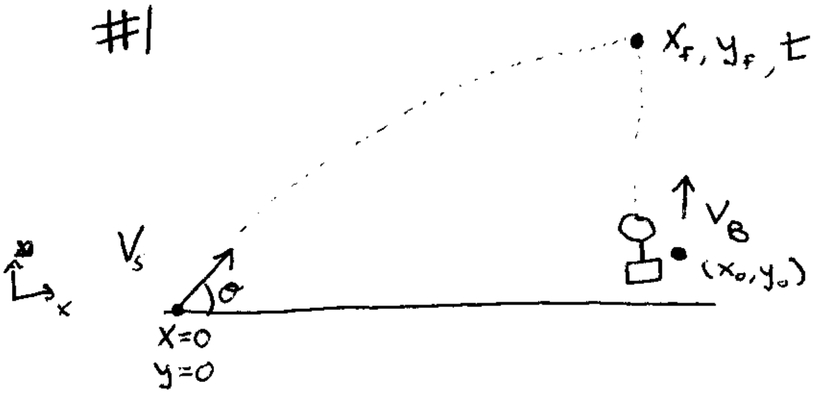
#1



$x_o = 1600\,m$ $\quad V_s = 400\,m/s$

$y_o = 600\,m$ $\quad \theta = 30°$

$m_b = 10_{grams}$

a) $X_s(t) = V_{os}\cos\theta\, t_F = X_f = X_o$

$$\boxed{t_F = \frac{X_o}{V_{os}\cos\theta} = 4.6\,s}$$

b) $y_s(t_F) = V_{os}\sin\theta\, t - 1/2\, g t^2$

$y_B(t_F) = y_o + V_B t$

$y_s(t_F) = y_B(t_F)$

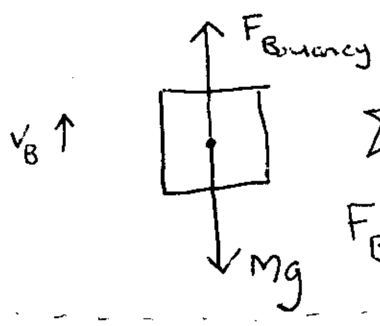
$V_{os}\sin\theta\, t - 1/2\, g t^2 = y_o + V_B t$

$V_B = V_{os}\sin\theta - 1/2\, g t - y_o/t$

$$\boxed{V_B = 47.5\,m/s}$$

c) $$\boxed{\begin{aligned} y_B(t_F) &= y_o + V_B t \\ &= 819\,m \end{aligned}}$$

d) Without Drag (constant velocity) $\qquad$ With Drag



$\sum F_y = M a_y$

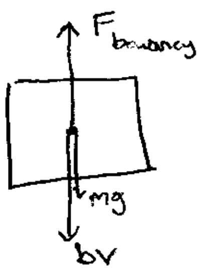
$F_B - Mg = 0$

$\sum F_y = m a_y$

$F_B - Mg - bV = ma$

$F_B - Mg - bV_t = 0$

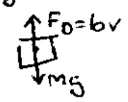
$-bV_t = 0$ $\quad \boxed{V_t = 0}$



$V_t \rightarrow a = 0$

Note IF you assume the balloon has popped:

then:



$\sum F_y = m a_y \Rightarrow bV_t - Mg = 0$

$$\boxed{V_t = mg/b = 980\,m/s}$$

2(a) This question was a bit confusing, however if you think about it for a little bit you will see that part (a) does make sense.

Clearly if we pull on m_3 with a very small force, since there is no friction between m_3 and the ground, the blocks will begin to move with very small static frictional forces holding them together. Eventually the external force will be large enough to cause the other two blocks to slide relative to the bottom one. However, there are two very different possible outcomes. Either m_1 will slide off of m_2 while m_2 moves along with m_3 held on by static friction between m_2 and m_3 (m_3 and m_2 basically behave like one block and m_1 slides off of them) in which case the problem no longer makes sense because now m_1 will be on top of m_3 and we have no idea what the coefficient of friction between m_1 and m_3 is, or the top two blocks will remain fixed together by static friction between m_1 and m_2 and they will slide off as a whole unit; this is the only way in which the question makes sense.

