1 Problem 1

a) The process is isobaric, or it occurs at constant pressure. We know this because the pressure is supplied entirely by the weight of the piston:

$$P_f = P_i = mgA$$

b) From the ideal gas law, we can also use this to get that

$$n = \frac{P_i V_i}{RT_i} = \frac{mgV_i}{ART_i}$$

Which is the number of moles of gas in the system. We know that when heat is added to an ideal monatomic gas at constant pressure:

$$Q = nC_p(T_f - T_i) = n\frac{5}{2}R(T_f - T_i)$$

We also know from the combined gas law:

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

We can also say that if $V_f = 2V_i$ this must imply that $T_f = 2T_i$ and therefore that:

$$T_f - T_i = T_i$$

Giving us the final result in terms of known parameters:

$$Q = n\frac{5}{2}R(T_i)$$

Or alternately:

$$Q = \frac{mgV_i}{ART_i}\frac{5}{2}R(T_i) = \frac{5mgV_i}{2A}$$

Part A

+4 used word isobaric or constant pressure

+1 explained isobaric situation arises from the constant pressure provided by the weight of the piston.

Part B

+4 Found number of moles in terms of given quantities using Ideal Gas Law.

+4 Used Q = n Cp dT to find heat, or plausibly applied the first law.

+4 Used ideal gas law to find delta T.

+2 identified pressure correctly as mg/A

+1 got the right answer

-1 Wrong units

-1 Blatantly incorrect statement

+5 if correctly determined heat input, assuming isothermal situation

2 Question 2

a) The system can provide work if the area enclosed by the loop in a clockwise manner is positive, that is that $T_1 > T_2$ and $V_2 > V_1$.

b) To start analyzing the system, we will first break it into sections:

AB: This process is isothermal so we can say from the First law:

$$U_B - U_A = 0$$

$$Q_{AB} = W_{AB} = nRT_1 ln(\frac{V_2}{V_1}) > 0$$

BC: This process is at constant volume, so we know that:

$$W_{BC} = 0$$

$$U_C - U_B = Q_{BC} = nC_v(T_2 - T_1) < 0$$

CD: Following the same reasoning from section AB:

$$U_D - U_C = 0$$

$$Q_{CD} = W_{CD} = nRT_2 ln(V_1/V_2) < 0$$

DA: This process is also at constant volume, so we can follow the reasoning from BC:

$$W_{BC} = 0$$

$$U_A - U_D = Q_{DA} = nC_v(T_1 - T_2) > 0$$

We can define the heat exhausted at low temperature as:

$$Q_L = -Q_{BC} - Q_{CD} = nC_v(T_1 - T_2) + nRT_2 ln(V_2/V_1)$$

And the heat added at high temperature:

$$Q_H = Q_{DA} + Q_{AB} = nC_v(T_1 - T_2) + nRT_1 ln(V_2/V_1)$$

And the net work done is:

$$W = Q_H - Q_L = nR(T_1 - T_2) ln(V_2/V_1)$$

The efficiency is:

$$E = \frac{W}{Q_H} = \frac{nR(T_1 - T_2)ln(V_2/V_1)}{nC_v(T_1 - T_2) + nRT_1 ln(V_2/V_1)}$$

c) Since entropy is a state function, we should expect that the total change in entropy of the gas is 0:

$$\Delta S_{gas} = 0$$

Whereas the change in entropy of the reservoirs:

$$\Delta S_H = -Q_H/T_1 < 0$$

$$\Delta S_L = Q_L/T_2 > 0$$

Note that since this engine is less efficient than a Carnot engine the total change in entropy of the reservoirs is net positive. This would be zero for a perfect Carnot engine.

1 Question 2

a) The system can provide positive net work if the area enclosed by the loop in a clockwise manner is positive, that is that $T_1 > T_2$ and $V_2 > V_1$.

