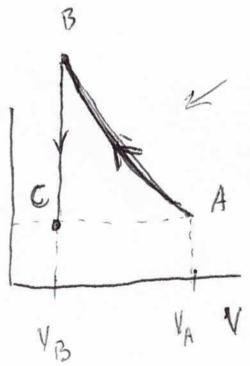
1. $n \to$ # moles , P_A , V_A ; monatomic $\to d=3$; $\gamma = \frac{3+2}{3} = \frac{5}{3}$

(a)



(b) $P_A V_A = nR \cdot T_A \Rightarrow \boxed{T_A = \frac{P_A V_A}{nR}}$; $\boxed{V_B = \frac{1}{5} V_A}$

Adiabatic $A \to B \Rightarrow P_A V_A^{\gamma} = P_B V_B^{\gamma}$

$\Rightarrow P_B = P_A \cdot \left(\frac{V_A}{V_B}\right)^{\gamma} = \boxed{P_A \cdot 5^{(5/3)} = P_B}$

$\boxed{V_c = \frac{1}{5} V_A}$ $\boxed{P_c = P_A}$

(c) $A \to B$, $\Delta U = \cancel{Q}^{0} - W$

$U = \frac{d}{2} nRT = \frac{3}{2} \cdot PV \to \Delta U = P_B V_B - P_A V_A = \left(5^{5/3} \cdot \frac{1}{5} - 1\right) P_A V_A$

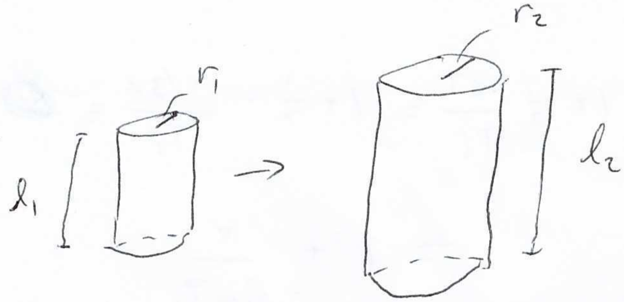
$= \left(5^{2/3} - 1\right) P_A V_A$

$\Rightarrow W = -\Delta U = \boxed{-\frac{3}{2}\left(5^{2/3} - 1\right) P_A V_A = W}$

$\left(W_{tot} = W_{A \to B} + \cancel{W_{B \to C}}^{0} \right)$

(d) $W_{\substack{A \to C \\ \text{isobar}}} = \int_{V_A}^{V_B} p \, dV = P_A (V_B - V_A) = \boxed{-\frac{4}{5} P_A V_A}$

3.



(Isotropic) length expansion coeff $= \alpha$

emissivity $\rightarrow \varepsilon$

(a) $\dfrac{\Delta V}{V} = (3\alpha)\Delta T \Rightarrow \Delta T = \dfrac{x}{3\alpha}$

(b) $\Delta l = l_0 \alpha \Delta T \Rightarrow l_2 = l_1 + l_1 \cdot \alpha \cdot \dfrac{x}{3\alpha} = \boxed{l_1\left(1 + \dfrac{x}{3}\right) = l_2}$

$\Delta A = A_0(2\alpha)\Delta T \Rightarrow A_2 = A_1 + A_1 \cdot (2\alpha) \cdot \dfrac{x}{3\alpha} = \boxed{A_1\left(1 + \dfrac{2}{3}x\right) = A_2}$

$\boxed{= \pi r_1^2\left(1 + \dfrac{2}{3}x\right)}$

(c) $P = \varepsilon \sigma A T^4$

$P_{net} = P_{out} - P_{in} = \varepsilon\sigma\,(\text{surface of cylinder}) \cdot \left(T_2^4 - T_1^4\right)$

$T_2 \rightarrow T_1 + \dfrac{x}{3\alpha}$

$2 \cdot \pi \cdot r_1^2\left(1 + \dfrac{2}{3}x\right)^2 + 2\pi\left(l_1\left(1 + \dfrac{x}{3}\right)\right)\left(r_1\left(1 + \dfrac{x}{3}\right)\right)$

$l_2 \gg r_2 \Rightarrow$ just $\cancel{x} \rightarrow 2\pi\left[l_1\left(1 + \dfrac{x}{3}\right) r_1\left(1 + \dfrac{x}{3}\right)\right]$

$\rightarrow \varepsilon\sigma \cdot \left[2\pi l_1 r_1\left(1 + \dfrac{x}{3}\right)^2\right] \cdot \left(\left(T_1 + \dfrac{x}{3\alpha}\right)^4 - T_1^4\right) = (2\pi l_1 r_1)\,\varepsilon\sigma T_1^4\left(1 + \dfrac{x}{3}\right)^2 \cdot [\star]$

