SOLUTIONS

Midterm 01

This is a 50 minute examination with 5 equally weighted problems.

INSTRUCTIONS

①........Do not open these pages until "START" is announced.

②........Print your name clearly in the box above.

③........Remember your Honor Code!

④........Silence and stow all cellphones and electronic devices.

⑤........Only *writing instruments / eraser / straightedge* are allowed.

⑥........There are *no questions allowed* (too disruptive) during the exam.

## Performance Summary

This exam was administered in class (245 Li Ka Shing Center) on Friday, 10/04/2013 to 220 students. The maximum possible score was 100 points.

High Score = **87**
Low Score = **14**
Average Score = **51**
Standard Deviation = **16**

1. Mechanical Behavior of Materials

a. Compare and contrast the toughness of the three alloys shown here. Explain your answer in detail by defining "toughness," the units by which it is measured, and how this plot supports your argument(s).
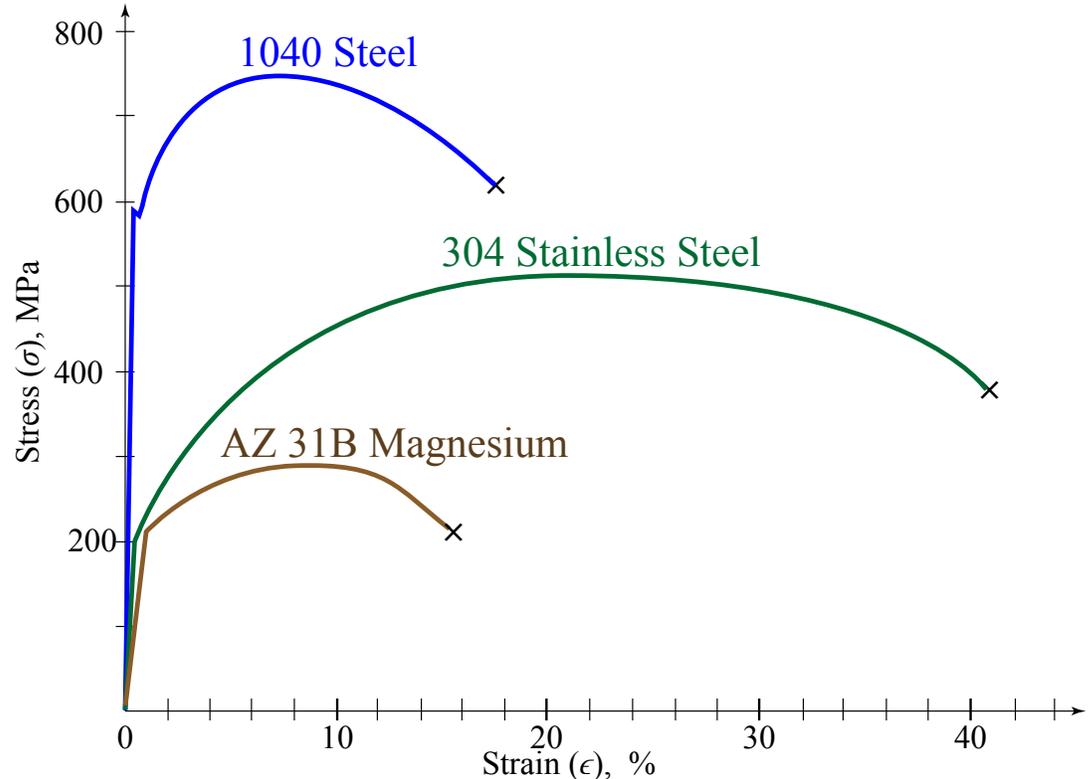
Toughness is defined as the **ability of a material to absorb energy under load**. It is measured in units of "energy" for this reason, and its relationship to the stress-strain plot is discerned by performing a simple dimensional analysis. Noting that stress has units of force/area, and strain has units of length/length, it is possible to generate units of energy by taking the product of these two terms,

[force/area] × [length/length] = [force × distance] / [area × length] = [energy] / [volume].

The product of stress × strain, or more precisely, $\int \sigma \, d\epsilon$, is represented by the **area under the stress-strain curve**; consequently, such **area** can be used as a reasonable indication of toughness.

