EE 20N
Fall 2013

Structure and Interpretation of Signals and Systems

Midterm 1

- You have 1 hours and 30 minutes.
- The exam is closed book, closed notes except a one-page cheat sheet.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.
- Note that the exam is worth 107 points, which means that you can get a 100 on the exam even if you drop 7 points.

First name	
Last name	
SID	

First and last name of student to your left	
First and last name of student to your right	

For staff use only:

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Q2.	I Haven't Plot a Clue	/15
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Q4.	A Tale of Two Series	/17
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Q1. [18 pts] Warmup

Are the following continuous-time real-valued systems linear or non-linear, and time-invariant or time-varying?
You must give an explanation to get any credit even if you circle the correct answer.

(a) [6 pts] $y(t) = x(t^2)$

◯ linear ◯ non-linear

◯ time-invariant ◯ time-varying

(b) [6 pts] $y(t) = x(3t)$

◯ linear ◯ non-linear

◯ time-invariant ◯ time-varying

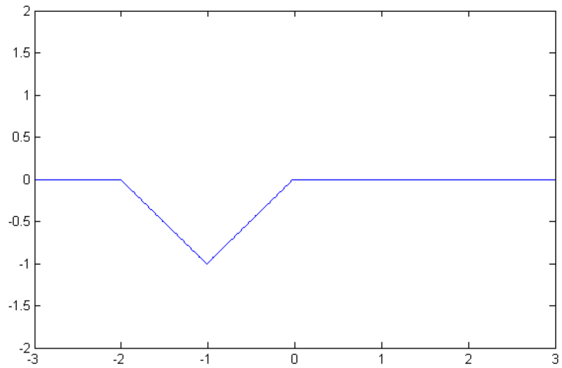
(c) [6 pts] $y(t) = |x(t)|$

◯ linear ◯ non-linear

◯ time-invariant ◯ time-varying

Q2. [15 pts] I Haven't Plot a Clue

Plot $y(t)$ on a well-labeled graph where $x(t)$ is given by the graph below:

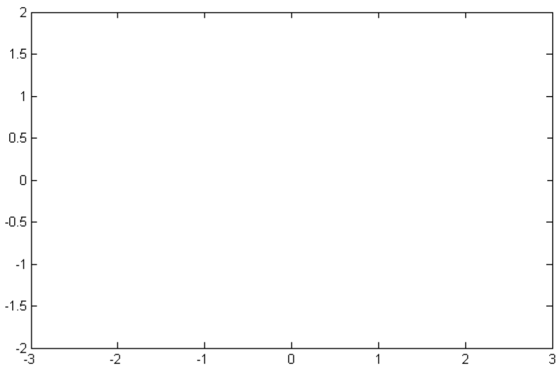


(a) [5 pts] $y(t) = x(t^2)$

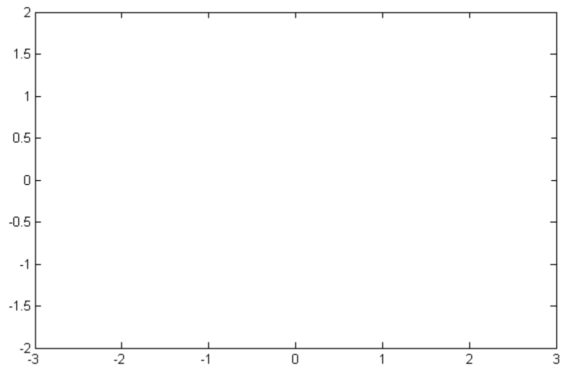
(b) [5 pts] $y(t) = x(3t)$

(c) [5 pts] $y(t) = 3 - x(t - 1)$

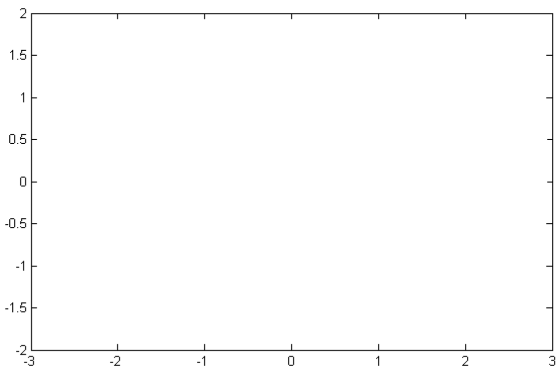
a)



b)