$\star = \left(1 + \dfrac{x}{3\alpha T_1}\right)^4 - 1 \simeq \dfrac{4}{3}\dfrac{x}{\alpha T_1}$

$\Rightarrow \boxed{(2\pi l_1 r_1)\,\varepsilon\sigma T_1^4\left(1 + \dfrac{2}{3}x\right) \cdot \dfrac{4}{3}\dfrac{x}{\alpha T_1}}$

(d) $\frac{1}{2} m_{N_2} V_{rms}^2 = \frac{3}{2} k_B T \rightarrow V_{rms} = \sqrt{\dfrac{3 k_B T}{m_{N_2}}}$

equipartition thm

$V_{rms}^2 = \langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

$= 3\left(\frac{1}{2} k_B T\right)$

$\cancel{\Rightarrow \Delta(V_{rms}) = \sqrt{\dfrac{3k_B}{m_{N_2}}}\underbrace{\left(\sqrt{T_2} - \sqrt{T_1}\right)}_{\sqrt{T_1 + \Delta T} - \sqrt{T_1}}}$

$\cancel{\sqrt{\dfrac{3k_B}{m_{N_2}}} \cdot \left(\sqrt{T_1 + \dfrac{x}{3\alpha}} - \sqrt{T_1}\right)}$

$$\frac{V_{rms,f}}{V_{rms,i}} = \frac{\sqrt{T_f}}{\sqrt{T_i}} = \sqrt{1+\frac{\Delta T}{T_i}} \simeq 1+\frac{1}{2}\frac{\Delta T}{T_i} = 1+\frac{1}{2}\cdot\frac{x/3\alpha}{T_1}$$

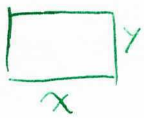
$$= 1 + \frac{x}{6\alpha T_1}$$

3. (a) $\frac{\Delta V}{V} = (3\alpha)\Delta T \Rightarrow \Delta T = \frac{x}{3\alpha}$ ...... 3 points

(b) $\Delta l = l_0 \alpha \Delta T$

$\Rightarrow l_2 = l_1 + l_1 \alpha \frac{x}{3\alpha} = l_1 \left(1 + \frac{x}{3}\right)$ ..... 2 points

Derive $\Delta A = A_0 (2\alpha) \Delta T$



$A = xy = x_0(1+\alpha\Delta T)\, y_0(1+\alpha\Delta T)$

$\approx x_0 y_0 (1 + 2\alpha\Delta T) = A_0 + A_0(2\alpha)\Delta T$

$\Rightarrow \Delta A = A_0(2\alpha)\Delta T$

$A_2 = A_1 + A_1 2\alpha \frac{x}{3\alpha} = A_1\left(1+\frac{2}{3}x\right) = \pi r_1^2 \left(1 + \frac{2}{3}x\right)$ ... 3 points

(c) $P = \varepsilon \sigma A T^4$

$P_{net} = P_{out} - P_{in} = \varepsilon \sigma A_{cylinder} (T_2^4 - T_1^4)$ .... 2 points

$A_{cylinder} = 2\pi r_2^2 + 2\pi r_2 l_2 \underset{r_2 \ll l_2}{=} 2\pi r_2 l_2 = 2\pi r_1 \left(1+\frac{x}{3}\right) l_1 \left(1+\frac{x}{3}\right) = 2\pi r_1 l_1 \left(1+\frac{x}{3}\right)^2$ ... 2 points

$T_2^4 - T_1^4 = (T_1 + \Delta T)^4 - T_1^4 = T_1^4 \left(1+\frac{\Delta T}{T_1}\right)^4 - T_1^4 = T_1^4\left(1 + \frac{4\Delta T}{T_1}\right) - T_1^4 = 4\Delta T\, T_1^3$

$= 4 \frac{x}{3\alpha} T_1^3$ ... 2 points

$P_{net} = P_{out} - P_{in} = \varepsilon\sigma\, 2\pi r_1 l_1 \left(1+\frac{x}{3}\right)^2 \frac{4x}{3\alpha} T_1^3 = \varepsilon\sigma\, 2\pi r_1 l_1 \left(1 + \frac{2x}{3}\right) \frac{4x}{3\alpha} T_1^3$

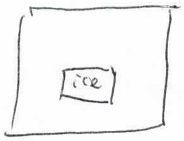
d) $\frac{1}{2} m_{N_2} v_{rms}^2 = \frac{3}{2} k_B T$

$v_{rms} = \sqrt{\frac{3 k_B T}{m_{N_2}}}$ ... 2 points

$\frac{v_{rms}^{final}}{v_{rms}^{initial}} = \sqrt{\frac{T_{final}}{T_{initial}}} = \sqrt{\frac{T_1 + \Delta T}{T_1}} = \sqrt{1 + \frac{\Delta T}{T_1}} = \left(1 + \frac{\Delta T}{2T_1}\right) = 1 + \frac{x}{6\alpha T_1}$

... 4 points

10 3.