Knowing that m_1 and m_2 slide off together we can now treat them as one object of mass $m_1 + m_2$ and we can ask, what is the minimum force needed to pull m_3 out from under this composite block, or equivalently, what is the maximum force we can exert on m_3 before the other two blocks start to slide relative to m_3.

If the other two blocks begin to slide, that means that the acceleration of m_3 in the horizontal direction, a, is larger than the maximum horizontal acceleration that can be imparted to the other two blocks. The only force that can be exerted on the other two blocks (treating them as one big block) is the frictional force between block m_2 and m_3 and the maximum value that frictional force can take is $\mu_2 N$, where N is the normal force exerted on the composite two block system by m_3. N is clearly equal to $(m_1 + m_2)g$ since gravity and the normal force are the only two forces that act on the top two blocks in the vertical direction, and since the blocks do not move in this direction they must cancel one another.

So the maximum force that can be exerted on the top two blocks is $\mu_2(m_1 + m_2)g$ which imparts an acceleration of $\mu_2 g$ to the top to blocks. Thus as soon as the acceleration of the three blocks as a whole reaches this value the top two blocks will begin to slip. The horizontal force exerted on the three blocks as a whole is just $F_{\text{ext}} \cos\theta$ and thus the acceleration is

$$a = \frac{F_{\text{ext}} \cos\theta}{m_1 + m_2 + m_3}$$

by Newton's second law. The blocks begin to slide when $a = \mu_2 g$, and this happens when

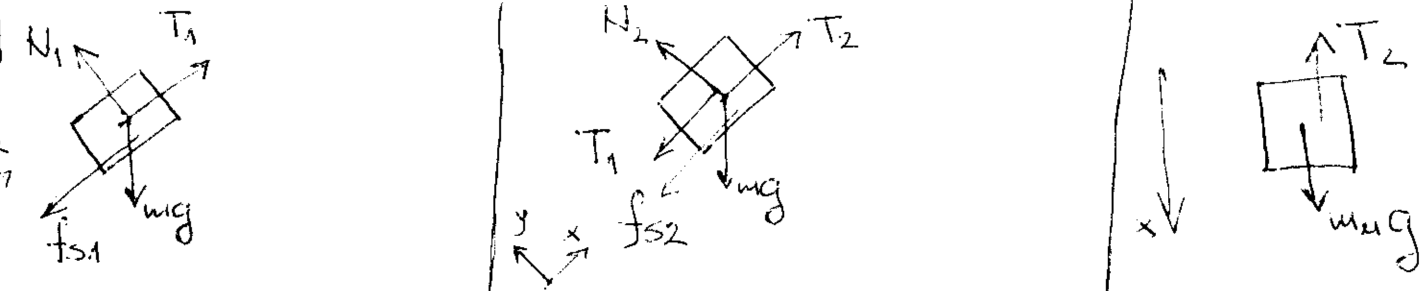
$$\frac{F_{\text{ext}} \cos\theta}{m_1 + m_2 + m_3} = \mu_2 g$$

$$\Rightarrow F_{\text{ext}} = \frac{\mu_2 g(m_1 + m_2 + m_3)}{\cos\theta}$$

Hence the minimum force necessary to be able to pull m_3 out from under the other two blocks is

$$\boxed{F_{\text{min}} = \frac{\mu_2 g(m_1 + m_2 + m_3)}{\cos\theta}}$$

a)



b)

x: $ma = T_1 - f_{s1} - mg\sin\phi$ | $ma = T_2 - T_1 - f_{s2} - mg\sin\phi$ | $m_\mu a = m_\mu g - T_2$

y: $0 = N_1 - mg\cos\phi$ | $0 = N_2 - mg\cos\phi$

The smallest m_μ that gets the blocks moving is the one that corresponds to maximal static frictions: $f_{s1} = \mu_s N_1$ and $f_{s2} = \mu_s N_2$

but no acceleration ($a=0$):

