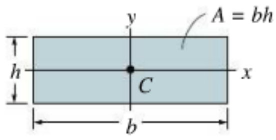
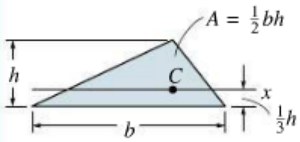
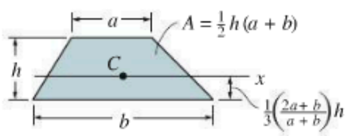
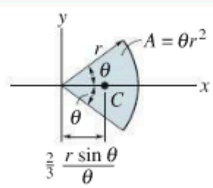
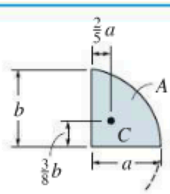
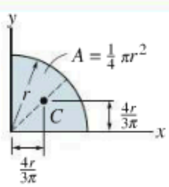
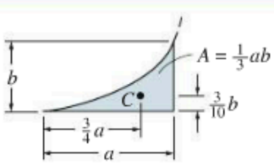
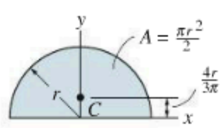
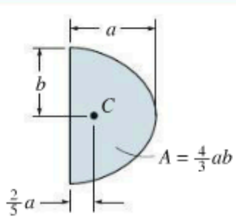
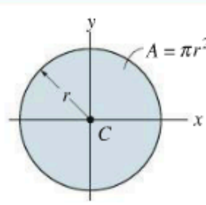
Introduction to Solid Mechanics
ME C85/CE C30

Midterm Exam 2

Fall, 2013

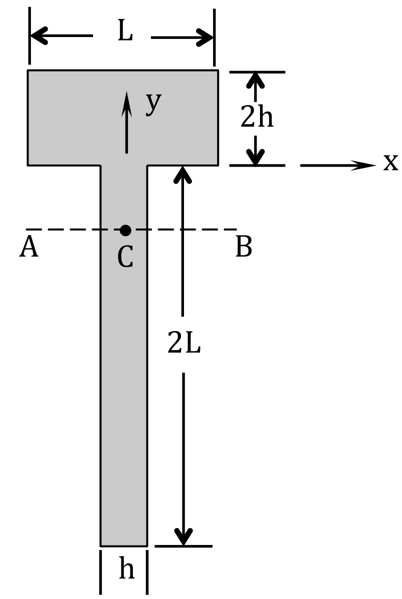
1. Do not open the exam until you are told to begin.
2. Put your name and SID on **every** page of your answer book.
3. Before you begin work on a problem, be sure to read it carefully. There is important information in the text that may help you to solve the problem itself.
4. You may not use a calculator, but you may use a straightedge to help you draw figures.
5. You may use one 8-1/2 x 11 sheet of notes, but not your book or any other notes.
6. Store everything else out of sight.
7. Turn off cell phones.
8. There will be no questions during the exam. Write your concerns or alternative interpretations in exam margins.
9. Write all answers in the answer book provided with this exam.
10. Be concise and write clearly. Identify your answer to a question by putting a box around it.
11. Use only the front sides of the answer sheets for your answers. You may use the backs of pages for "scratch" paper, but if there is work that we should see, be sure to point that out in the main body of the exam.
12. Time will be strictly enforced. At 9:00, you must put down your pencil or pen and immediately turn in your exam. Failure to do so may result in loss of points.

Centroid Location	Area Moment of Inertia
 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Triangular area	$I_x = \frac{1}{36}bh^3$

Centroid Location	Centroid Location	Area Moment of Inertia
 Trapezoidal area	 Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Semiparabolic area	 Quarter circular area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Exparabolic area	 Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Parabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$

Problem 1. (20 Points)

(a) Determine the location of the centroid C of the composite area with respect to the x and y axes shown.