Referring to the given plots, it can be concluded that **type 304 stainless steel has the highest toughness, followed by type 1040 steel, then magnesium alloy AZ 31B**, judged by the representative areas, largest to smallest, under their respective stress-strain curves.

1. Mechanical Behavior of Materials

b. When subjected to mechanical testing by the Charpy V-notch method, 1040 steel exhibits an impact energy of 180 Joules. But when tested at 0°C, its impact energy drops below that of magnesium alloy AZ 31B. How do you reconcile these observations with the stress-strain behavior documented above? Explain.
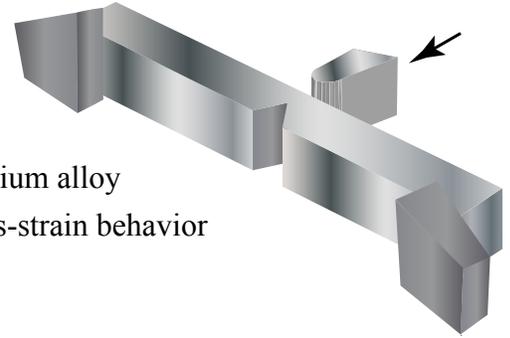
This behavior is a consequence of the **ductile-to-brittle transition** exhibited by most steels. At a critical temperature, known as the ductile-to-brittle transition temperature (DBTT), the amount of stress that causes catastrophic failure plunges, sometimes precipitously, to dangerously low values, at times lower that the yield point recorded under higher temperature (above DBTT) conditions. This type of brittle failure is most prominently linked to "impact" loading, where the load is applied at high strain rates, as effected in the Charpy test by a swinging hammer. The notched sample concentrates stress at the notch, ensuring that failure will occur there, allowing a measurement to be made of the "impact energy" absorbed by the sample.

Nonferrous alloys, like AZ 31B, do **not** exhibit a ductile-to-brittle transition, explaining why this alloy with much lower overall toughness, as indicated by the plots in (a) above, would exhibit a higher impact energy at 0°C than the much tougher (above DBTT) 1040 steel.

2. Bonding in Engineering Materials

a. Using concepts of metallic bonding, explain the transition from elastic to plastic behavior exhibited by an Al-Cu alloy in this stress-strain plot. Be specific.
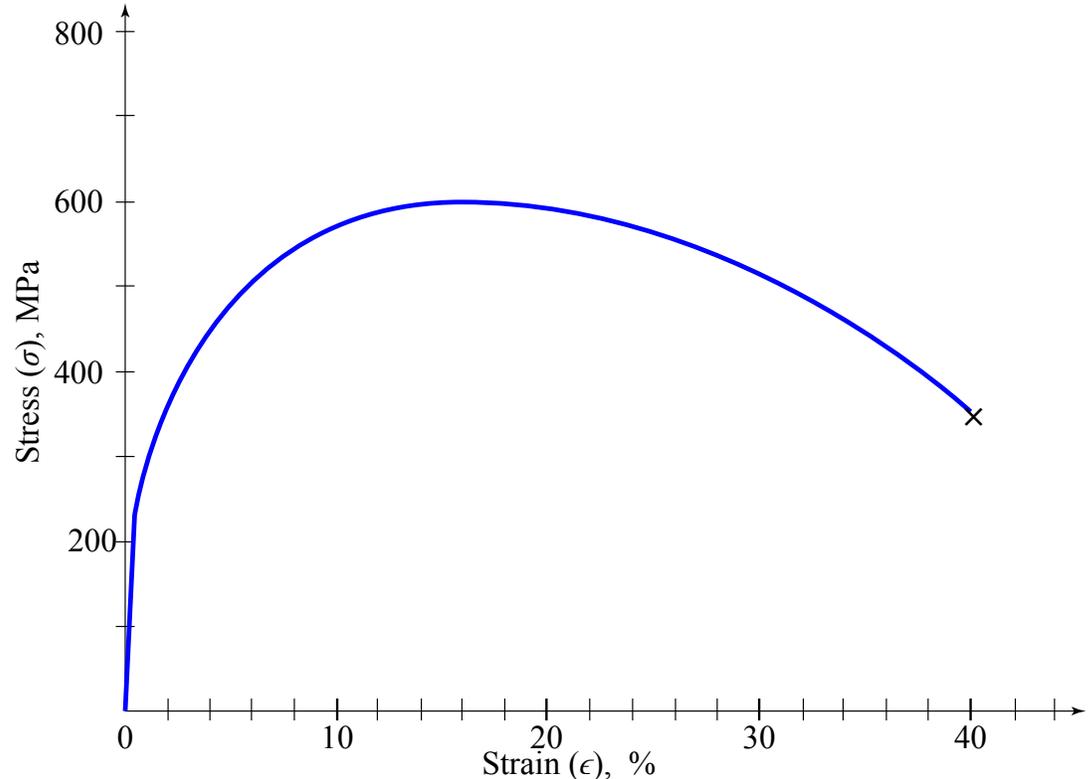
The pertinent concepts of metallic bonding at play here are the **non-directionality** of metallic bonding and the opportunity for **extended orbital overlap** of bonding electrons during deformation. Applied to the Al-Cu alloy in the problem, these concepts explain elastic behavior, characterized by fully recoverable strain, as the **stretching** of atomic bonds, and plastic behavior, characterized by a non-recoverable strain resulting in permanent deformation as the **breaking** and **restoration** (at different sites) of atomic bonds (the "dislocation" mechanism of plastic deformation). Non-directionality of orbital overlap enables an extended range of plasticity in most metallic alloys.

