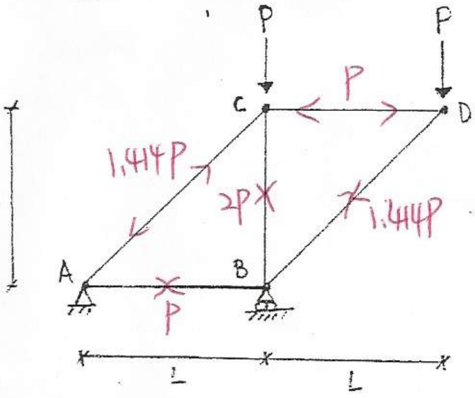
2 magnitudes
-2 for tension/comp

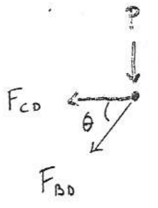
Problem 1:

(a) Determine the force in each member of the timber truss. Express your results in terms of P and state whether each member is in tension or compression.

(b) If all the members have the same cross-section (6"x6"), determine the maximum load that can be applied with a factor of safety of 2.5. The ultimate strength of timber is 10 ksi.



(a) FBD (Node D)



$\theta = \tan^{-1}\left(\frac{L}{L}\right) = 45°$

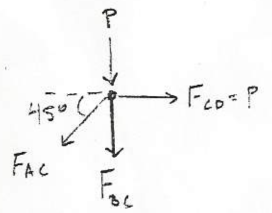
$\Sigma F_y = 0: \; -P - F_{BD}\sin 45° = 0$

$F_{BD} = -\frac{P}{\sin 45°} = \underline{\underline{-1.414\,P,\ \text{Comp.}}}$

$\Sigma F_x = 0: \; -F_{CD} - F_{BD}\cos 45° = 0$

$F_{CD} = -(-1.414P)\cos 45° = \underline{\underline{P,\ \text{Ten.}}}$

FBD (Node C)



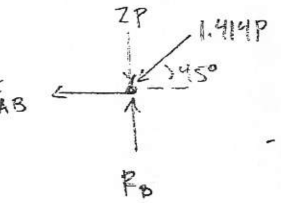
$\Sigma F_x = 0: \; P - F_{AC}\cos 45° = 0$

$F_{AC} = \frac{P}{\cos 45°} = \underline{\underline{1.414P,\ \text{Ten.}}}$

$\Sigma F_y = 0: \; -P - F_{AC}\sin 45° - F_{BC} = 0$

$F_{BC} = -P - 1.414P\sin 45° = \underline{\underline{-2P,\ \text{Comp.}}}$

FBD (Node B)



$\Sigma F_x = 0:$

$-F_{AB} - 1.414P\cos 45° = 0$

$F_{AB} = \underline{\underline{-P,\ \text{Comp.}}}$

(b) $\sigma = \frac{P}{A}$, $A = 6" \times 6" = 36\,in^2$

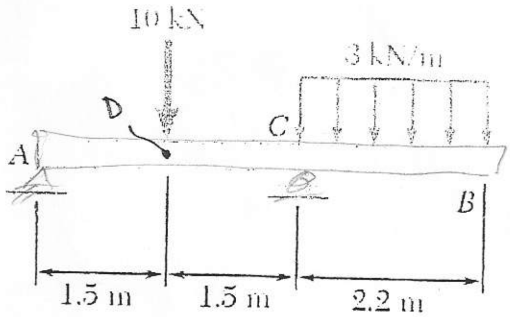
$\sigma_{max} = \frac{P_{max}}{A}$, $P_{max} = F_{BC} = -2P$, $\sigma_{max} = -10$ ksi (Because member is in compression)

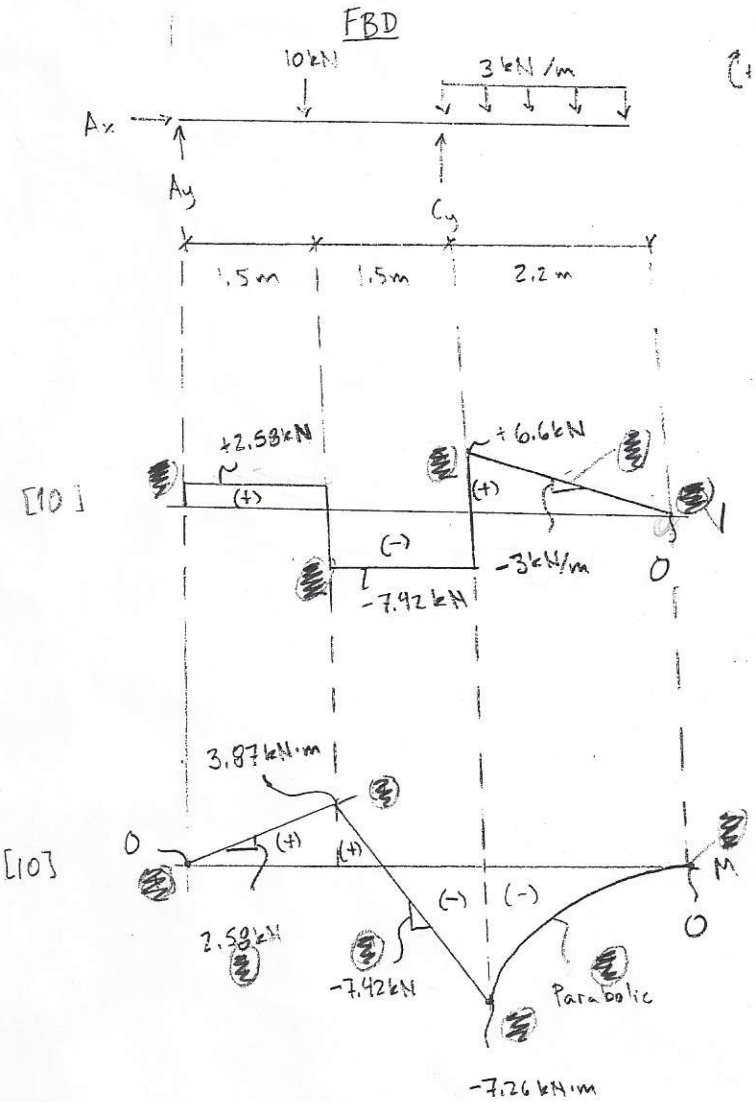
$-10\,ksi = \frac{-2P}{36\,in^2} \rightarrow P_{max} = 180$ kips