$$0 = T_1 - \mu_s mg\cos\phi - mg\sin\phi$$
$$0 = T_2 - T_1 - \mu_s mg\cos\phi - mg\sin\phi$$
$$+\quad 0 = m_\mu g - T_2$$

$$0 = m_\mu g - 2\mu_s mg\cos\phi - 2mg\sin\phi$$

minimal $\boxed{m_\mu = 2m(\mu_s\cos\phi + \sin\phi)}$

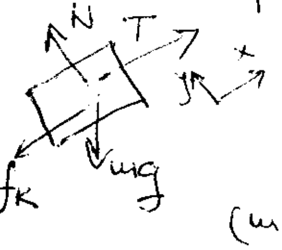
c) after cutting the lower block slides down:

x: $ma = mg\sin\phi - \mu_k N$

y: $0 = N - mg\cos\phi \qquad N = mg\cos\phi$

$\boxed{a = g(\sin\phi - \mu_k\cos\phi)}$

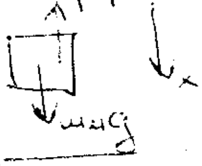
the top block slides up the inclined plane:



x: $ma = T - \mu_k N - mg\sin\phi$

$0 = N - mg\cos\phi$

$ma = T - \mu_k mg\cos\phi - mg\sin\phi$



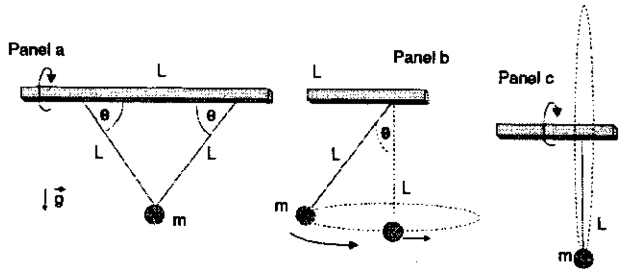
x: $m_\mu a = m_\mu g - T$

$(m + m_\mu)a = g(m_\mu - m(\mu_k\cos\phi + \sin\phi))$

$\boxed{a = g\,\dfrac{m_\mu - m(\mu_k\cos\phi + \sin\phi)}{m_\mu + m}}$

Problem 4 – (25 points)

A mass m is attached to a pole by two massless strings as shown (panel a). The length of the strings and their horizontal separation along the pole are all L. The mass is circling the pole with a speed v that is high enough to keep the strings taut at an angle $\theta = 60°$. The acceleration of gravity **g** points downward on the page. The direction of rotation is counterclockwise, but this doesn't matter.



a) (5pts) Draw the free body diagram for the mass.

b) (5pts) Find the tensions T_1 and T_2 for each string.

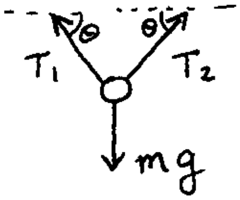
c) (5pts) What is the magnitude and direction of the acceleration of the mass at the instant shown?

d) (5pts) We now cut one string.

The mass will be revolving around as a conical pendulum (see panel b) at an angle θ, where θ is the angle the string makes with the vertical to the pole. Calculate the velocity and the period of the ball.

e) (5pts) If we now let the mass m swing around the pole (panel c) how will its velocity change? Describe how this case differs from d)?

(a)



$\uparrow a = \frac{v^2}{r}$

(b). $y \uparrow \oplus$, $\rightarrow x$

① x: $T_2 \cos\theta - T_1 \cos\theta = 0$

② y: $T_1 \sin\theta + T_2 \sin\theta - mg = m\frac{v^2}{r}$

①$\Rightarrow$ $T_1 = T_2 = T$, $(r = L\sin\theta)$

② $\Rightarrow$ $2T\sin\theta = m\frac{v^2}{L\sin\theta} + mg$.

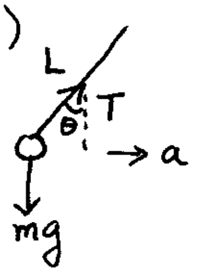
$$\Rightarrow \boxed{T = m\frac{v^2}{2L\sin^2\theta} + \frac{mg}{2\sin\theta}}.$$

(c).