Consequently the **transition** from elastic behavior, manifested in the initial linear portion of the stress-strain curve, to plastic behavior, evidenced by the deviation from linearity at the yield point of approximately 240 MPa , and continuing through approximately 40% strain exhibited by this alloy at failure, is succinctly explained as the **transition from bond stretching to bond breaking**.

2. Bonding in Engineering Materials

b. A Google search on "steel" generates the following passage from eHow™
< http://www.ehow.com/about_6638014_atomic-structure-steel.html > :

"Steel is a crystalline structure of iron molecules interspersed with carbon molecules. This is properly known as "cementite." The hardness and malleability of steel depends not only on the carbon content, but on how the carbon and iron molecules are arranged to one another."

Critique this description using concepts of primary and secondary bonding.

This passage has **several** errors rooted in a misunderstanding of primary and secondary bonding. Primary bonds occur between individual atoms (or ions), while secondary bonds occur between groups of atoms (or ions), which could be "molecules." Steel is indeed crystalline, a consequence of metallic bonding, one of the primary bond types, and it is indeed an alloy of Fe and C. However, iron does **not** form "molecules" the way carbon does (C_{60} for example), so there is no chance for secondary bonding between molecules of iron and carbon, the only type of bonding that would hold molecules together. Secondary bonding is very weak, inconsistent with the high strength of metalically-bonded steel. So at the outset it must be stated that **there are no molecules of any kind in steel**!

The structure of steel is based upon a Bravais lattice (most often cubic), with Fe atoms on lattice sites, and C atoms located interstitially, between lattice sites. "Cementite" is one of the solid **phases** found in steel, a carbide with a strict 1:3 stoichiometry or Fe_3C; but it is not a "molecule," nor is it the only solid phase found in steel. Steel is therefore **not** "properly known as cementite!" All microconstituent phases of steel are products of primary bonds, sometimes purely metallic ("ferrite" or "austenite") and sometimes mixed metallic and covalent character ("cementite").

Finally, while it is true that "the hardness and malleability of steel depends (*sic*) not only on the carbon content," the other attributes are prior austentite grain size, amount and distribution of microconstituent phases, and dislocation content, among others, but **NOT** "how the carbon and iron molecules are arranged to one another!"