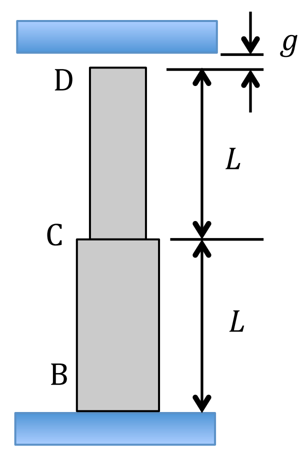


(b) Determine the moment of inertia about a horizontal axis AB passing through the centroid of the area. Your answer should be written in the form:

$$I_{AB} = \frac{Lh}{\alpha}(\beta h^2 + \gamma Lh + \delta L^2)$$

where α, β, γ and δ are integers that may be positive or negative.

Problem 2. (40 Points) The bar shown rests on a rigid base at B and is made of a material with Young's modulus E and linear coefficient of thermal expansion α. (This α is not related in any way to the parameter in Problem 1.) The top segment CD has cross sectional area A and the bottom segment BC has cross sectional area $2A$. Both segments are of length L. When the bar is at temperature T_0 there is a small gap $g \ll L$ between the top of the bar and a rigid restraint. When the bar is at temperature $T_1 > T_0$, the top of the bar just touches the rigid restraint. The rigid restraint causes the bar to remain the same length for any temperature $T_2 > T_1$.



(a) Determine the temperature T_1 in terms of the other parameters of the problem: A, E, L, α, g and T_0. Note: your answer need not involve all six of these parameters.

(b) Determine the force that the rigid restraint exerts on the top of the bar at D when the temperature is $T_2 > T_1$. Your solution should be written in terms of the parameters A, E, L, α, g, T_0 and T_2, again recognizing that your answer may not involve all parameters.

(c) Determine the deflection of the center plane C under the total change in temperature from T_0 to T_2. Your solution should be written in terms of the parameters A, E, L, α, g, T_0 and T_2, again recognizing that your answer may not involve all parameters. Be sure to identify whether this point moves up, down, or does not move at all.

Problem 3. (40 Points) Torque T is applied to the system shown at hub A. The components at B and C are gears, while D and H are pulleys. A belt passes over the pulley at D and under the pulley at H. The end of shaft CDE is supported by an ideal bearing at E. Not shown in this figure are additional ideal bearings near points A, B and C, and the shaft to which the lower pulley is attached.

For this problem, we let the lower pulley at H be fixed (no rotation of H is possible) and do not allow the belt to slip on either pulley. As such, the belt is the only component of this system acting to limit the rotation of the system. However, the belt is elastic, so that it may stretch if it is placed in tension.

Shafts AB, CD and DE are identical, with shear modulus G, length L and polar moment of inertia J. Gear B has radius r_B, and gear C and the two pulleys at D and H all have radius r_C. The belt around D and H has Young's modulus E and cross sectional area A. The distance between the centers of the two pulleys is h.

(a) From the point of view of moment equilibrium about the axes of the shafts, and ignoring any forces that the bearings exert on the shafts, is this system statically determinate? Justify your answer.

(b) Let "F" and "G" denote the two sides of the belt that might be in tension. Which side of the belt (F or G) will be in tension for the counterclockwise direction given for the applied torque T?

(c) Under the applied torque T, how much will the side of the belt that is in tension stretch (i.e., how much longer will that side of the belt get)?

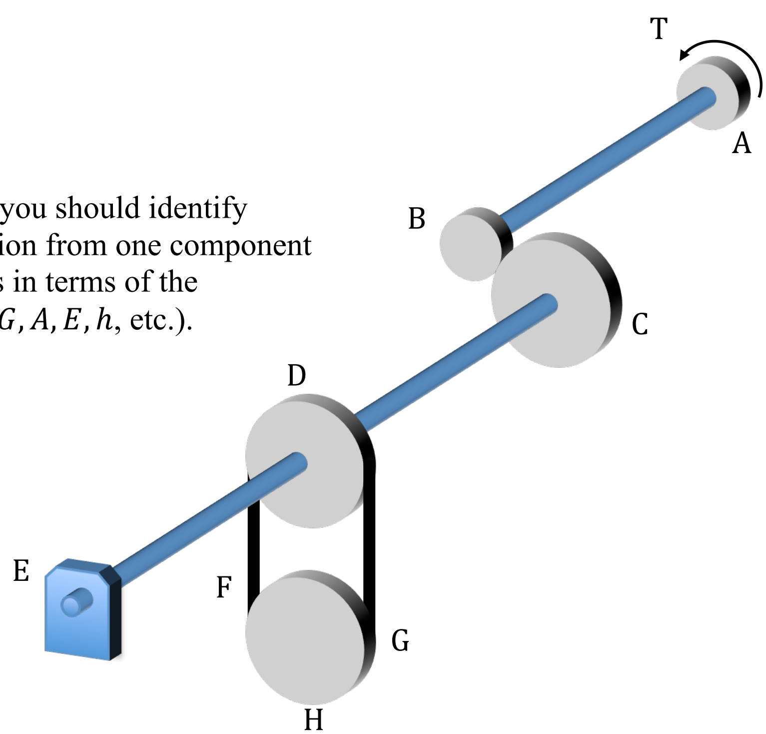
(d) Let the rotation of all components of the system be zero when T = 0. Under the action of T $\neq$ 0, determine the rotations of:

i. Pulley D
ii. Gear C
iii. Gear B
iv. Hub A

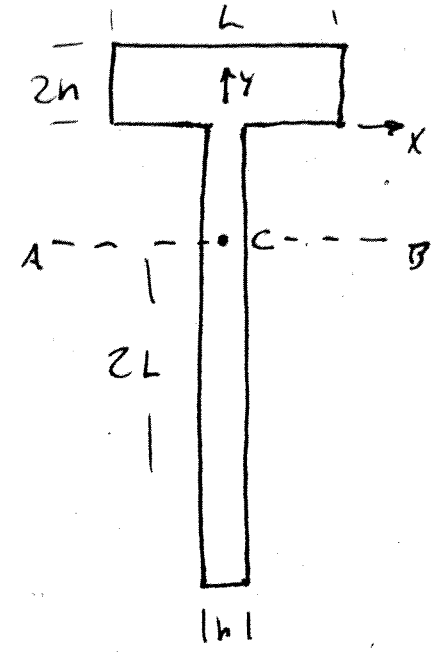
In expressing your answers to Part (d), you should identify the individual contributions to the rotation from one component to the next before showing the rotations in terms of the parameters of the problem (i.e., T, L, J, G, A, E, h, etc.). That is, you should first write

$$\phi_C = \phi_D + \phi_{C/D}$$

then express ϕ_D and $\phi_{C/D}$ in terms of T, L, J, G, A, E, h, etc.



1.



LET TOP SECTION BE 1, BOTTOM 2

a) CENTROID:

$$A_1 = A_2 = 2Lh,\quad A = 4Lh$$

$$\bar{x}_1 = \bar{x}_2 = 0$$

$$\bar{y}_1 = h,\quad \bar{y}_2 = -L$$

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A} = 0$$

$$\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A} = \frac{1}{2}(h-L)$$

b) MOMENT OF INERTIA ABOUT HORIZONTAL AXIS THROUGH C:

$$I_{\bar{x}_1} = \frac{1}{12} L (2h)^3,\quad I_{\bar{x}_2} = \frac{1}{12}(2L)^3 h$$

LET d_1 + d_2 BE THE DISTANCES FROM C TO THE CENTROIDS OF THE RESPECTIVE SECTIONS.

$$I_{AB} = \left(I_{\bar{x}_1} + A_1 d_1^2\right) + \left(I_{\bar{x}_2} + A_2 d_2^2\right)$$

$$\left.\begin{aligned} d_1 &= |\bar{y}| + h = \tfrac{1}{2}(L+h) \\ d_2 &= L - |\bar{y}| = \tfrac{1}{2}(L+h) \end{aligned}\right\} d_1^2 = d_2^2 = \frac{1}{4}(L+h)^2$$

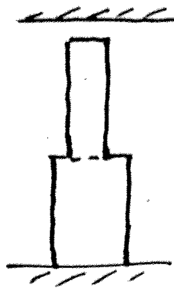
$$I_{AB} = \frac{2}{3}Lh^3 + (2Lh)\left(\frac{1}{4}(L+h)^2\right) + \frac{2}{3}L^3h + (2Lh)\left(\frac{1}{4}(L+h)^2\right)$$

$$= \frac{2}{3}Lh^3 + \frac{2}{3}L^3h + (Lh)(L^2+h^2)$$

$$= \frac{2}{3}(Lh^3 + L^3h) + L^3h + 2L^2h^2 + Lh^3$$

$$\boxed{I_{AB} = \frac{Lh}{3}\left(5h^2 + 6Lh + 5L^2\right)}$$

2.