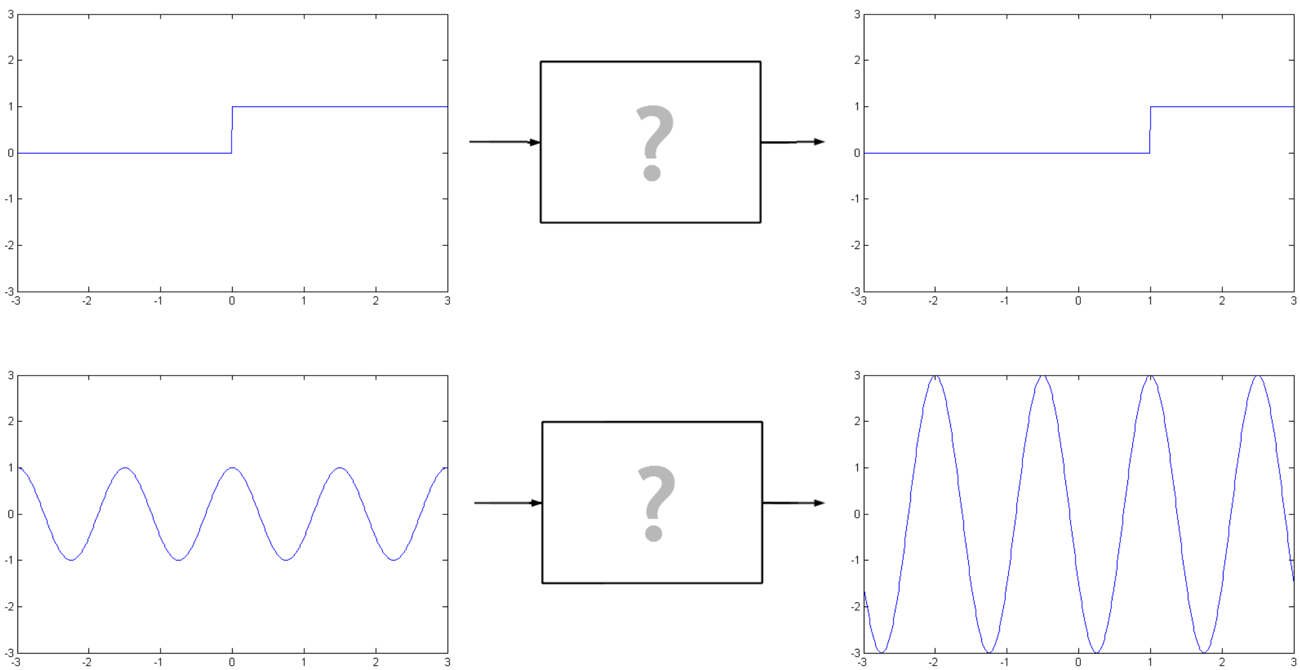


c)



Q3. [8 pts] A Brief Mystery of Time

We want to know if a certain black box is a continuous-time LTI system. We make the following input-output pair observations, where the two input signals below are $x_1(t)$ and $x_2(t)$ respectively, producing the corresponding outputs $y_1(t)$ and $y_2(t)$. The x-axis is in units of time (sec.) and the y-axis in units of Volts.