1 point was given for each inequality, or two points was given for clockwise or an equivalent thermodynamic argument

b) To start analyzing the system, we will first break it into sections:

AB: This process is isothermal so we can say from the First law:

$$U_B - U_A = 0$$

$$Q_{AB} = W_{AB} = nRT_1 ln(\frac{V_2}{V_1}) > 0$$

1 Point

BC: This process is at constant volume, so we know that:

$$W_{BC} = 0$$

$$U_C - U_B = Q_{BC} = nC_v(T_2 - T_1) < 0$$

1 Point

CD: Following the same reasoning from section AB:

$$U_D - U_C = 0$$

$$Q_{CD} = W_{CD} = nRT_2 ln(V_1/V_2) < 0$$

1 Point

DA: This process is also at constant volume, so we can follow the reasoning from BC:

$$W_{BC} = 0$$

$$U_A - U_D = Q_{DA} = nC_v(T_1 - T_2) > 0$$

1 Point. Here you needed to calculate the heats and works for individual processes. You needed either four heats or two heats and two works for a total of four points.

We can define the heat exhausted at low temperature as:

$$Q_L = -Q_{BC} - Q_{CD} = nC_v(T_1 - T_2) + nRT_2 ln(V_2/V_1)$$

And the heat added at high temperature:

$$Q_H = Q_{DA} + Q_{AB} = nC_v(T_1 - T_2) + nRT_1 ln(V_2/V_1)$$

1 Point

And the net work done is:

$$W = Q_H - Q_L = nR(T_1 - T_2) ln(V_2/V_1)$$

1 Point. For this section, you needed to define either net work and heat added, or heat released and heat added for a total of two points.

The efficiency is:

$$E = \frac{W}{Q_H} = \frac{nR(T_1 - T_2) ln(V_2/V_1)}{nC_v(T_1 - T_2) + nRT_1 ln(V_2/V_1)}$$

1 Point for a correct final answer. A correct answer written in the form $1 - Q_L/Q_H$ *was also accepted.*

c) Since entropy is a state function, we should expect that the total change in entropy of the gas is 0:

$$\Delta S_{gas} = 0$$

4 points. Some reasoning was required, but we did not require explicitly showing the changes in S for the individual processes summed to 0.

Whereas the change in entropy of the reservoirs:

$$\Delta S_H = -Q_H/T_1 < 0$$

3 points. We did not require explicitly solving and simplifying this provided that Q_H *and* T_H *have been defined correctly above.*

$$\Delta S_L = Q_L/T_2 > 0$$

3 points. We did not require explicitly solving and simplifying this provided that Q_L *and* T_L *have been defined correctly above.*

Note that since this engine is less efficient than a Carnot engine the total change in entropy of the reservoirs is net positive. This would be zero for a perfect Carnot engine.

3.

(a)

the heat engine undergoes a cornot cycle. The heat (Q_0) extracted from water, with ice, through engine, and some heat (Q_1) flow out to gas. Becuase it is a carnot cycle,

$$\frac{Q_L}{Q_H} = \frac{Q_1}{Q_0} = \frac{T_1}{T_0}$$

$$Q_0 = ML \Rightarrow Q_1 = ML\left(\frac{T_1}{T_0}\right)$$

during the isothermal expansion,

$$\Delta E_{int} = 0 \Rightarrow Q_1 = \Delta W_1 = \int P\, dV = \int_{V_1}^{V_2} \frac{nRT_1}{V}\, dV = nRT_1 \operatorname{Log}\left[\frac{V_2}{V_1}\right]$$

$$\Rightarrow ML\left(\frac{T_1}{T_0}\right) = nRT_1 \operatorname{Log}\left[\frac{V_2}{V_1}\right]$$

$$\Rightarrow \frac{V_2}{V_1} = \exp\left(\frac{ML}{nRT_0}\right)$$

percentage of volume change :

$$\frac{V_2 - V_1}{V_1} = \exp\left(\frac{ML}{nRT_0}\right) - 1$$

(b)

reservoir with ice : the temperature stay fixed during the whole cycle

$$\Delta S_0 = \frac{-Q_0}{T_0} = \frac{-ML}{T_0}$$

reservoir made of ideal gas : (temperature is changing)

heat flow out during the isothermal compression (no heat flows during adiabatic compression)