3. Lattice Geometry

a. Draw on the following template one member of each of the following two families of planes, then show and specify the lattice direction along which they intersect: $(\bar{2}12)$ and $(2\bar{1}0)$.
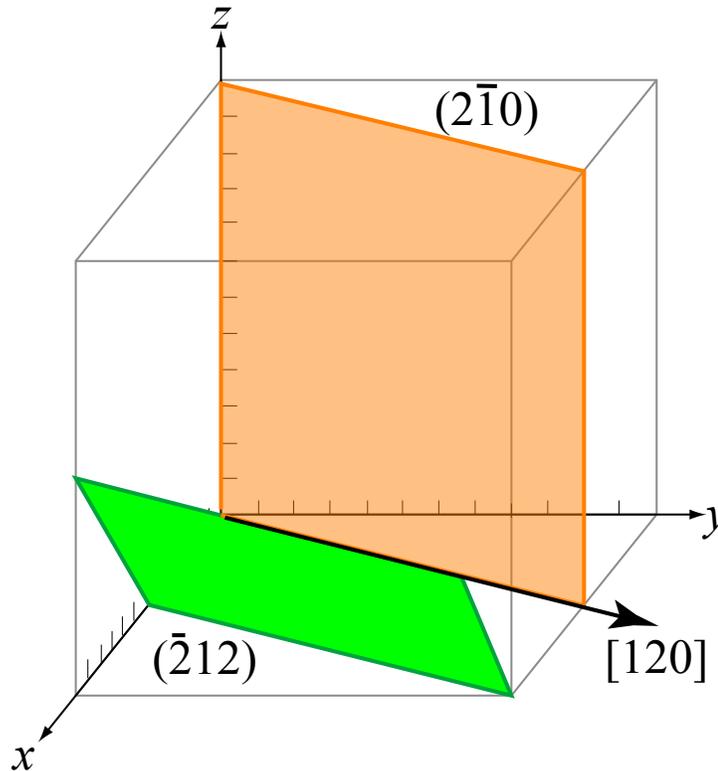
These planes intersect along this crystallographic direction   **[120]**

The direction of intersection can be readily discerned from the **drawing** if done precisely, as demonstrated above, or it can be **calculated** by taking the vector cross product of the normals to the two planes given here. In the cubic system, the directions normal to a family of planes (*hkl*) bears the same indices [*hkl*]. Taking the cross product of the two normals confirms the direction along which the associated planes intersect.

$$\begin{vmatrix}\bar{2}\\2\end{vmatrix}\begin{vmatrix}1 & 2 & \bar{2} & 1\\\bar{1} & 0 & 2 & 1\end{vmatrix}\begin{vmatrix}\bar{2}\\0\end{vmatrix} = ([1]\times[0])-([-1]\times[2]) \quad ([2]\times[2])-([0]\times[-2]) \quad ([-2]\times[-1])-([2]\times[1])$$

$$= \textbf{[240]} \text{ or equivalently, } \textbf{[120]}, \text{ as shown on the drawing.}$$

Of course, the antiparallel direction $[\bar{1}\bar{2}0]$ is also correct.

3. Lattice Geometry

b. Draw on the following template one member of each of the following two families of planes, then show and specify the lattice direction along which they intersect: $(\bar{4}220)$ and $(\bar{1}010)$.
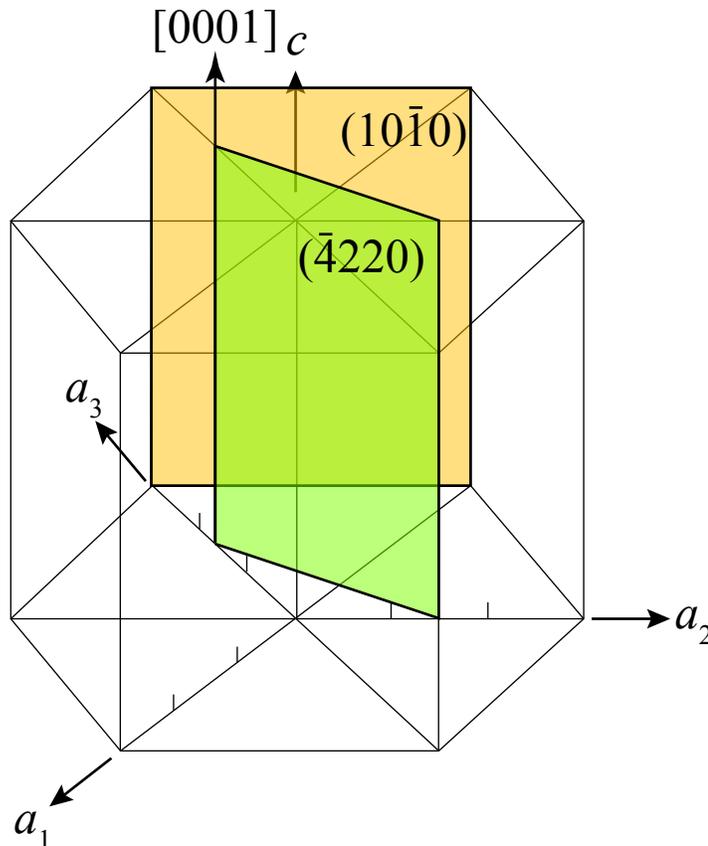