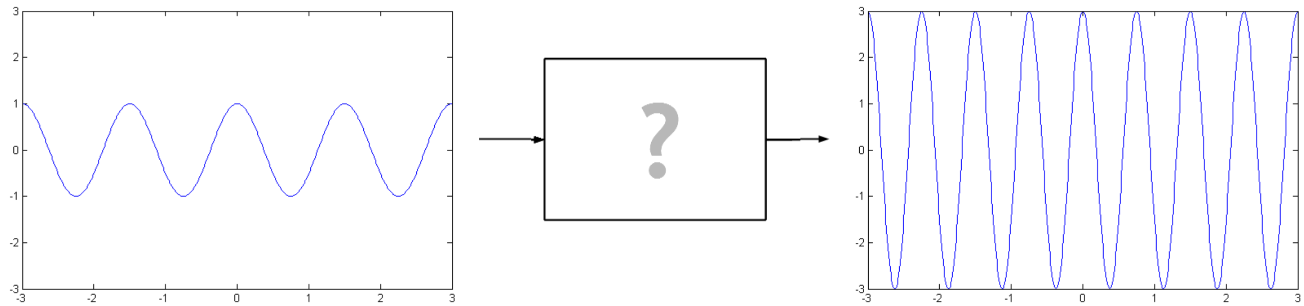
(a) [4 pts] Which of the following is correct? *You must give a short explanation of your answer (1-2 sentences) to get any credit even if you circle the correct answer.*

◯ The system is LTI

◯ The system is not LTI

◯ There isn't enough information to make a conclusion

◯ I haven't studied well enough to answer this



(b) [4 pts] Now suppose that instead of observing the input-output pair $x_2(t)$, $y_2(t)$ in Problem 3 a), you observe the input-output pair $\tilde{x_2}(t), \tilde{y_2}(t)$ shown above. Which of the following is now true? *You must give a short explanation to get any credit even if you circle the correct answer.*

◯ The system is LTI

◯ The system is not LTI

◯ There isn't enough information to make a conclusion

◯ I haven't studied well enough to answer this

Q4. [17 pts] A Tale of Two Series

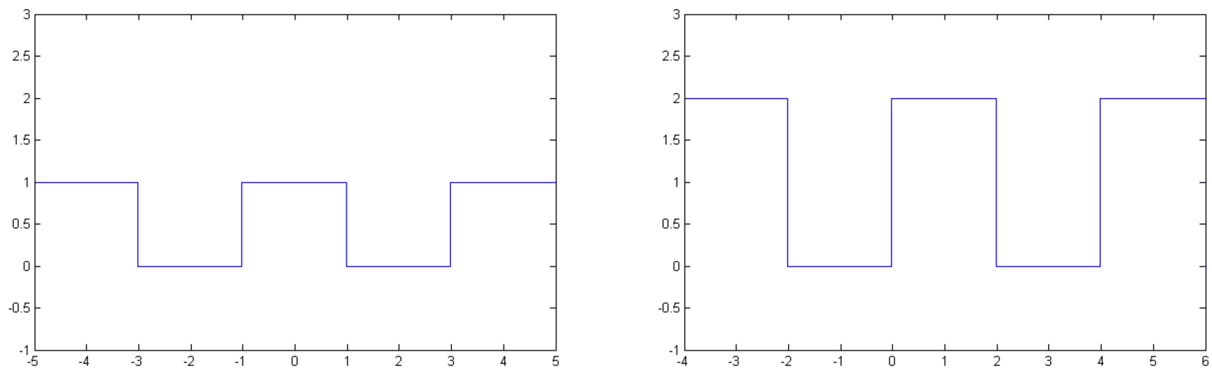
The periodic CT signal $x(t)$ shown below has a CTFS given by

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} B_k \sin(k\omega_0 t)$$

and $y(t)$ has a CTFS expansion given by

$$y(t) = C_0 + \sum_{k=1}^{\infty} C_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} D_k \sin(k\omega_0 t).$$

The plots of $x(t)$ and $y(t)$ are shown below.



(a) [2 pts] Find A_0 and ω_0

(b) [3 pts] Show that $B_k = 0 \ \forall \ k$ **without doing any integration**

(c) [5 pts] Find C_0, C_k, D_k **without doing any integration** in terms of A_0, A_k, B_k, the Fourier series coefficients of $x(t)$.

