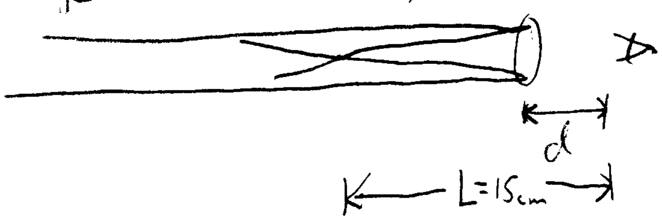
Phys 7c - Spring 2004 - Sect 2 (Lee) - Final Solutions

1) Let the lens be a distance d from the person's eye.
An image infinitely far away need to make an image $L=15$ cm from the persons eye,



Assume $d<L$ (only pinnochio could have otherwise)

Lens eqn:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

$$d_o \to \infty$$

$$\frac{1}{d_i}=\frac{1}{f}$$

d_i in this case is (a) negative (on same side of lens as object) and (b) $|d_i|=|L-d| \Rightarrow d_i=-(L-d)$

$$-\frac{1}{L-d}=\frac{1}{f}$$

$$f=d-L$$

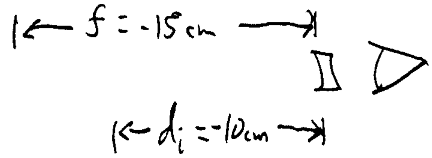
For those who chose contact lenses ($d=0$)

$f=-L=$ $\boxed{-15\text{cm}=f}$ (before lens)

negative focal length = $\boxed{\text{Concave lens}}$ (so my drawing should have)(instead of ())

Oops! I just read part (b) "disregard the eye-to-lens distance".
Oh well. That's how you'd do it *with* an eye-to-lens distance.

b) This person can see an object 10 cm infront of his eyeball. Therefore, the image that is 10 cm in front of his eyeball will be in focus. We need to find the object distance that will put an image 10 cm in front.



d_o?

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{d_i - f}{f d_i} \Rightarrow d_o = \frac{f d_i}{d_i - f} = \frac{(-15\text{cm})(-10\text{cm})}{(-10\text{cm}) - (-15\text{cm})} = \frac{150\text{cm}^2}{5\text{cm}} = 30\text{cm}$$

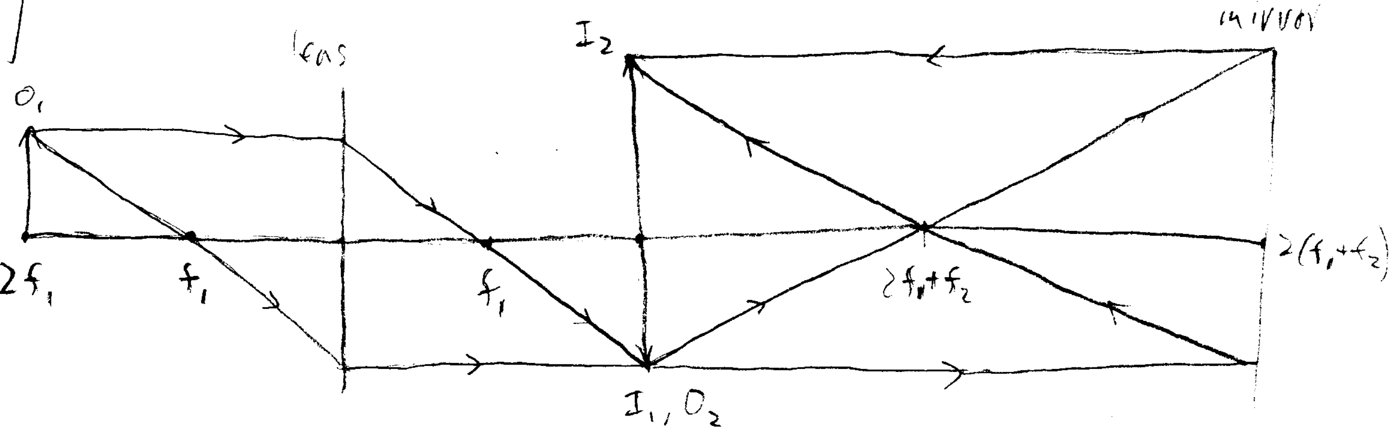
Positive! Does this mean they can see behind their head?
No, Object distances are positive on the "left" side of the lens.
Remember chart:

f −	f +
d_i −	d_i +
d_o +	d_o −

$\boxed{d_o = 30\text{ cm}}$ (before lens)

