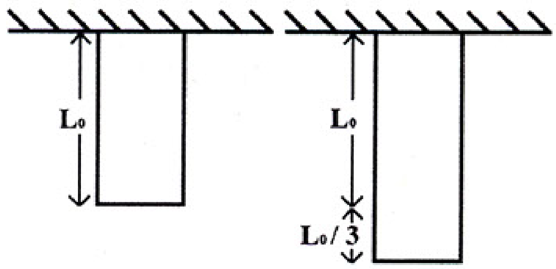
Name Key

SID

BioE 102 Fall 2013
Midterm #1

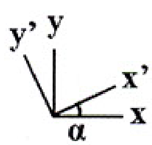
Instructions: Please write legibly; write your name and SID on the upper right corner of each page.

1. A beam with length L_0 is anchored to the ceiling as shown in the diagram below. A stress is applied such that it extends by a length of ${}^{L_0}/_3$. You can disregard any gravitational force. (1) Calculate the strain in the y direction due to the applied stress. (2) Sketch plots for $\varepsilon_{xx}'(\alpha)$, $\varepsilon_{yy}'(\alpha)$, and $\varepsilon_{xy}'(\alpha)$ for α from 0° to 90°. Be sure to have the correct curvature for each plot. Label maximum and minimum values for all plots. (30 points)



$x - x_0 = \frac{L_0}{3}$

$x_0 - 0 = L_0$



6 points

$$\boxed{\varepsilon_{yy} = \frac{x - x_0}{x_0 - 0} = \frac{L_0/3}{L_0} = \frac{1}{3} \qquad \varepsilon_{xx} = 0 \qquad \varepsilon_{xy} = 0}$$

$$\varepsilon_{xx}' = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos(2\alpha) + \varepsilon_{xy}\sin(2\alpha)$$

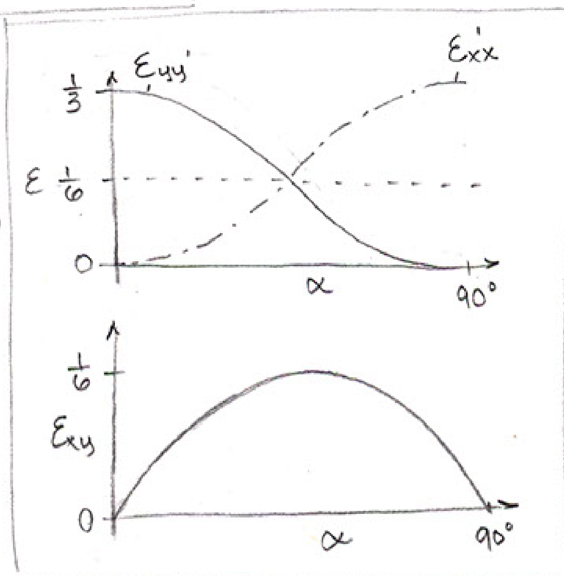
$$= \frac{1}{6} - \frac{1}{6}\cos(2\alpha)$$

$$\varepsilon_{yy}' = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{yy}-\varepsilon_{xx}}{2}\cos(2\alpha) + \varepsilon_{xy}\sin(2\alpha)$$

$$= \frac{1}{6} + \frac{1}{6}\cos(2\alpha)$$

$$\varepsilon_{xy}' = \frac{\varepsilon_{yy}-\varepsilon_{xx}}{2}\sin(2\alpha) + \varepsilon_{xy}\cos(2\alpha)$$

$$= \frac{1}{6}\sin(2\alpha)$$



Axes variables Labelled (1 pt ea) × 3
Axes Max/Min (3 pt ea) × 3
Correct curvature (4 pt ea) × 3

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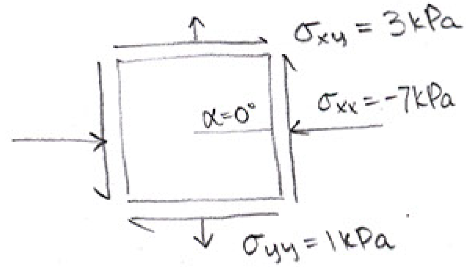
2. Given $\sigma_{xx} = -7\ kPa$, $\sigma_{yy} = 1\ kPa$, and $\sigma_{xy} = 3\ kPa$ at point p, calculate the principle stresses and maximum shear stress. What are the values of α_p and α_s? Draw a 2D representation of the stresses at rotations of α_p and α_s. (40 points)



$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

$$= \frac{-7+1}{2} \pm \sqrt{\left(\frac{-7-1}{2}\right)^2 + 3^2}$$

$$= -3 \pm \sqrt{16 + 9}$$

$$= -3 \pm 5 = \boxed{2kPa, -8kPa}$$

$$\tau_m = \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{-7-1}{2}\right)^2 + 3^2}$$

$$= \boxed{\pm 5kPa}$$

$$\alpha_p = \frac{1}{2} \tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}}\right)$$

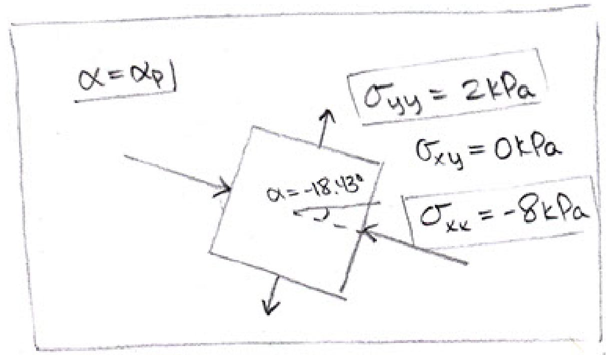
$$= \frac{1}{2} \tan^{-1}\left(\frac{2 \cdot 3}{-7-1}\right)$$