Now $x(t)$ is passed through system H, and the resulting output is $z(t)$. The frequency response $H(\omega)$ and the Fourier series representation of $z(t)$ are given below where $\omega_c = \frac{4\pi}{3}$ radians/sec.

$$H(\omega) = \begin{cases} 1 & : |\omega| < \omega_c \\ 0 & : |\omega| \geq \omega_c \end{cases}$$

$$z(t) = E_0 + \sum_{k=1}^{K} E_k \cos(k\omega_0 t) + \sum_{k=1}^{K} F_k \sin(k\omega_0 t)$$

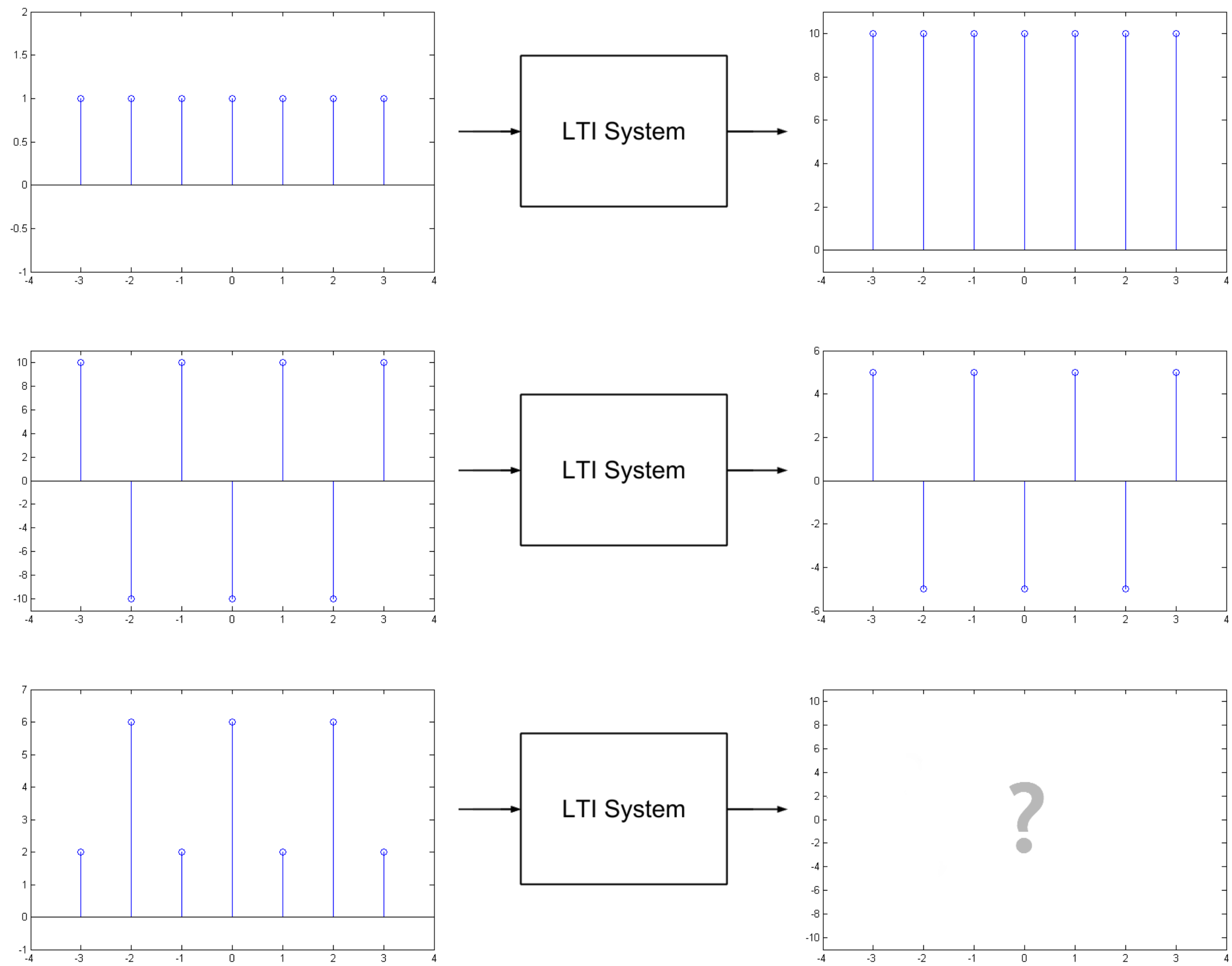
(d) [4 pts] What is the smallest finite value of K needed in the Fourier expansion of $z(t)$?

