EE40 Midterm 1 Solutions
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$$\mathrm{par}(R_1, R_2) := \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Problem 1 a

$$V_{oc} := \alpha \cdot V_x \qquad I_{sc} := \beta \cdot V_x \qquad R_{th} := \frac{V_{oc}}{I_{sc}} \rightarrow \frac{\alpha}{\beta}$$

The above parameters specify a Thevenin source with voltage source Voc and resistance Rth. We need to find an equivalent circuit that has voltage Vx rather than Voc on the left hand side. There are many possible solutions, but the simplest one is a voltage divider.

For a voltage divider with series resistance R1 and load R2, the Thevenin parameters are:

$$V_{oc} := \frac{R_2}{R_1 + R_2} \cdot V_x \qquad I_{sc} := \frac{V_x}{R_1}$$

Equating these with the above, we can find α and β by inspection.

$$\text{Given} \qquad \alpha = \frac{R_2}{R_1 + R_2} \qquad \beta = \frac{1}{R_1}$$

$$\mathrm{Find}(R_1, R_2) \text{ simplify} \rightarrow \begin{bmatrix} \frac{1}{\beta} \\ -\frac{\alpha}{\beta \cdot (\alpha - 1)} \end{bmatrix}$$

With three resistors in a T, R1, R2, R3

$$V_{oc} := \frac{R_2}{R_1 + R_2} \cdot V_x \qquad I_{sc} := \frac{\frac{\mathrm{par}(R_2, R_3)}{R_1 + \mathrm{par}(R_2, R_3)} \cdot V_x}{R_3} \text{ simplify} \rightarrow \frac{R_2 \cdot V_x}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}$$

$$\text{Given} \qquad \alpha = \frac{R_2}{R_1 + R_2} \qquad \beta = \frac{R_2}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}$$

$$\text{Find}(R_2, R_3)\ \text{simplify} \rightarrow \begin{pmatrix} -\frac{R_1 \cdot \alpha}{\alpha - 1} \\ \frac{\alpha}{\beta} - R_1 \cdot \alpha \end{pmatrix}$$

You're given full credit if you assumed α=1 and used one resistor in series

$$R_{th} := \frac{V_x}{\beta \cdot V_x} \rightarrow \frac{1}{\beta}$$

Full credit is also given for a dependent source and series resistor.

$$V_d := \alpha \cdot V_x \qquad R_{th} := \frac{\alpha}{\beta}$$

Problem 1 b

Let b be our reference voltage (ground). The open circuit voltage at node a is

$$V_{aoc} := V_1 \cdot \frac{R_6}{R_5 + R_6}$$

The short circuit current obtained by grounding a is

$$I_{scab} := \frac{V_1}{R_5}$$

Then

$$R_{thb} := \frac{V_{aoc}}{I_{scab}} \rightarrow \frac{R_5 \cdot R_6}{R_5 + R_6} \qquad V_{thb} := V_{aoc} \rightarrow \frac{R_6 \cdot V_1}{R_5 + R_6}$$

Problem 1 c

Ground the node in the middle. The open circuit voltage is

$$V_{aoc} := V_1 \cdot \frac{R_6}{R_5 + R_6} \qquad V_{boc} := 0$$

$$V_{aboc} := V_{aoc} - V_{boc} \rightarrow \frac{R_6 \cdot V_1}{R_5 + R_6}$$

The short circuit current is

$$I_{scab} := \frac{V_{thb}}{R_{thb} + \frac{R_3 \cdot (R_1 + R_2)}{R_3 + R_1 + R_2}} \text{ simplify } \rightarrow \frac{R_6 \cdot V_1}{\left[\frac{R_5 \cdot R_6}{R_5 + R_6} + \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}\right] \cdot (R_5 + R_6)}$$

where we used the Thevenin source from part b.

$$\frac{V_{aboc}}{I_{scab}} \text{ simplify } \rightarrow \frac{R_5 \cdot R_6}{R_5 + R_6} + \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}$$

The solution is more easily found by subsituting the result from part b directly into the circuit of part c. Then by inspection,

$$V_{oc} := V_{thb}$$

$$R_{th} := R_{thb} + \text{par}(R_3, R_1 + R_2) \rightarrow \frac{R_5 \cdot R_6}{R_5 + R_6} + \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3}$$

Problem 2

First, choose a ground. Node 4 is convenient because it gets rid of the supernode.

$$\frac{V_1 - V_x}{R_7} + \frac{V_1 - V_3}{R_4} + \frac{V_1 - V_2}{R_5} = 0$$

$$\frac{V_2 - V_1}{R_5} - \alpha \cdot V_o - I_A = 0$$

$$I_A - I_B + \frac{V_3 - V_1}{R_4} = 0$$

$V_o = V_1 - V_3$ substitute this into the second equation to get

$$\frac{V_2 - V_1}{R_5} - \alpha \cdot (V_1 - V_3) - I_A = 0$$

Rewriting the above:

$$V_1 \cdot \left(\frac{1}{R_7} + \frac{1}{R_4} + \frac{1}{R_5}\right) + V_2 \cdot \left(\frac{-1}{R_5}\right) + V_3 \cdot \left(\frac{-1}{R_4}\right) = \frac{V_x}{R_7}$$

$$V_1 \cdot \left(\frac{-1}{R_5} - \alpha\right) + V_2 \cdot \left(\frac{1}{R_5}\right) + V_3 \cdot (\alpha) = I_A$$

$$V_1 \cdot \left(\frac{-1}{R_4}\right) + V_3 \cdot \left(\frac{1}{R_4}\right) = I_B - I_A$$

Problem 3

First, redraw the circuit so there are no overlaps.

Let IR2 be the current flowing through R2 consistent with the direction of Iy

KVL: $$I_{R2} \cdot R_2 + (I_{R2} - i_o) \cdot R_5 + \beta \cdot i_o + (I_{R2} - I_y) \cdot R_6 = 0$$

KCL at bottom node: $$-I_x + i_o - I_y = 0$$

$$I_{R2}(R_2 + R_5 + R_6) + i_o(-R_5 + \beta) = I_y \cdot (R_6)$$

$$i_o = I_x + I_y$$

Note: depending on how you redrew the circuit and labeled the meshes, you may end up with several different forms of the above equations.

Problem 4 a

Define a ground. The bottom node is convenient.

Given

$$\frac{V_x}{1\Omega} - 1A - 1A - \alpha \cdot V_R = 0$$

$$\frac{V_R}{1\Omega} + 1A = 0$$

$$V_x = 500mV$$

$$\begin{pmatrix} \alpha \\ V_x \\ V_R \end{pmatrix} := \text{Find}(\alpha, V_x, V_R) \rightarrow \begin{pmatrix} -\frac{500 \cdot mV - 2 \cdot A \cdot \Omega}{A \cdot \Omega^2} \\ 500 \cdot mV \\ -A \cdot \Omega \end{pmatrix}$$

$$\alpha \rightarrow -\frac{500 \cdot mV - 2 \cdot A \cdot \Omega}{A \cdot \Omega^2} = 1.5 \cdot S$$