$T_0 = 0°C$ $\quad m = 20\ kg$ $\quad L_f = 300\ kJ/kg$

$T_1 = 20°C$ $\quad R = 5\ W/K$

(a) $R = \dfrac{\text{heat transfer}}{\text{temp. diff.} \cdot \text{time}} = \boxed{R = \dfrac{Q}{\Delta T \cdot \Delta t}} \longleftarrow \boxed{5\ pts}$

(b) $Q = mL_f$ $\cancel{= R\,\Delta T\,\Delta t}$

$= (20\ kg)(3 \cdot 10^5\ J/kg) = \boxed{6 \cdot 10^6\ J} \longleftarrow \boxed{5\ pts}$

(c) $Q = R\,\Delta T \cdot \Delta t \Rightarrow \Delta t = \dfrac{Q}{R \cdot \Delta T} = \dfrac{6 \cdot 10^6\ J}{5 \frac{J}{s \cdot K} \cdot (20 - 0\ °C)}$ $\rightarrow 1°C = 1K$

$\downarrow$

heat needed to melt the ice $\boxed{3\ pts}$

$= \boxed{6 \cdot 10^4\ s} \leftarrow \boxed{2\ pts}$

(d) $m L_f = R\,\Delta T \cdot \Delta t$

$\Rightarrow m = \dfrac{R \cdot \Delta T \cdot \Delta t}{L_f} = \dfrac{(5\ W/K) \cdot (20\ K) \cdot (20 \cdot 60 \cdot 60\ s)}{300 \cdot 10^3\ J/kg} = \boxed{24\ kg}$

$\uparrow \boxed{3\ pts}$ $\qquad \uparrow \boxed{2\ pts}$

Problem 4

(a) This process is irreversible because it does not occur though a set of equilibrium states.

(b) The heat released by the meteor is $Q = mC\Delta T$. To calculate the entropy of an irreversible process we note that S is a state function and is thus path independent. Choosing a reversible path on which to evaluate the change we can apply the equation for reversible entropy changes. The heat is transfered at a changing temperature so we get,

$$\Delta S_{thing} = \int \frac{dQ}{T} = \int_{3T_2}^{T_2} \frac{d(mC(T - T'))}{T} = mC \int_{3T_2}^{T_2} \frac{dT}{T} = -mC\ln(3)$$

(c) The environment is so large that it receives essentially all the heat at its temperature T_2:

$$\Delta S_{env} = \frac{mC(3T_2 - T_2)}{T_2} = 2mC$$

(d) The net entropy change of the system is $mC(2 - \ln 3) > 0$. This agrees with the Second Law of Thermodynamics.

Probelm 4 Rubric

Part A

2pts – Stating that the process is irreversible
4pts – For the explanation.

Specifically, you get all 4 if you mention that the process is not quasistatic, there is not time for thermalization, the process is not slow, does not have well defined thermodynamic values, and or does not pass through equilibrium states. If you state that the entropy increases you get 2 points and if you state that no real processes are irreversible you get 1 pt.

The purpose of this question is to test whether you know the definition of reversibility. There was a very common answer that is false. Reversibility is not equivalent to spontaneity. For a process to be spontaneous it must have a negative Gibbs free energy change, $\Delta G = \Delta H - T\Delta S$. In particular there are spontaneous processes that have zero, positive and negative entropy changes. But you must show that the process has non-zero entropy change to be irreversible so noting that it is spontaneous or that the inverse process is not spontaneous is not sufficient.

You get no points at all if you state that the process is reversible.

Part B

2pts – Explaining why the formula $\int \frac{dQ}{T}$ is valid for an irreversible process.
2pts – Showing that the infinitesimal heat change in the process is $dQ = mCdT$.
2pts – For doing the calculation correctly, integration..etc.

Because we know that the process is irreversible, it is not immediately clear that the formula $\int \frac{dQ}{T}$ can be applied. It is only valid for reversible transformations. You need to justify this by stating that entropy is a state variable and that you can choose another reversible path along which you evaluate ΔS.

Although it may be noted in the exam, there were no points removed for incorrect signs in this part.

Part C

2pts – Explaining why the environment acts like a heat bath and thus all the heat is transferred at the temperature of the ocean. That is, $\Delta S = \frac{Q}{T}$.
2pts – For the correct calculation.

Again, here signs were not penalized.