(e) [3 pts] Do the Fourier series representation of either $x(t)$ or $z(t)$ exhibit Gibbs ringing? *You must give a short explanation for each signal to get any credit even if you circle the correct answer.*

○ Only $x(t)$ ○ Only $z(t)$ ○ Both $x(t)$ and $z(t)$

Q5. [7 pts] To LTI or Not to LTI?

The following black box LTI system exhibits the following behavior for 3 input signals $x_1[n]$, $x_2[n]$ and $x_3[n]$, and their respective outputs $y_1[n]$, $y_2[n]$ and $y_3[n]$.



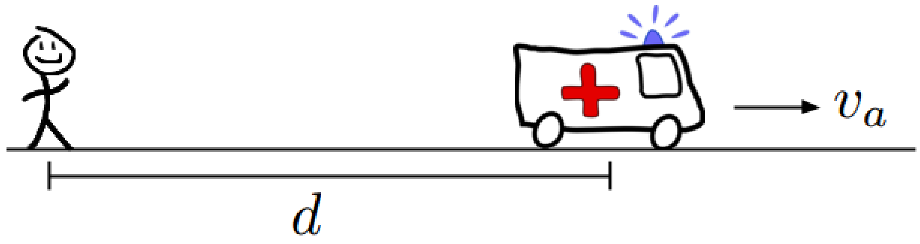
(a) [3 pts] Find $y_3[n]$ and plot it above.

(b) [4 pts] If the LTI system is $y[n] = a_1 x[n-1] + a_2 x[n-2]$, what are a_1 and a_2?

Q6. [10 pts] Ambulance Revisited

An ambulance is speeding away from you at a speed of $v_a = 34$ meters/second. At $t = 0$, the ambulance is $d = 374$ meters away from you. Let $x(t) = \cos(2\pi \times 10^6 t)$ be the sound emitted by the ambulance, and $y(t)$ be the received signal. The relationship between $x(t)$ and $y(t)$ can be described as

$$y(t) = x(\frac{t}{a} - b).$$



(a) [5 pts] Assuming the speed of sound is 340 m/s, find a and b.

(b) [3 pts] Is the system time-invariant? Why or why not?

(c) [2 pts] What is the frequency of the received signal? What do you call the shift in frequency between the signal emitted by the ambulance and the signal you receive?

Q7. [10 pts] Calculation Does Not Imply Causation

Let's assume we know the Fourier series coefficients for

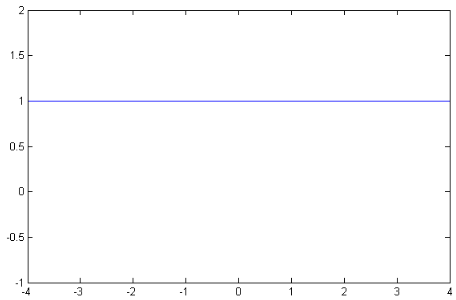
$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + B_k \sin(k\omega_0 t).$$

Now let's assume we have a system $H\{x(t)\}$ which we pass $x(t)$ through. The output $H\{x(t)\} = y(t) = x(2t)$. Solve for the Fourier series coefficients for $y(t)$ in terms of the Fourier series coefficients of $x(t)$.

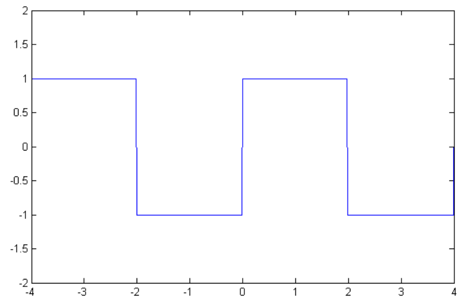
Q8. [10 pts] Odd Job

For each of the following $x(t)$, indicate which set of Fourier series coefficients (A_0, A_k, B_k) can be guaranteed to be zero for all values of k where A_k and B_k are in $x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} B_k \sin(k\omega_0 t)$. *You must provide a short explanation to get any credit even if you circle the correct answer.*

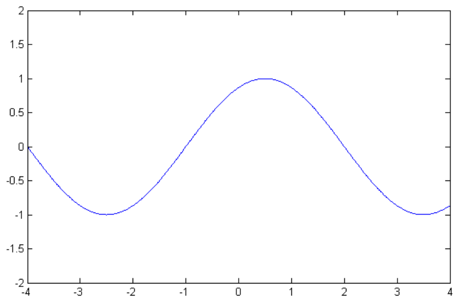
a)



