MIDTERMI 1 Tuesday October 1, 2013

Instructor: Prof. A. LANZARA

TOTAL POINTS: 100

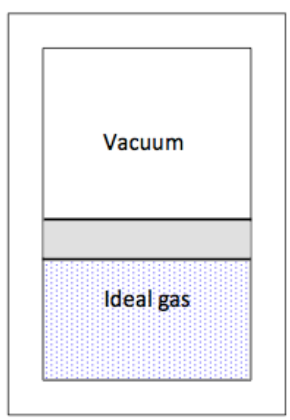
TOTAL PROBLEMS: 5

Show all work, and take particular care to explain what you are doing. Partial credit are given. Please use the symbols described in the problems, define any new symbol that you introduce and label any drawings that you make. If you get stuck, skip to the next problem and return to the difficult section later in the exam period.
All answers should be in terms of variables.

GOOD LUCK!

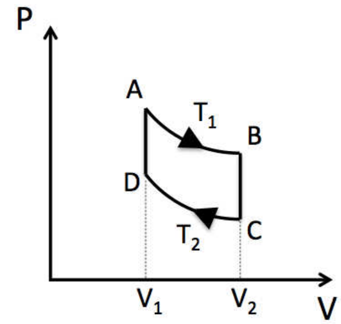
PROBLEM 1 (Points 20)

An ideal monoatomic gas is confined in a cylindrical container with a movable piston. The container is divided in two parts and the gas occupies only one side. On the other side of the container there is vacuum.
The piston has mass M, area A and is frictionless.
The initial volume occupied by the gas is V_1 and the temperature of the gas is T_1.
Heat is given to the system in a reversible way, so that the gas can expand to twice its initial volume.
a) Describe the process that describes the expansion of the gas (5pts).
b) Find the amount of heat absorbed by the gas in order for the expansion to take place (Neglect the specific heat of the cylinder and of the piston) (15 pts).



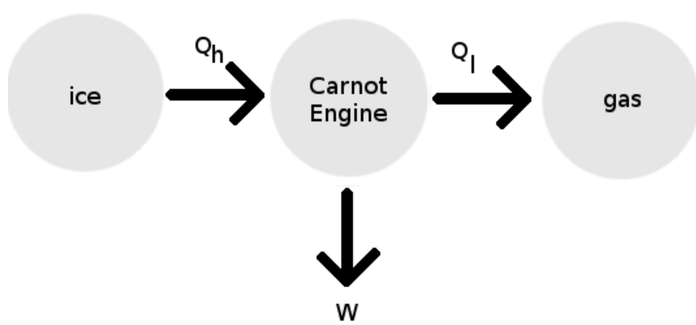
PROBLEM 2 (Points 20)

An heat engine works between two reservoirs T_1 and T_2. The working substance is an ideal monoatomic gas (n moles). The cycle is shown below. The processes AB and CD are isothermal, whereas BC and AD are at constant volume, V_2 and V_1 respectively.
a) Specify under which condition your engine can provide work (2pts).
b) Find the efficiency of the cycle (8pts)
c) Find the change of entropy of the gas + reservoir at the end of the cycle (10pts).



PROBLEM 3 (Points 20)

A reversible Carnot engine works in cycle by extracting heat from a mixture of melting ice (T_0=0^0C) and depositing heat to another heat reservoir made of one mole of ideal gas at temperature $T_1 > T_0$.
The temperature of the gas is kept constant during the entire process. The latent heat of ice is L.
a) Determine the percentage increase of the volume of the gas at the end of the cycle, corresponding to solidification of a mass M of water (10pts)
b) Determine the change of entropy of the two reservoirs (10pts)



PROBLEM 4 (Points 20)

Two metal discs of mass M are enclosed in a cylinder filled by one mole of a monoatomic ideal gas. The specific heat of each disc is C. Neglect the specific heat of the container. The system is isolated.
The system is initially in equilibrium at a temperature T_0. The two discs are then rubbed one against the other and an energy E is released in the process.
Determine the final equilibrium temperature of the system.

PROBLEM 5 (Points 20)

Twelve equal charges +q are situated in a circle with radius R and they are equally spaced (see figure a). A charge –Q of mass M is at the center of the circle.

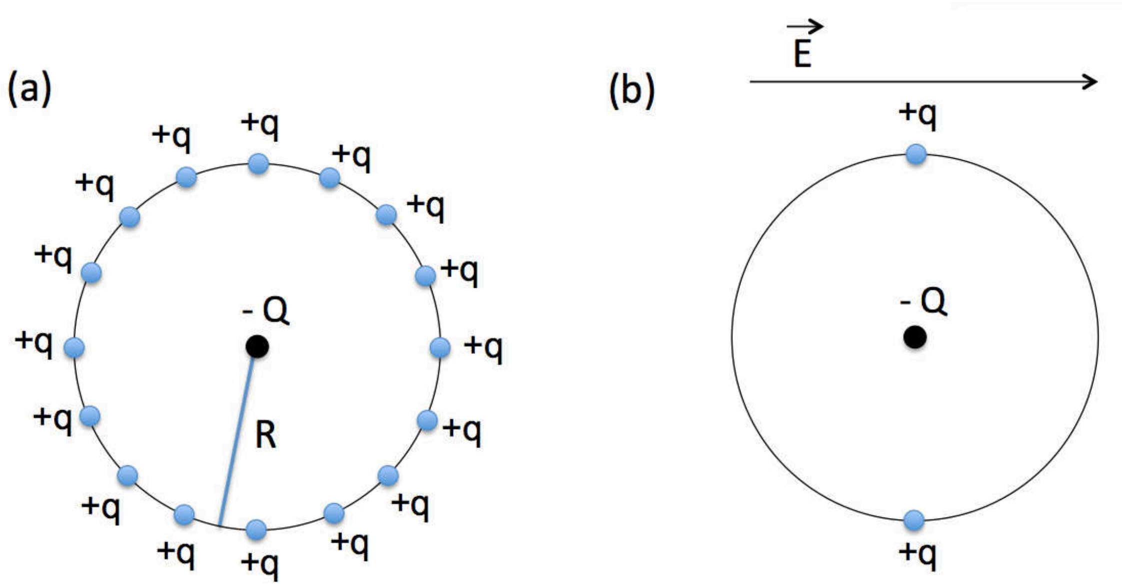
a) What is the net force (magnitude and direction) on the charge -Q? Explain your answer (5pts)

We remove only the +q charge, which is located at "3-o'clock".

b) What is the net force (magnitude and direction) on the charge -Q at the center of the circle? (5 pts)

I now remove 10 charges, and am left only with two charges located at "12-o'clock" and 6-o'clock". I turn on an electric field E (see figure b). Neglect the field generated by each individual charge +q and –Q.

c) In which direction are the two remaining charges (+q) and the central charge (-Q) moving? (Explain your answer) (5pts)
d) What is the velocity of charge –Q after it has travelled a distance 2R? (5pts)



$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

$$f_{Maxwell}(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$E_{int} = \frac{d}{2} NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{int} = Q - W$$

$$W = \int P dV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For an adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2} R$$

$$e = \frac{W}{Q_h}$$

$$e_{ideal} = 1 - \frac{T_L}{T_H}$$

$$S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\rho = \frac{dQ}{dV}$$

$$\sigma = \frac{dQ}{dA}$$

$$\lambda = \frac{dQ}{dl}$$

$$\overline{g(v)} = \int_0^\infty g(v) \frac{f(v)}{N} dv \ \ (f(v) \text{ a speed distribution})$$

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4} \right) \right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2} x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$