These planes intersect along this crystallographic direction   **[0001]**

The solution here follows directly from the drawing. There is no anomaly associated with the depiction of lattice planes in Miller-Bravais notation; indices are the reciprocals of the fractional intercepts with the four crystallographic axes. The direction of the intersection of these planes is evident because both families of planes have vertical edges confined by the unit cell that are parallel to the c axis. Here the direction is shown as **[0001]**, but as above, its antiparallel direction along the $-c$ axis direction is also appropriate. The cross product can also be computed, although it is more complex in four indices, and it should be noted that the normals to both families of planes bearing the same indices as their Miller-Bravais designations, have a **zero dot product** with [0001].

4. Crystal Structure

Crystal structures are sometimes indicated in *Strukturbericht* notation, from an original publication of the *Akademische Verlagsgesellschaft m.b.H.*, Leipsing, Germany, released in 1913, and continued under the name "Structure Reports" by the International Union of Crystallography from 1940 until today. Pure copper has an "A1" designation, CsCl has a "B2" designation. The designation "D8$_4$" is used for a more complex structure exhibited by $Cr_{23}C_6$, with 116 atoms per unit cell and the diffraction pattern shown here.
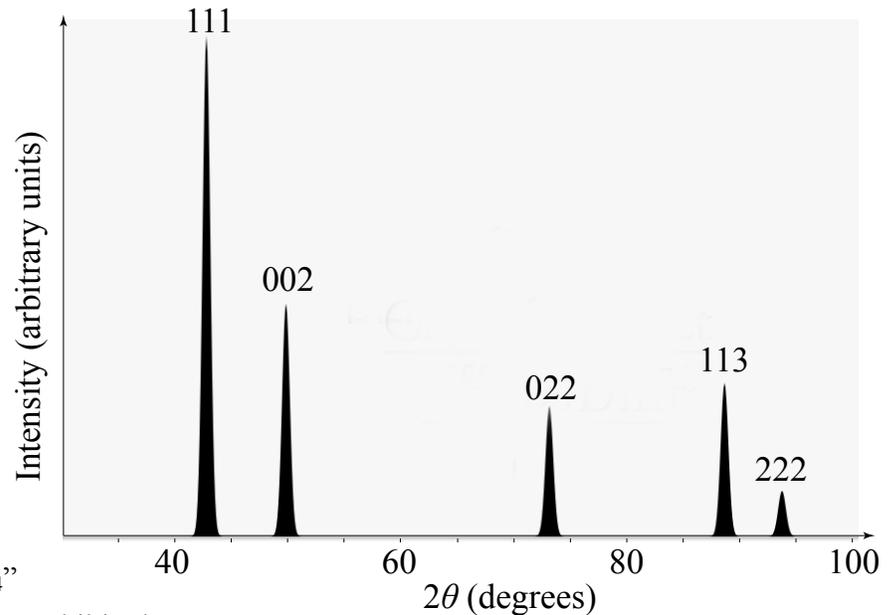
a. What is the Bravais lattice appropriate to define the D8$_4$ structure? Explain.