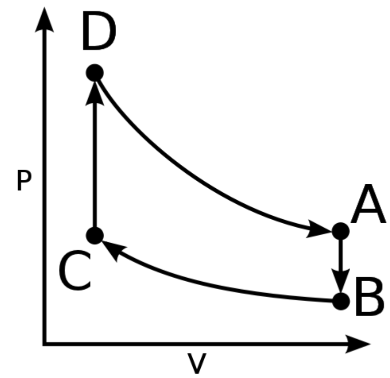
Part D

2pts – For summing the entropies with **correct** signs, and getting a positive result.
2pts – For a justification of the net positive entropy change. You get all of the points if you mention that this is in accordance with the 2nd Law of Thermodynamics. You get 1 point if you mention something about irreversibility or entropy increasing.

Here it is very important that you get a positive answer. If you write that the net change in the entropy of the universe is negative for some process, then you do not understand the 2nd Law. Many people got negative answers because they assumed that the ocean entropy change was 0. But this is wrong,it's actually greater than the meteor and not negligible, so the physical approximation is invalid. You are getting the wrong physics. You should have returned to (c) and fixed that.

Problem 5

The cycle is:



The efficiency of an ideal Carnot engine operating between these two temperatures is:

$$e_{carnot} = 1 - \frac{T_C}{T_A}$$

To calculate the efficiency of this engine we need to use the formula:

$$e = \frac{W}{Q_{in}}$$

To calculate the work done we need only calculate the area enclosed by the cycle in PV space. Hence we integrate the difference of the two isotherm curves:

$$\begin{aligned} W = \int_{V_C}^{V_A} P_{DA} - P_{BC} dV &= \int_{V_C}^{V_A} \frac{nRT_A}{V} - \frac{nRT_C}{V} dV \\ &= nR(T_A - T_C) \ln\left(\frac{V_A}{V_C}\right) \end{aligned}$$

To find the heat that flows in we notice that this only occurs on paths CD and DA. This can be seen by the First Law, $\Delta U = Q - W$. On CD $W = 0$ but temperature increases and on DA work is done but internal energy does not change because $U \propto T$. A similar argument determines that heat flows out on the other two paths.

Now, CD is a constant volume process so we can use the relation for the heat, $Q_V = nC_V \Delta T_{CD}$. On DA we know $\Delta U = 0 = Q - W$ and we calculated this work above. Therefore we have

$$Q_{in} = nC_V(T_A - T_C) + nRT_A \ln\left(\frac{V_A}{V_C}\right) = n\frac{d}{2}R(T_A - T_C) + nRT_A \ln\left(\frac{V_A}{V_C}\right)$$

Finally we have:

$$e = \frac{(T_A - T_C)\ln\left(\frac{V_A}{V_C}\right)}{\frac{d}{2}(T_A - T_C) + T_A \ln\left(\frac{V_A}{V_C}\right)} = \frac{(1 - \frac{T_C}{T_A})\ln\left(\frac{V_A}{V_C}\right)}{\frac{d}{2}(1 - \frac{T_C}{T_A}) + \ln\left(\frac{V_A}{V_C}\right)} = e_{carnot}\frac{\ln\left(\frac{V_A}{V_C}\right)}{\frac{d}{2}(1 - \frac{T_C}{T_A}) + \ln\left(\frac{V_A}{V_C}\right)}$$

$$\implies \frac{e}{e_{carnot}} = \frac{\ln\left(\frac{V_A}{V_C}\right)}{\frac{d}{2}(1 - \frac{T_C}{T_A}) + \ln\left(\frac{V_A}{V_C}\right)} < 1$$

You can easily see that even in the $T_A \to \infty$ or the $T_C \to 0$ limits the efficiency is not as great as Carnot.

Problem 5 (20 points), Midterm 1, Bordel (Zach Fisher)

- Expression of Carnot efficiency in terms of T_A, T_C (+1)

- Efficiency $= W/Q_{in}$ (+3)
- Work $= \int p\,dV$ ~~+4 for all~~ (+6 for all)
  - $= 0$ for $C \to D$, $A \to B$ (+2)
  - $= \int_{V_i}^{V_f} \frac{nRT}{V} dV$ (+3)
  - Net work $= nR(T_A - T_C)\ln(V_A/V_C)$ (+1)
- Heat (+7 for all)
  - Isothermal legs:
    - $\Delta U = Q - W$ (+1)
    - $\Delta U \propto \Delta T = 0$ (+1)
    - $\therefore Q = W$ (+1)
  - Isochoric legs:
    - $Q = \begin{cases} nC_V \Delta T \\ mC_V \Delta T \\ C_V \Delta T \end{cases}$ (+1)
  - Net heat in $= nC_V(T_A - T_C) + nRT_A \ln\left(\frac{V_A}{V_C}\right)$
    - (only using heat in) (+3)

- Proving $e_{sterling} < e_{carnot}$ by algebra (+3)

Note: −1 for algebra mistakes that result in mistaken units
−1 for assuming $d=3$ or $C_V = \frac{3}{2}R$ and never mentioning monatomic
−2 for not noticing if $e_{Carnot} \leq e_{sterling}$