2

a)



b) $\frac{1}{f_1} = \frac{1}{O_1} + \frac{1}{I_1} = \frac{1}{2f_1} + \frac{1}{I_1} \Rightarrow I_1 = 2f_1$

$\frac{1}{f_2} = \frac{1}{O_2} + \frac{1}{I_2} = \frac{1}{2f_2} + \frac{1}{I_2} \Rightarrow I_2 = 2f_2$

$\rightarrow$ $I_2 = 2f_1$ BEYOND LENS

c) $I_1 = O_1$ & $I_2 = O_2$, SO $h_I = h$, i.e. $m = 1$

d) RAYS CONVERGE TO FORM I_2, SO REAL

Lee Spring 04 final $\boxed{\#3 \text{ Solution}}$

(a). $t' = \gamma\left(t - \frac{vx}{c^2}\right)$.

$\beta = v/c$

+v is in the +x direction
(i.e. S' moves →)
−v is in the −x dir
(i.e. S' moves ←).

$t_1' = \gamma(1 - \beta(1))$

$t_2' = \gamma(.5 - \beta(2))$

$t_1' = t_2' \implies 1 - \beta = .5 - 2\beta$

$.5 = -\beta$

$\beta = -1/2$

so $v = -c/2$

so S' moves in the −x direction at speed $c/2$.

(b). $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3/4}} = \frac{2}{\sqrt{3}}$

so $t_1' = \frac{2}{\sqrt{3}}\left(1 + (1/2)(1)\right) = \frac{3}{2}\frac{2}{\sqrt{3}} = \sqrt{3}$ yrs

$t_2' = \frac{2}{\sqrt{3}}\left(\frac{1}{2} - (-1/2)(2)\right) = \frac{2}{\sqrt{3}}\,3/2 = \sqrt{3}$ yrs

so, yes $t_1' = t_2'$. and, $\sqrt{3} \approx 1.73$ yrs.

Lee spring final 04 | #4 solution |

(a) Find E_γ in lab frame.

we know $E_{\gamma_1} = E_{\gamma_2}$.

and, $E_{\text{initial, total}} = KE_{e^-} + m_e c^2 + m_e c^2$

$= 1\,\text{MeV} + 2(511\,\text{KeV})$

$= 2.022\,\text{MeV}$

$E_{\text{init tot}} = 2E_\gamma$

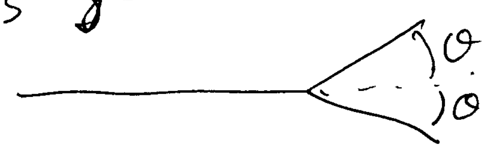
so $E_\gamma = 1.011\,\text{MeV}$ (per photon).

(b). for a photon, $|\vec{p}_\gamma| = \frac{E_\gamma}{c}$.

so $|\vec{p}_\gamma| = 1.011\,\text{MeV}/c$

(c) We note by consv. of momentum $\phi_1 = \pi + \phi_2$.

(I.e. the γ's go off in the same plane:



).

$z \rightarrow$

so, we know $p_{\gamma_z} = |\vec{p}_\gamma| \cos\theta$

and, by z direction momentum consv,

$p_{e^-\text{initial}} = 2p_{\gamma_z}$

$c\,p_{e^-_i} = 2|\vec{p}_\gamma| c \cos\theta$.

and p_{e^-} solves

$p_{e^-}^2 c^2 + m_e^2 c^4 = E_{e^-}^2$.

$E_{e^-} = 1\,\text{MeV} + m_e c^2$

so $E_{e^-}^2 = 1(\text{MeV})^2 + 2\,\text{MeV}(mc^2) + m_e^2 c^4$

so $p_{e^-}^2 c^2 = 1(\text{MeV})^2 + 2(\text{MeV})(mc^2)$

so $p_{e^-} c = 1.422\,\text{MeV}$.

Thus, $\frac{c\,p_{e^-_i}}{2|\vec{p}_\gamma| c} = \cos\theta \simeq .703$

so $\theta \simeq 45.3^\circ$ or $.791$ radians.