The answer is presented in the diffraction data. Note that the labeled peaks on the $Cr_{23}C_6$ diffraction pattern have indices *hkl* that are either **all even** or **all odd**, satisfying the "Reflection Rules" for an FCC "crystal type" given in the Worksheet table on page 1 of this file. Remember also that diffraction indices are "unadorned" by any type of bracket or parentheses, so they can be associated with families of lattice planes that satisfy Bragg's Law or a higher order reflection from those families, or both. In the pattern shown here, the 222 reflection can appear as the second order reflection ($n = 2$) from the {111} families of planes, even if there are no atoms on {222} planes in the structure. Ultimately, there is only one Bravais lattice associated with an FCC crystal, consequently the D8$_4$ structure is most appropriately defined by an **FCC Bravais lattice**.

4. Crystal Structure

b. Using the Bravais lattice determined in (**a**) above, how many Cr atoms and C atoms comprise the motif appropriate to define the $Cr_{23}C_6$ structure? Explain.

The relevant concept here is that the motif **must preserve the stoichiometry of the structure it represents**. Consequently, the motif associated with $Cr_{23}C_6$ must maintain a ratio of 23 Cr atoms to 6 C atoms, meaning that multiples of 23:6 (46:12, 69:18, 92:24, *etc.*) are also allowed. Deciding among these options requires more information, such as that provided in the problem statement, "116 atoms per unit cell."

This information alone would immediately favor the fourth option in the list above (92:24), since 92+24=116. However, another important concept is at play here, namely, that the **same motif must be assigned to every lattice point**.

From part (**a**) above, the Bravais lattice of $Cr_{23}C_6$ is FCC, a single unit cell of which contains **four lattice points**, all decorated by the same motif. The "116 atoms per unit cell" of $Cr_{23}C_6$ are indeed the stoichiometric sum of 92 Cr atoms and 24 C atoms distributed across four lattice points.

ANSWER: the motif contains **23 Cr atoms and 6 C atoms**.

5. Crystal Defects

a. Locate the dislocation in this figure, label it, and trace a Burgers circuit appropriate to define its Burgers vector using the FSRH convention.
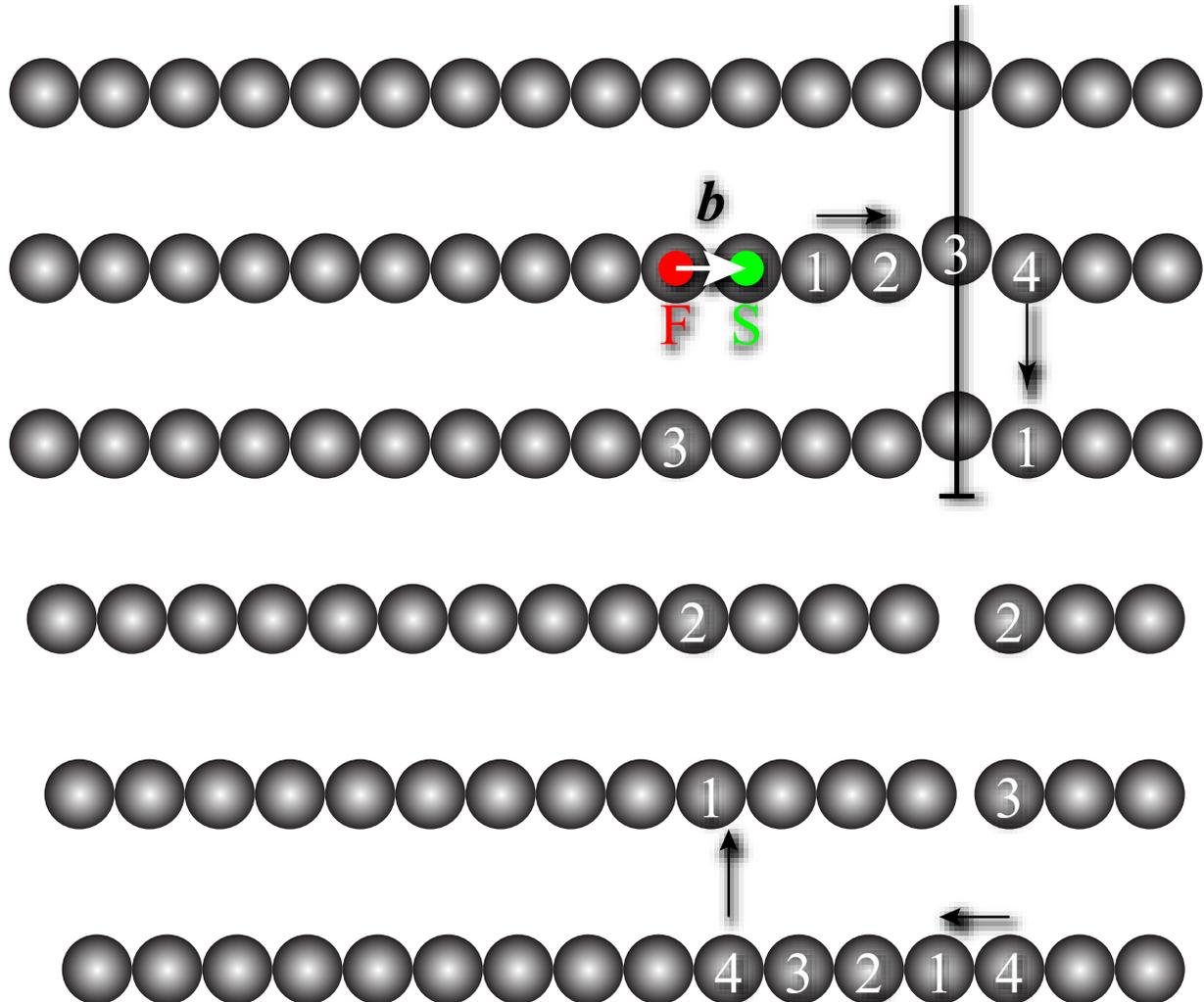
The start (S) and finish (F) points on any Burgers circuit may have multiple locations, and the number of steps is arbitrary, but the **circuit must enclose the dislocation line**. If the circuit misses the dislocation core, there will be no "closure failure" and the Burgers vector will not be revealed. The answer shown above shows closure failure, with the Burgers vector connecting F and S locations after a clockwise path, as required by the finish-to-start-right-hand (FSRH) convention.