$\Delta S_1 =$

$$\int \frac{1}{T}\, dQ = \int \frac{1}{T}\, dW = \int \frac{P}{T}\, dV = \int_{V_1}^{V_2} \frac{nR}{V}\, dV = nR\operatorname{Log}\left[\frac{V_2}{V_1}\right] = \frac{ML}{T_0} \text{ (from part (a))}$$

thus total entropy :

$$\Delta S = \Delta S_1 + \Delta S_0 = \frac{-ML}{T_0} + \frac{ML}{T_0} = 0$$

Problem 3 Solution and Rubric

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(a)

The heat released by water with mass M during solidification is

$$Q_0 = ML \tag{1}$$

The Carnot engine has efficiency

$$e = 1 - \frac{T_1}{T_0} \tag{2}$$

Also by definition of engine efficiency,

$$e = \frac{W}{Q_0} = \frac{Q_0 - Q_1}{Q_0} = 1 - \frac{Q_1}{Q_0} \tag{3}$$

Comparing Eq (2) and Eq (3) gives

$$Q_1 = Q_0 \frac{T_1}{T_0} \tag{4}$$

Since the ideal gas in the low temperature reservoir remains the same temperature T_1, its internal energy E_{int} remains the same. By the first law of thermal dynamics,

$$Q_1 = W_1 \tag{5}$$

where W_1 is the work done by the ideal gas (during the expansion) . W_1 is given by

$$\begin{aligned} W_1 &= \int P dV \\ &= \int_{V_i}^{V_f} \frac{N k_B T_1}{V} dV \\ &= N_A k_B T_1 \ln V \mid_{V_i}^{V_f} \\ &= R T_1 \ln \left(\frac{V_f}{V_i} \right) \end{aligned} \tag{6}$$

where V_f and V_i are the final and initial volume of the ideal gas in the low temperature reservoir, R is the ideal gas constant. In the above steps, we have used the ideal gas law, the fact that T_1 is a constant, and that the gas has 1 mole molecules. Hence

$$Q_1 = R T_1 \ln \left(\frac{V_f}{V_i} \right) \tag{7}$$

Comparing Eq (4) with Eq (7), and substituting Eq (1) yields

$$\frac{V_f}{V_i} = \exp \left(\frac{ML}{RT_0} \right) \tag{8}$$

Therefore the percentage increase of the volume of the gas is

$$\frac{\Delta V}{V_i} \times 100\% = \frac{V_f - V_i}{V_i} = \frac{V_f}{V_i} - 1 = \exp(\frac{ML}{RT_0}) - 1 \tag{9}$$

(b)

$$\Delta S_{ice} = \int \frac{dQ}{T_0} = \frac{\Delta Q}{T_0} = \frac{-Q_0}{T_0} = -\frac{ML}{T_0} \tag{10}$$

The minus sign in the last step indicates that heat flows out of the reservoir, and that the entropy decreases.

$$\Delta S_{gas} = \int \frac{dQ}{T_1} = \frac{\Delta Q}{T_1} = \frac{Q_1}{T_1} = Q_0 \frac{T_1}{T_0} \times \frac{1}{T_1} = \frac{Q_0}{T_0} = \frac{ML}{T_0} \tag{11}$$

We used Eq (4) in the last step.

Grading Rubric

Regrading leads to a complete reevalation of the solution, and may result in grade deduction. Think carefully before asking for regrading.

Different approaches from the above one are also accepted, if clearly formulated. We emphasize that your understanding of the physical process needs to be clearly demonstrated in equations and sentences that you put down on the blue/green book. It is your responsibility to make your argument transparent to the graders.

We try to ensure fairness by assigning certain points to each important steps. For part (a), Eq (1) - (5), (8) and (9) each is assigned 1 point. Eq (6) is assigned 2 points. A last point is given for correct final answer. 5 points is given for correctly arriving at Eq (10), and another 5 points for (11). But we also want to stress the complexity of judging real student responses, and the necessary flexibility in grading. A student's clarity is not merely reflected by the equations he/she puts down. We also takes into account the context, i.e., your descriptions, the connections with the equations before and after, and the clarity manifested by the notations. Hence do not feel that you must get certain amount of points for writing down certain equations. For instance, many of you would put down $W = \int PdV = ... = NKT \ln(V_f/V_i)$ in your solution. But not all of you would get the full 2 points assigned for this equation. This example is elaborated in "common mistakes" (so common that I referred to it as "standard") below.