(5) IMAGINE A UNIVERSE WHERE THE POTENTIAL BETWEEN A PROTON AND AN ELECTRON WAS $V(r) = Cr^4$ RATHER THAN THAT GIVEN BY COULOMB'S LAW. CONSTRUCT A BOHR-LIKE THEORY FOR HYDROGEN. (REMEMBER THAT $F = -\frac{d}{dr}V(r)$, AND SINCE C IS POSITIVE, THE FORCE IS ATTRACTIVE.)

(a) PROVE THAT THE ALLOWED ENERGIES OF THE STATIONARY STATES ARE $E_n = R \cdot n^{4/3}$ FOR $n = 1, 2, 3, \cdots$ DETERMINE THE COMPLETE EXPRESSION FOR R IN TERMS OF m, C AND h (25 POINTS)

$$\vec{F} = -\frac{d}{dr}V(r)\hat{r} = -4cr^3\hat{r} = -\frac{mv^2}{r}\hat{r}$$

$$\Rightarrow \quad v = \left(\frac{4cr^4}{m}\right)^{1/2}$$

$$E_{TOT} = E_K + V = \frac{1}{2}mv^2 + cr^4 = 2cr^4 + cr^4 = 3cr^4$$

QUANTIZATION OF ANGULAR MOMENTUM: $\quad mvr = n\hbar \quad n = 1, 2, \cdots$

$$\Rightarrow \quad m^2v^2r_n^2 = 4mcr_n^6 = n^2\hbar^2$$

$$\Rightarrow \quad r_n = \left(\frac{n^2\hbar^2}{4mc}\right)^{1/6}$$

$$E_n = 3cr_n^4 = 3c\cdot\left(\frac{n^2\hbar^2}{4mc}\right)^{\frac{2}{3}} = \underbrace{\left(\frac{27\cdot c\cdot h^4}{256\cdot m^2\pi^4}\right)^{1/3}}_{= R}\cdot n^{4/3} \quad *$$

(b) IF THE RADIUS FOR $n=1$ IS DENOTED BY $r_1 = a$, DETERMINE THE QUANTUM NUMBER n FOR WHICH $r_n = 3a$ (15 POINTS)

$$r_n = \left(\frac{n^2\hbar^2}{4mc}\right)^{1/6}$$

$$\Rightarrow \quad n = \frac{2\sqrt{mc}}{\hbar}r_n^3$$

$$1 = \frac{2\sqrt{mc}}{\hbar}\cdot a^3$$

$$n = \frac{2\sqrt{mc}}{\hbar}\cdot(3a)^3 = 27\cdot\underbrace{\frac{2\sqrt{mc}}{\hbar}a^3}_{= 1} = \underline{\underline{27}} \, *$$

(6) CONSIDER AN INFINITE WELL OF WIDTH L WITH $V(x)=0$ FOR $0<x<L$

(a) SOLVE SCHRÖDINGER'S EQUATION FOR THIS PROBLEM. FIND THE GENERAL SOLUTIONS, AND THEN APPLY BOUNDARY CONDITIONS TO FIND THE EXACT SOLUTIONS FOR THE COEFFCIENTS AND WAVENUMBER k. YOU DO NOT HAVE TO NORMALIZE THE WAVEFUNCTION.

TIME-INDEPENDENT SCHRÖDINGER EQUATION:

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right]\psi(x)=E\,\psi(x)$$

FOR $x\leq 0$ or $x\geq L$, $V(x)=\infty \rightarrow \psi(x)=0$.

FOR $0<x<L$, $V(x)=0$.

$$\rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)=E\cdot\psi(x)$$

GENERAL SOLUTION TO THIS DIFFERENTIAL EQUATION:

$$\psi(x)=A\cdot\cos(kx)+B\cdot\sin(kx)$$

$$(*)\quad -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)=\frac{\hbar^2k^2}{2m}\psi(x)=E\,\psi(x)$$

USING THE BOUNDARY CONDITIONS: $\psi(x=0)=\psi(x=L)=0$ ($\because$ $\psi(x)$ SHOULD BE CONTINUOUS.)