UNCONSTRAINED THERMAL EXPANSION

$$\varepsilon_T = \alpha(T - T_0)$$

$$\delta_T = (2L)\,\varepsilon_T = 2L\alpha(T - T_0)$$

a) $\delta_T = g$ WHEN $T = T_1$

$$g = 2L\alpha(T_1 - T_0) \Rightarrow \boxed{T_1 = T_0 + \frac{g}{2\alpha L}}$$

b) AT TEMPERATURE T_2, THE UNCONSTRAINED EXPANSION WOULD BE

$$\delta_{T_2} = 2L\alpha(T_2 - T_0) \qquad (1)$$

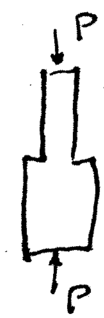
THE FORCE THAT THE UPPER CONSTRAINT SUPPLIES TO THE TOP BAR MUST BE SUFFICIENT TO MOVE THE TOP BACK DOWN BY AN AMOUNT

$$\delta_P = g - \delta_{T_2} \quad (\star) \qquad (2)$$

SO THAT THE TOTAL DEFLECTION OF THE TOP AT T_2 IS

$$\delta_P + \delta_{T_2} = g$$

FBD OF BODY UNDER ACTION OF P:



THE TOP SECTION SHORTENS BY AN AMOUNT

$$\delta_{CD} = -\frac{PL}{AE}$$

THE BOTTOM SECTION SHORTENS BY AN AMOUNT

$$\delta_{BC} = \frac{-PL}{2AE}$$

SO THE TOTAL CHANGE IN LENGTH DUE TO P IS

$$\delta_P = -\frac{3}{2}\frac{PL}{AE} \qquad (3)$$

USING (1) + (3) IN (2) $\Rightarrow$

$$-\frac{3}{2}\frac{PL}{AE} = g - 2L\alpha(T_2 - T_0)$$

$$\boxed{P = \frac{4}{3}AE\alpha(T_2 - T_0) - \frac{2gAE}{3L}}$$

(POSITIVE ACTING DOWN AT THE TOP)

c) DEFLECTION OF POINT C:

$$\delta_C = \underbrace{\delta_{TC}}_{\text{THERMAL}} + \underbrace{\delta_{BC}}_{\text{MECHANICAL}}$$

$$\delta_{TC} = \alpha L(T_2 - T_0)$$

$$\delta_{BC} = \frac{-PL}{2AE} = \frac{L}{2AE}\left(\frac{2gAE}{3L} - \frac{4}{3}AE\alpha(T_2 - T_0)\right)$$

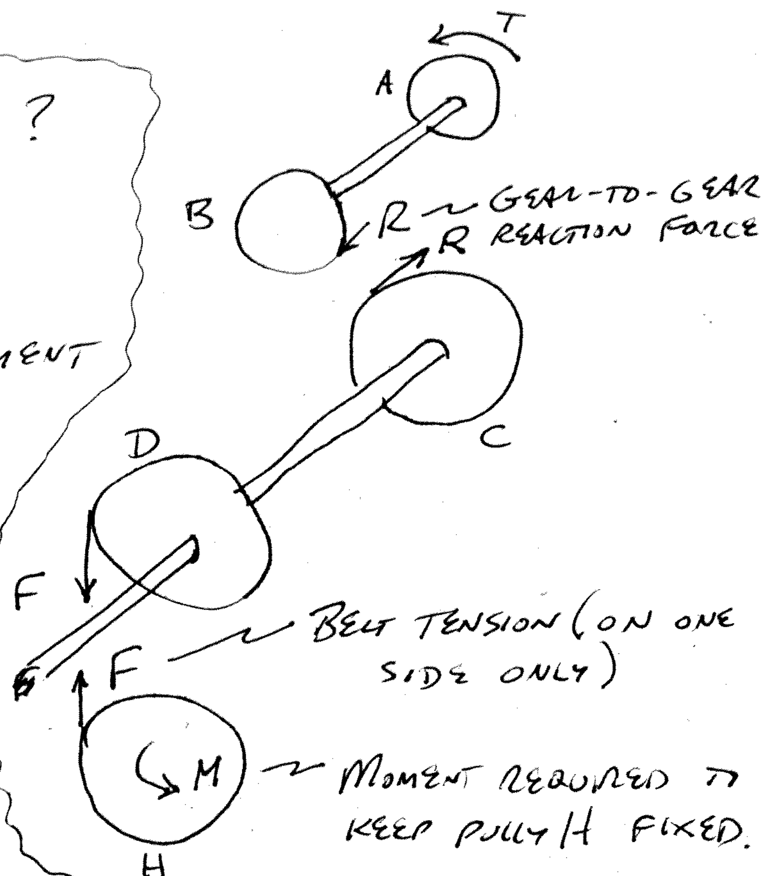
$$= \frac{g}{3} - \frac{2}{3}\alpha L(T_2 - T_0)$$