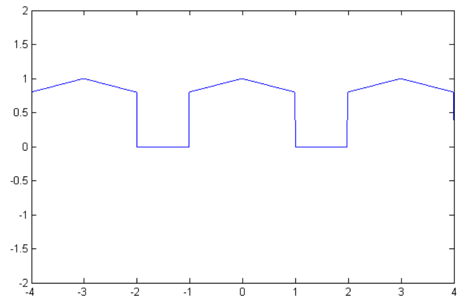
b)



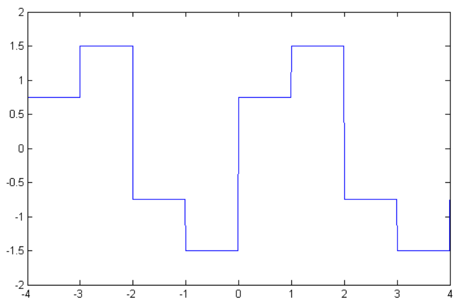
c)



d)



e)



(a) [2 pts] ○ $A_0 = 0$ ○ $A_k = 0$ ○ $B_k = 0$
○ $A_0 \neq 0$ ○ $A_k \neq 0$ ○ $B_k \neq 0$

(b) [2 pts] ○ $A_0 = 0$ ○ $A_k = 0$ ○ $B_k = 0$
○ $A_0 \neq 0$ ○ $A_k \neq 0$ ○ $B_k \neq 0$

(c) [2 pts] ○ $A_0 = 0$ ○ $A_k = 0$ ○ $B_k = 0$
○ $A_0 \neq 0$ ○ $A_k \neq 0$ ○ $B_k \neq 0$

(d) [2 pts] ○ $A_0 = 0$ ○ $A_k = 0$ ○ $B_k = 0$
○ $A_0 \neq 0$ ○ $A_k \neq 0$ ○ $B_k \neq 0$

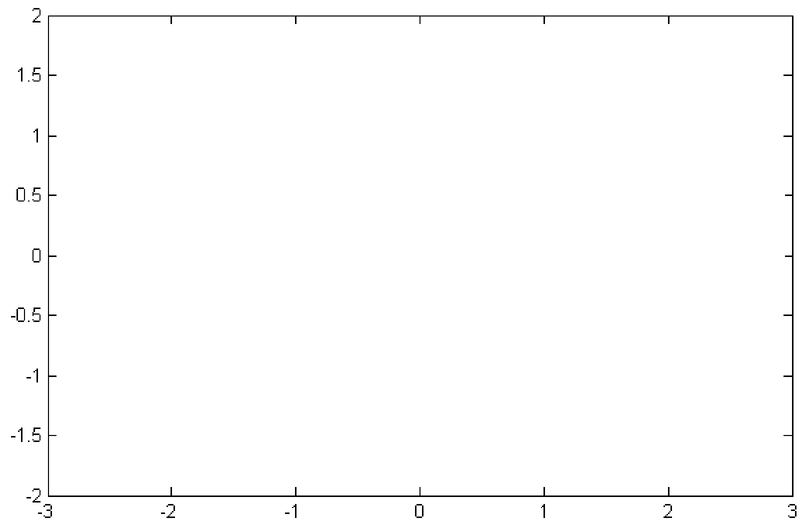
(e) [2 pts] ○ $A_0 = 0$ ○ $A_k = 0$ ○ $B_k = 0$
○ $A_0 \neq 0$ ○ $A_k \neq 0$ ○ $B_k \neq 0$

Q9. [12 pts] What's a Frequency Response?

The input-output relation of continuous-time LTI system F is given as

$$y(t) = 0.5y(t-1) + 0.3x(t)$$

(a) [6 pts] Determine $F(\omega)$ and then plot $|F(\omega)|$.



(b) [6 pts] If the input is $x(t) = 5\sqrt{2} + \frac{5}{3}\cos(\pi t - \frac{7\pi}{12})$, determing the corresponding output.

Useful Formulae

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

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