5. Crystal Defects

b. The Burgers vector in (**a**) above is claimed to have the crystallographic designation $\vec{b} = \dfrac{a_0}{2}[\bar{1}11]$, and to be associated with a $(\bar{1}01)$ slip plane. Is this a legitimate expectation for a BCC alloy? Explain.
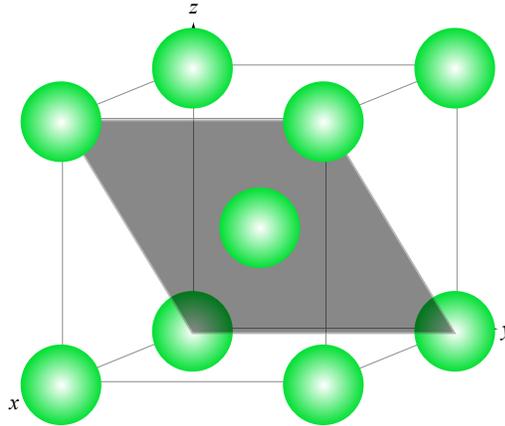
Answer: **NO**.

Conceptual understanding of dislocation "slip" includes the definition of a **slip system**, a combination of a close-packed plane and a close-packed direction along which dislocations move under the action of a shear stress. In the BCC crystal system, the closest packed direction is the <111> direction, and the closest packed planes are the {011} planes. The given information would therefore seem to be appropriate for a BCC alloy, but there is more to the concept of significance here.

First, it is important to understand that **the slip direction is the direction of the Burgers vector.** And most importantly, the slip direction, and therefore the Burgers vector, **must be a direction in the slip plane**.

Recall that the normal to any (*hkl*) plane in a cubic crystal system has indices [*hkl*], so by vector algebra, any direction [*uvw*] contained in an (*hkl*) plane will express a dot product of zero with the normal to that plane, specifically, [*uvw*] • [*hkl*] = 0.

Applying this test to the problem at hand, it is seen that $[\bar{1}11] \bullet [\bar{1}01] = 2$, failing the test, signifying that the Burgers vector is not in the slip plane. So **this combination of Burgers vector and slip plane is NOT a legitimate expectation for a BCC alloy**.