$F.S. = 2.5 = \frac{P_{max}}{P_{all.}} \rightarrow P_{all} = \frac{180}{2.5} = \underline{\underline{72\ \text{kips}}}$

Problem 2:

Draw the shear and bending moment diagrams for the beam shown. Indicate all key values (min/max values, values at ends and supports, slopes, linear, parabolic/cubic distributions) with their algebraic sign.





$\Sigma F_x = 0: \; A_x = 0$

$\circlearrowleft + \Sigma M_A = 0: \; 10(1.5) + 3(2.2)(3 + \frac{2.2}{2}) - C_y(3) = 0$

$C_y = 14.02 \text{ kN}$

$\Sigma F_y = 0: \; A_y + 14.02 - 10 - 3(2.2) = 0$

$A_y = 2.58 \text{ kN}$

$V_a = A_y = 2.58 \text{ kN}$

$V_d = V_a - 10 \text{ kN} = -7.42 \text{ kN}$

$V_c = V_d + C_y = 6.6 \text{ kN}$

$V_b = V_c - 3 \text{ kN} \times 2.2 \text{ m} = 0$

$M_a = 0$

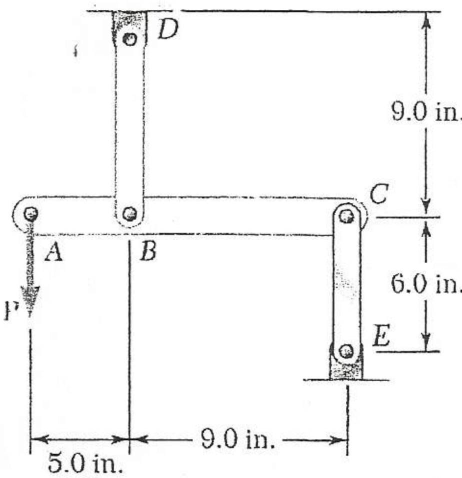
$M_d = 2.58 \text{ kN} \times 1.5 \text{ m} = 3.87 \text{ kN·m}$

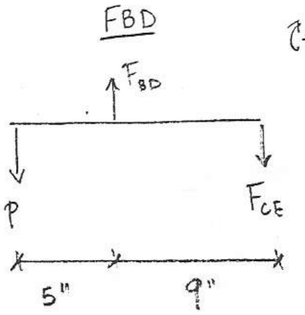
$M_c = M_d - 7.42 \times 1.5 \text{ m} = -7.26 \text{ kN·m}$

$M_b = M_c + \frac{1}{2}(6.6 \text{ kN})(2.2 \text{ m}) = 0$

Problem 3:

Link BD is made of brass ($E=15\text{x}10^6$ psi) and has a cross-sectional area of 0.40 in^2. Link CE is made of aluminum ($E=10.4\text{x}10^6$ psi) and has a cross-sectional area of 0.50 in^2. Determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.014 in.





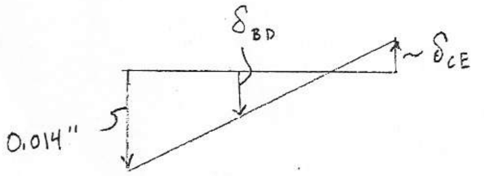
$\circlearrowleft + \Sigma M_C = 0: \; F_{BD}(9) - P(14) = 0$

$F_{BD} = \frac{14}{9} P$

$\Sigma F_y = 0: \; \frac{14}{9} P - P - F_{CE} = 0$

$F_{CE} = \frac{5}{9} P$

$\delta_{BD} = \dfrac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$, $L_{BD} = 9"$, $E_{BD} = 15\times10^6$ psi, $A_{BD} = 0.4 \text{ in}^2$



$\delta_{CE} = \dfrac{F_{CE} L_{CE}}{E_{CE} A_{CE}}$, $L_{CE} = 6"$, $E_{CE} = 10.4\times10^6$ psi, $A_{CE} = 0.5 \text{ in}^2$

Using Similar Triangles: $\dfrac{\delta_{CE} + 0.014"}{14"} = \dfrac{\delta_{BD} + \delta_{CE}}{9"}$

$$9"\left[\frac{5/9\,P\,(6")}{(10.4\times10^6\text{ psi})(0.5\text{ in}^2)} + 0.014"\right] = 14"\left[\frac{14/9\,P\,(9")}{(15\times10^6\text{ psi})(0.4\text{ in}^2)} + \frac{5/9\,P\,(6")}{10.4\times10^6\text{ psi}\,(0.5\text{ in}^2)}\right]$$

$$5.769\times10^{-6}\,P\left(\tfrac{\text{in}}{\text{lb}}\right) + 0.126" = 4.164\times10^{-5}\left(\tfrac{\text{in}}{\text{lb}}\right)$$

$$P = \frac{0.126"}{3.587\times10^{-5}(\text{in}/\text{lb})} = 3{,}512.5\text{ lb}$$

$\underline{\underline{P = 3.51 \text{ kips}}}$