigma_{22})\big)$

$\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{12} = 0$

From $\varepsilon_{22} = 0 \rightarrow \boldsymbol{\sigma_{22}} = \upsilon\sigma_{11} = \frac{1}{3} * -20\ MPa = \mathbf{-6.67\ MPa}$

Then,

$\varepsilon_{33} = \frac{1}{E}\big(-\upsilon(\sigma_{11} + \sigma_{22})\big)$

$\varepsilon_{33} = \frac{1}{70{,}000\ MPa}\left(-\frac{1}{3}(-20\ MPa - 6.67\ MPa)\right)$

$\boldsymbol{\varepsilon_{33} = 1.27 * 10^{-4}}$

Finally,

$\varepsilon_{11} = \frac{1}{E}(\sigma_{11} - \upsilon\sigma_{22})$

$\varepsilon_{11} = \frac{1}{70{,}000\ MPa}\left(-20\ MPa + \dfrac{6.67\ MPa}{3}\right) = -2.54 * 10^{-4}$

Where, $\varepsilon_{11} = \frac{\Delta L}{\mathrm{L}}$

$\rightarrow \Delta\mathrm{L} = L * \varepsilon_{11} = 20\ cm * -2.54 * 10^{-4} = -.005\ cm$

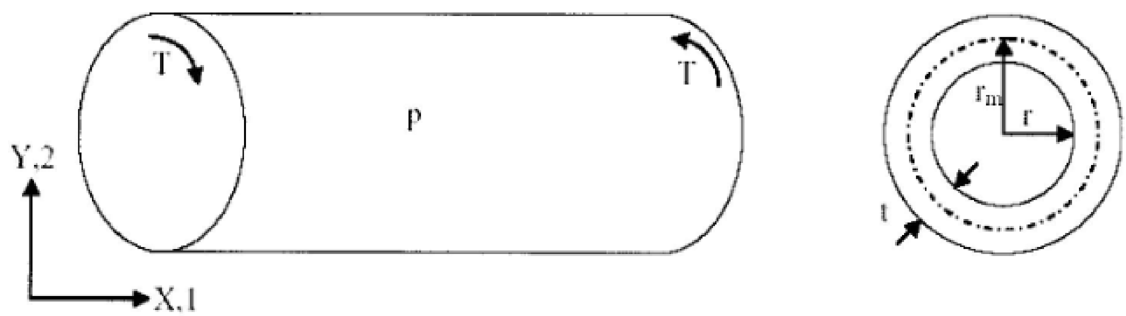
$\boldsymbol{L'} = L + \Delta L = 20\ cm - 0.005\ cm = \mathbf{19.995\ cm}$

Problem 3 *35 points*

The thin-walled cylinder, shown below, has an internal pressure of p = 1000 kPa, and is subjected to a twist of T = 10 MNm. The inner radius of the cylinder is 2 m with a wall thickness of 20 mm. The shear stress on a thin-walled tube can be approximated by:

$$\tau = \sigma_{\theta z} = \frac{T}{2\,\pi\,{r_m}^2\,t}$$

where T is the applied torsional moment, r_m is the radius to the median line, and t is the thickness of the cylinder. See below:



a) Assuming that the ends have no effect on the stresses near the center of the cylinder,
   i. Determine the principal stresses and show them on a sketch of a properly oriented element, i.e. rotated by the principal angle.
   ii. Determine the maximum shear stress.

b) Is there any evidence of plastic yielding in the cylinder if the yield strength of the material used is 180 MPa? (Consider only the θ-z plane, i.e. 2D)

c) If the cylinder is punctured to leave a tiny pinhole in the wall thickness, check if yielding will occur at the edge of the hole using the Tresca and von Mises criteria.

Solution

$$r_m = r + \frac{t}{2} = 2 + \frac{0.02}{2}\ m = 2.01\ m$$

$$\tau = \sigma_{\theta z} = \frac{T}{2\,\pi\, {r_m}^2\, t} = \frac{10 * 10^6 \cdot N \cdot m}{2\pi(2.01\ m)^2 * 0.02\ m} = 1.97 * 10^7\ Pa = 19.7\ MPa$$

$$\sigma_{\theta\theta} = \frac{pr}{t} = \frac{1\ MPa * 2\ m}{0.02\ m} = 100\ MPa$$

$$\sigma_{zz} = \frac{pr}{2t} = \frac{1\ MPa * 2\ m}{2 * 0.02\ m} = 50\ MPa$$

a.