$a = \frac{v^2}{r} = \frac{v^2}{L\sin\theta}$.

going up.

(d)



$y \uparrow \oplus$, $\rightarrow x$

① y: $T\cos\theta - mg = 0$

② x: $T\sin\theta = ma = m\frac{v^2}{r}$

$r = L\sin\theta$: ① $\Rightarrow$ $T = \frac{mg}{\cos\theta}$,

② $\Rightarrow$ $v^2 = \frac{r \cdot T\sin\theta}{m} = \frac{\cancel{m} g L \sin^2\theta}{\cancel{m}\cos\theta}$

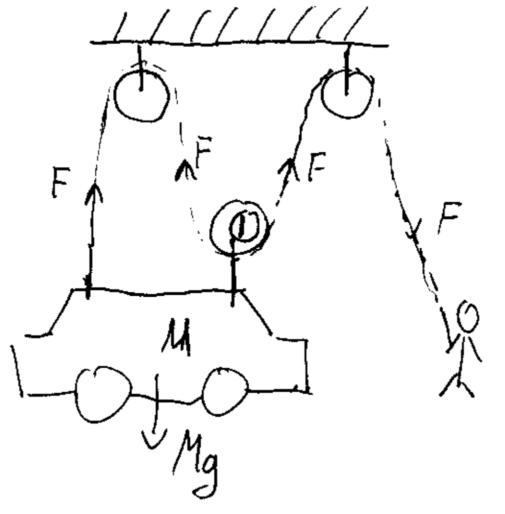
$$\Rightarrow \boxed{v = (gL\tan\theta \cdot \sin\theta)^{1/2}}$$

$$\boxed{T_0 = \frac{2\pi r}{v} = \frac{2\pi L\sin\theta}{(gL\tan\theta \cdot \sin\theta)^{1/2}} .}$$

(e) In panel c, the velocity decreases when the ball is swinging up, due to the deceleration from gravity; the velocity then increases when the ball is swinging down. So the speed is not constant in panel c, whereas the speed is constant in panel b.

Problem 5

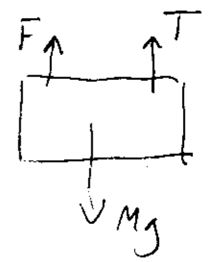
a)



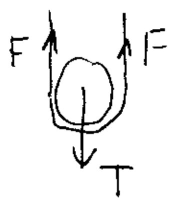
Use a system of 3 pulleys

one pulley is attached to the car
two pulleys are attached to the ceiling
One end of the rope is fixed to the Car's roof. The man pulls on the other end of the rope.

b) FBD (car)



FBD (pulley 1)



move car at constant speed $(a = 0)$

for the car $\quad F_{net} = F + T - Mg = Ma = 0$

for pulley 1 $\quad F_{net} = 2F - T = 0 \quad$ (pulley is massless, no acceleration in the system)

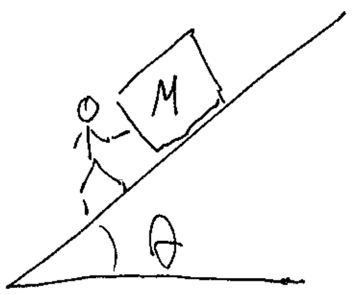
$\Rightarrow T = 2F$

$0 = F + T - Mg = F + 2F - Mg = 3F - Mg.$

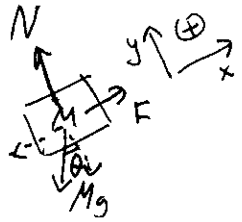
$\Rightarrow F = Mg/3$

second scheme

Man pushes the car up the inclined plane



FBD (car)



$$F_{net,x} = F - Mg\sin\theta = Ma$$

move car at constant speed

$$a = 0$$

$$F = Mg\sin\theta$$

If α $\sin\theta \le \frac{1}{3}$ $\quad F \le \frac{Mg}{3}$