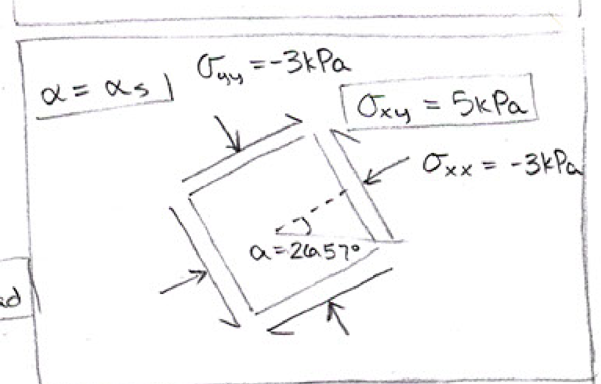
$$= \frac{1}{2} \tan^{-1}\left(\frac{-3}{4}\right) = \boxed{-18.43°} \text{ or } \boxed{-.322 \text{ rad}}$$

$$\alpha_s = \frac{1}{2} \tan^{-1}\left(\frac{\sigma_{yy} - \sigma_{xx}}{2\sigma_{xy}}\right)$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{1+7}{2 \cdot 3}\right)$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{4}{3}\right) = \boxed{26.57°} \text{ or } \boxed{0.464 \text{ rad}}$$





Correct Rotation (α indicated) (2 pt ea) × 2
Correct Arrow Convention (5 pt ea) × 2
Values indicated (3 pt ea) × 2

Numerical Answers: (4 pt ea) × 5
↳ −2 ea for lack of units

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3. You're placing strain gauges when you accidentally realized you have misplace one. As the savvy engineer you are, you decide you can spare yourself the extra work by quickly recalculating equations for the principle strains using the gauges as they are.

The gauges are placed at angles of $\alpha_1 = 0°$, $\alpha_2 = 30°$, and $\alpha_3 = 90°$. (1) Solve for ε_{xx}, ε_{yy}, and ε_{xy} in terms of $\varepsilon_{0°}$, $\varepsilon_{30°}$, and $\varepsilon_{90°}$. (2) Determine general equations for the principle strains in terms of $\varepsilon_{0°}$, $\varepsilon_{30°}$, and $\varepsilon_{90°}$. (30 points)

Numerical Answers

7pt ea. ×3

$$\varepsilon_{0°} = \varepsilon_{xx}\big|_{\alpha=0°} = \varepsilon_{xx}\cos^2(\alpha) + 2\varepsilon_{xy}\sin(\alpha)\cos(\alpha) + \varepsilon_{yy}\sin^2(\alpha)$$
$$= \varepsilon_{xx}(1)^2 + 2\varepsilon_{xy}(0)(1) + \varepsilon_{yy}(0)^2$$
$$= \varepsilon_{xx}$$
$$\Rightarrow \boxed{\varepsilon_{xx} = \varepsilon_{0°}}$$

$$\varepsilon_{90°} = \varepsilon_{xx}\big|_{\alpha=90°} = \varepsilon_{xx}\cos^2(\alpha) + 2\varepsilon_{xy}\sin(\alpha)\cos(\alpha) + \varepsilon_{yy}\sin^2(\alpha)$$
$$= \varepsilon_{xx}(0)^2 + 2\varepsilon_{xy}(1)(0) + \varepsilon_{yy}(1)^2$$
$$= \varepsilon_{yy}$$
$$\Rightarrow \boxed{\varepsilon_{yy} = \varepsilon_{90°}}$$

$$\varepsilon_{30°} = \varepsilon_{xx}\big|_{\alpha=30°} = \varepsilon_{xx}\cos^2(\alpha) + 2\varepsilon_{xy}\sin(\alpha)\cos(\alpha) + \varepsilon_{yy}\sin^2(\alpha)$$
$$= \varepsilon_{xx}\left(\tfrac{\sqrt{3}}{2}\right)^2 + 2\varepsilon_{xy}\left(\tfrac{1}{2}\right)\left(\tfrac{\sqrt{3}}{2}\right) + \varepsilon_{yy}\sin\left(\tfrac{1}{2}\right)^2$$
$$= \tfrac{3}{4}\varepsilon_{xx} + \tfrac{\sqrt{3}}{2}\varepsilon_{xy} + \tfrac{1}{4}\varepsilon_{yy}$$
$$\Rightarrow \boxed{\varepsilon_{xy} = \tfrac{2}{\sqrt{3}}\varepsilon_{30°} - \tfrac{\sqrt{3}}{2}\varepsilon_{0°} - \tfrac{1}{2\sqrt{3}}\varepsilon_{90°}}$$

Equation

9 pt

$$\varepsilon_{1,2} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}^2}$$
$$= \boxed{\frac{\varepsilon_{0°} + \varepsilon_{90°}}{2} \pm \sqrt{\left(\frac{\varepsilon_{0} - \varepsilon_{90}}{2}\right)^2 + \left(\tfrac{2}{\sqrt{3}}\varepsilon_{30°} - \tfrac{\sqrt{3}}{2}\varepsilon_{0°} - \tfrac{1}{2\sqrt{3}}\varepsilon_{90°}\right)^2}}$$
$$= \frac{\varepsilon_{0} + \varepsilon_{90}}{2} \pm \frac{\sqrt{3}}{3}\sqrt{3\varepsilon_0^2 - 6\varepsilon_0\varepsilon_{30} + 4\varepsilon_{30}^2 - 2\varepsilon_{30}\varepsilon_{90} + \varepsilon_{90}^2}$$