$$\psi(x=0)=A=0$$

$$\psi(x=L)=B\cdot\sin(k\cdot L)=0 \rightarrow kL=n\pi$$

$$\rightarrow \underline{k=\frac{n\pi}{L}}$$

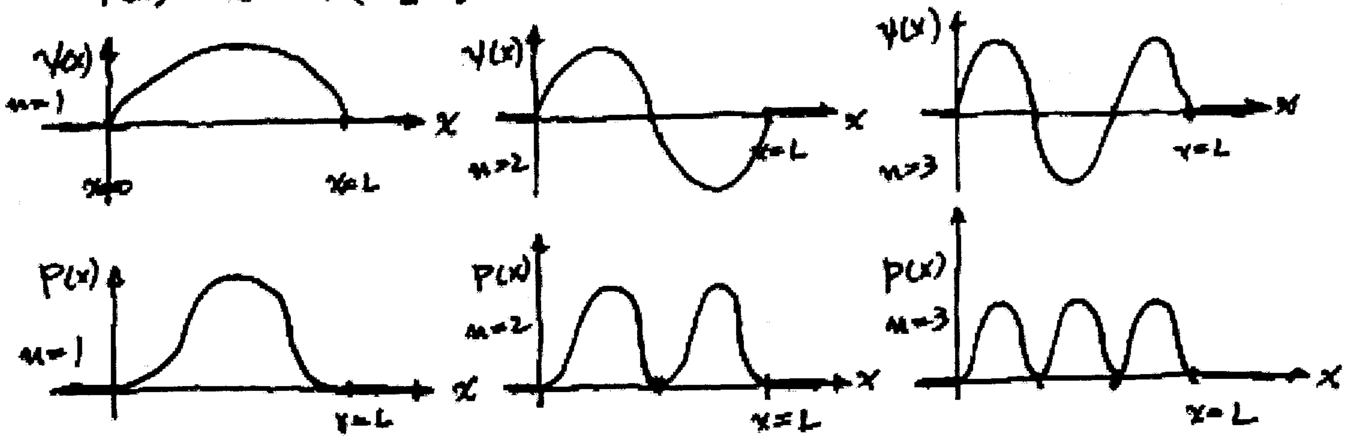
(b) WHAT ARE THE ENERGY STATES OF THIS SYSTEM?

THE WAVEFUNCTION $\psi_n(x)=B\cdot\sin\left(\frac{n\pi}{L}\cdot x\right)$ $\quad\left(k=\frac{n\pi}{L}\right)$

USING $(*)$ IN (a); $E_n=\frac{\hbar^2k^2}{2m}=\underline{\frac{\hbar^2\pi^2}{2mL^2}\cdot n^2}$

(c) SKETCH THE WAVE FUNCTIONS AND PROBABILITIES FOR THE LOWEST 3 ENERGY STATES

$\psi(x)=B\cdot\sin\left(\frac{n\pi x}{L}\right)$ $\qquad P=|\psi(x)|^2=\psi^*(x)\psi(x)=B^2\sin^2\left(\frac{n\pi x}{L}\right)$



(d) IF 5 ELECTRONS ARE PLACED IN THIS SYSTEM, WHAT IS THE ENERGY OF THE 5TH ELECTRON (ASSUME ELECTRONS HAVE SPIN)?

ELECTRONS HAVE SPIN 1/2 $\Rightarrow$ ACCORDING TO PAULI'S EXCLUSION PRINCIPLE, EACH ENERGY STATE CAN HAVE TWO ELECTRONS.

$\Rightarrow$ THE 5TH ELECTRON IS AT $n=3$ QUANTUM STATE

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2mL^2} = \underline{\frac{9}{2}\,\frac{\pi^2\hbar^2}{mL^2}}$$

$\boxed{7}$

a) $KE = \frac{hc}{\lambda} - \phi = \frac{1240\ eV\text{-}nm}{410\ nm} - 1.85eV = 1.17eV$

$\rightarrow \boxed{V_{stop} = 1.17\ V}$

b) $\frac{KE}{m_e c^2} \ll 1$, SO NON-RELATIVISTIC

$KE \simeq \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \cdot 1.17eV}{0.511e6\ eV}}\ c$

$= 2.14e{-3} \times 3e8\ m/s$

$\boxed{v = 642\ km/s}$

8) a) (ii)

b) (i) T

(ii) T

(iii) T

(iv) F

(v) T

c) (ii)

d) (i) T

(ii) F

(iii) T

(iv) T

(v) T