The most common wrong argument takes the below form:

> "$Q_0 = ML$, $Q = W$ for isotherm, hence $ML = \int PdV = ... = RT_1 \ln(V_f/V_i)$. Then $V_f/V_i = \exp(ML/RT_1)$"

It seems that no big mistakes were made, and that the final result is off by only one subscript. However, the small difference in final result cannot obscure the deep misunderstanding of the Carnot engine and the physical process. Unconsciously, the students who wrote the above solution assume that Carnot engine has ideal gas as its working substance. While the assumption is wrong, this unimportant mistake (or narrow discussion) should not yield a wrong result. Notice that by assuming ideal gas as working substance, TWO ideal gases occur in the problem. One in the low temperature reservoir, and the other in the engine. The latter is actually invented by the students themselves, and finally confused themselves. While the problem says the ideal gas (in the reservoire) would expand, the students wrote down equations for the gas in the engine. They have no idea that V_f and V_i are the volume for the gas in the engine, and nevertheless use it to calculate volume change. We give at most 5 points for part (a) if we decide that a wrong solution falls into the above category.

Some other ambiguities are summarized below.

1. In (a), if you put down $\frac{Q_1}{Q_0} = \frac{T_1}{T_0}$ without Eq (2) and (3), at most 2 points will be given for Eq (2) - (4). Less points will be given if you put down the relation without any context.
2. In (a), if you write down $Q_1 = W$ without even implicitly referring to the 1st law (e.g. "$\Delta E_{int} = 0 \Rightarrow Q_1 = W$ "), no point will be given for Eq (5).
3. In (b), if you progress to $\Delta S_{ice} = \int \frac{dQ}{T_0} = \frac{\Delta Q}{T_0} = \pm \frac{Q_0}{T_0}$, 2 or 3 points will be given.
4. In (b), if you obtain $\Delta S_{ice} = ML/T_0$, i.e., with the wrong sign, at most 3 points will be given. 2 points are taken for misconception.

5. In (b), if you start with $S = Q/T$ (rather than ΔS) without any explanation, at most 2 points could be given for each calculation of entropy changes.

6. In (b), if you assume that the Carnot engine has ideal gas as its working substance and obtain the correct expression for ΔS, at most 4 points will be given.

7. In (b), if you do not calculate ΔS_{ice} and ΔS_{ice} and simply state that $\Delta S = 0$ for a reversible process in an isolated system, 1 or 2 points might be given.

8. In (b), if you calculate ΔS_{ice}correctly, and clearly explains why $\Delta S_{ice} + \Delta S_{gas} = 0$, and obtain $\Delta S_{gas} = -\Delta S_{ice}$, full marks will be given. Explanations like below is acceptable. Ice, gas and the engine together form an isolated system. After a complete cycle, $\Delta S_{ice} + \Delta S_{engine} + \Delta S_{gas} = 0$. Because S is a state function, and the engine returns to its initial state, ΔS_{engine} vanishes. Therefore $\Delta S_{ice} + \Delta S_{gas} = 0$.

4.

the energy (E) released by rubbing the two metal raises the (internal energy) temperature of metal and gas

$E = 2\,\Delta E_m + \Delta E_g$

$\Delta E_m = \Delta Q - \Delta W = \Delta Q = MC\Delta T$

$\Delta E_g = nC_v\,\Delta T$

$\Rightarrow E = MC\Delta T + nC_v\,\Delta T$

$$\Rightarrow \Delta T = \frac{E}{2\,MC + nC_v} = T - T_0$$

$$\Rightarrow T = T_0 + \frac{E}{2\,MC + nC_v}$$

Problem 4 Grading Rubric

+ 6 points for correct logic relating E to changes in energy of disks and gas ($E = \Delta E_{gas} + \Delta E_{disks}$)

- +3 given if only correct written explanation is present

+3 points for finding correct ΔE for the disks

+ 6 points for finding correct ΔE of the gas with appropriate reasoning

- -2 is no Δ is present in the expression
- -2 for no logic relating ΔE of the gas to Q (W = 0)
- -2 for not plugging in 3/2R for Cv, or nR for Nk