$$\delta_C = \alpha L(T_2 - T_0) + \frac{g}{3} - \frac{2}{3}\alpha L(T_2 - T_0)$$

$$\boxed{\delta_C = \frac{1}{3}\left(g + \alpha L(T_2 - T_0)\right)}$$

MOVES UP (EVEN IF $g = 0$)

3. FBD'S OF THE VARIOUS SEGMENTS



a) STATICALLY DETERMINATE?
YES. ALL UNKNOWNS (R, F, M) CAN BE DETERMINED FROM MOMENT EQUILIBRIUM EQUATIONS.

b) WITHOUT THE BELT, GEAR B WOULD ROTATE COUNTERCLOCKWISE (CCW) AND GEAR C WOULD ROTATE CLOCKWISE (CW) WITHOUT RESTRAINT. TO PREVENT THAT FREE ROTATION, THE TENSION IN THE BELT MUST CAUSE A CCW MOMENT. THUS, THE SIDE "F" WILL BE IN TENSION, AND SIDE "G" WILL BE SLACK.

c) EQUILIBRIUM OF AB: $\Sigma M = 0 = T - R r_B$

$$R = T / r_B$$

EQUILIBRIUM OF CDE: $\Sigma M = 0 = -R r_c + F r_c$

$$F = R = T / r_B$$

THE PORTION OF THE BELT THAT IS BEING STRETCHED IS OF LENGTH h, SO THE EXTENSION OF THAT SIDE OF THE BELT IS

$$\boxed{\delta_F = \frac{Fh}{AE} = \frac{Th}{r_B AE}}$$

TOPS.

d) ROTATIONS: i. THE ROTATION OF PULLEY D IS LIMITED BY THE STRETCH OF THE BELT, SO

$$r_c \phi_D = \delta_F \Rightarrow \boxed{\phi_D = \frac{Th}{r_B r_c AE}} \quad (CW)$$

ii. $\phi_c = \phi_D + \phi_{c/D}$, $\phi_{c/D} = \frac{T_{CD} L}{JG}$, $T_{CD} = R r_c = \frac{T r_c}{r_B}$

$$\phi_{c/D} = \frac{T r_c L}{r_B JG}$$

$$\boxed{\phi_c = \frac{Th}{r_B r_c AE} + \frac{TL}{JG}\left(\frac{r_c}{r_B}\right)} \quad (CW)$$

iii. $\phi_B r_B = \phi_c r_c \Rightarrow \phi_B = \phi_c \left(\frac{r_c}{r_B}\right)$

$$\boxed{\phi_B = \frac{Th}{r_B^2 AE} + \frac{TL}{JG}\left(\frac{r_c}{r_B}\right)^2} \quad (CCW)$$

iv. $\phi_A = \phi_B + \phi_{A/B}$

$$\phi_{A/B} = \frac{TL}{JG}$$

$$\boxed{\phi_A = \frac{Th}{r_B^2 AE} + \frac{TL}{JG}\left[1 + \left(\frac{r_c}{r_B}\right)^2\right]}$$