For the following equations,

$\sigma_x = \sigma_{\theta\theta} = 100\ MPa$

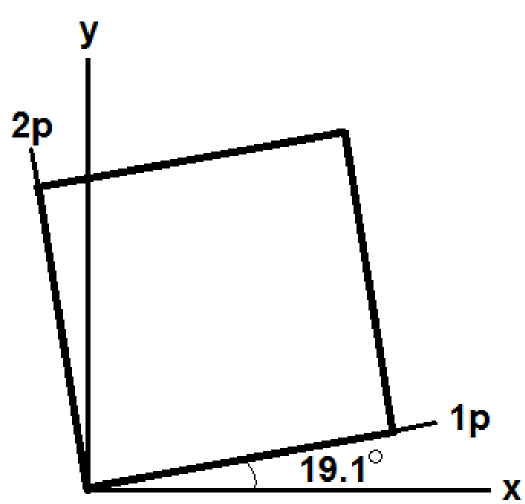
$\sigma_y = \sigma_{zz} = 50\ MPa$

$\tau_{xy} = \tau = 19.7\ MPa$

$$\boldsymbol{\sigma_{1p}} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \mathbf{106.8\ MPa}$$

$$\boldsymbol{\sigma_{2p}} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \mathbf{43.2\ \ MPa}$$

$$\tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \rightarrow \theta_p = 19.1°$$



$$\boldsymbol{\tau_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \mathbf{31.8\ MPa}$$

b.

Tresca

$$k = \frac{\sigma_y}{2} = \frac{180\ MPa}{2} = 90\ MPa\ >\ \tau_{max} = 31.8\ MPa$$

Therefore, does **not** yield.

Von Mises

Consider the stress state in the θ-z plane, called the 1-2 plane here. Note that the stresses in the 3rd direction are zero since this is a 2D problem.

$$\sigma = \sqrt{\frac{1}{2}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + 3({\sigma_{12}}^2 + {\sigma_{23}}^2 + {\sigma_{31}}^2)}$$

$$\sigma = \sqrt{\frac{1}{2}[(100 - 50)^2 + (50 - 0)^2 + (0 - 100)^2] + 3(19.7^2 + 0^2 + 0^2)}$$

$$\sigma = 93.1\ MPa\ <\ \sigma_y = 180\ MPa$$

Therefore, does **not** yield.

c.

$$S_1 =\ \sigma_{1p} = 106.8\ MPa \qquad S_2 =\ \sigma_{2p} = 43.2\ MPa$$

$$\sigma_{A\theta\theta} =\ 3S_2 -\ S_1 = 22.7\ MPa \qquad \sigma_{B\theta\theta} =\ 3S_1 -\ S_2 = 277.3\ MPa$$

$\sigma_{Arr} =\ \sigma_{Brr} = \sigma_{Ar\theta} = \sigma_{Br\theta} = 0$ Note that at the edge of a hole there is no material radially inward. If there is no material present, then there can't be can't be a stress inward. In order to have force balance, there can't be any radial stresses at the edge of a hole.

<u>At point A</u>

Tresca

$$\tau_{max} = \frac{\sigma_{A\theta\theta} - \sigma_{Arr}}{2} = 11.34\ MPa < 90\ MPa = k$$

Von Mises

$$\sigma = \sqrt{\frac{1}{2}[(22.7 - 0)^2 + (0 - 0)^2 + (0 - 22.7)^2] + 3(0^2 + 0^2 + 0^2)} = 11.34 < k$$

Yielding does not occur at point A.

<u>At point B</u>

Tresca

$$\tau_{max} = \frac{\sigma_{A\theta\theta} - \sigma_{Arr}}{2} = 138.7\ MPa > 90\ MPa = k$$

$$\sigma = \sqrt{\frac{1}{2}[(138.7-0)^2+(0-0)^2+(0-138.7)^2]+3(0^2+0^2+0^2)} = 138.7\ MPa > k$$

Yielding occurs at Point B.