+ 5 for correct final answer

- Credit only given if original expression for E was correct
- -2 for algebraic mistake
- -2 if ΔT is left in the answer

Problem 5

a)

The force on Q due to any particular charge +q on the right is exactly balanced by the force due to the charge +q diametrically opposite. So the net force on Q is zero.

b)

With the three o'clock charge removed, only the nine o'clock charge is unbalanced. By Coulombs law the force will be:

$$\vec{F} = \frac{-qQ}{4\pi\epsilon_0 R^2}\hat{x}$$

Where $\hat{x}$ points to the right.

c)

Positive charges move along the electric field, so the +q charges will move to the right. Negative charges move against it, so -Q moves to the left.

d)

The force is:

$$\vec{F} = M\vec{a} = -Q\vec{E}$$

his means:

$$\vec{a} = \frac{-Q}{M}\vec{E}$$

This is a constant acceleration so if you remember 1-D kinematics from 7A, you can write down that:

$$v_f^2 = v_0^2 - 2|\vec{a}|\Delta x = \frac{4QER}{M}$$

So the final speed is:

$$v_f = \sqrt{\frac{4QER}{M}}$$

If you didn't remember this you'd have to integrate the acceleration equation out, which isn't too bad since the acceleration is a constant:

$$v_f - v_0 = v_f = \frac{QE}{M}t$$

$$\Delta x = \frac{1}{2}\frac{QE}{M}t^2$$

So, I get that the time for when $\Delta x = 2R$ is:

$$t = \sqrt{\frac{4RM}{QE}}$$

Plugging this into the final velocity equation:

$$v_f = \frac{QE}{M}t = \frac{QE}{M}\sqrt{\frac{4RM}{QE}} = \sqrt{\frac{4QER}{M}}$$

Question 5 Rubric

5 points max. for each part. Partial credits are given as follows:

A.

Point Values by Responses

i. 2 points for stating **F=0** without correct explanation
ii. Max. 2 points for correct statements regarding individual contributions of each charge to the total force, without arriving at the fact that the net force is **0**
iii. Max. 1 point for correct statements in a generally incorrect answer

Additional Deductions

iv. -1 for ambiguities, lack of clarity or imprecise statements

B.

Point Values by Response

i. 3 points for getting magnitude of force correct but direction wrong
ii. 2 points for recognizing force is horizontal, but wrong magnitude
iii. 2 points for an unsuccessful attempt in summing the correct contributions from each charge
iv. Max. 1 point for correct statements in a generally incorrect answer

Additional Deductions

v. -1 for ambiguous coordinate system
vi. -1 for typo/small algebra mistake
vii. -1 for giving **E** instead of **F**

C.

Point Values by Response

i. 3 points for getting all directions correct but no/bad explanation
ii. 3 points for a good explanation of why they move in opposite directions without actually specifying the direction of each charge or getting the direction wrong
iii. 2 points for correctly recognizing how the charges affect one another, but effect of E-field is neglected or not understood
iv. Max. 2 points for knowing they go in opposite directions, but getting the directions wrong and not explaining/ explaining poorly
v. Max. 1 point for correct statements in a generally incorrect answer

Additional Deductions

vi. -1 penalty for coordinate ambiguity
vii. -1 for ambiguities, lack of clarity or imprecise statements

D.

Point Values by Response

i. 2 points for knowing how to find the force on –Q due to **E**, but not knowing the mechanics
ii. 3 points for getting correct relevant equations (work, force), but not putting them together to get a final answer
iii. Max. 2 points for correct mechanics but wrong E&M
iv. Max. 1 point for correct statements in a generally incorrect answer

Additional Deductions

v. -1 point for recalling relevant mechanics equation incorrectly
vi. -1 for minor algebra mistakes
vii. Max. -1 for poor presentation (e.g. minus sign or vectors under radical)
viii. -1 for only giving speed, not velocity (OK if stating the component along the direction, say v_x, is negative; points deducted if suggesting the magnitude, say v_f, is negative)