Physics H7C Midterm 1 Solutions

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1 Qualitative Questions

a) You'll never see such "thick film" interference with white light. There are several reasons for this; here's one. White light is composed of many components with a range of wavelengths. While a component with one exact wavelength may interfere with itself over arbitary distances, over a distance large compared to the wavelength the interference pattern will vary rapidly with wavelength. You won't be able to tell there's interference at all.

b) Light is wave motion in electric and magnetic fields, which can set polarizable molecules oscillating. Light is absorbed and reemitted by such molecules and this repeated scattering process slows the macroscopic progress of light through a medium relative to its motion in vacuum. The gas molecules in our atmosphere have resonances at high energies compared with visible light, so higher energy visible light interacts better with these molecules. More interaction means more scattering, so when you look at the atmosphere away from the sun you will see higher energy (blue) light more than lower energy (red) light.

c) Total internal reflection occurs for all angles beyond some critical angle when the index of refraction of the "transmitting" medium is less than that of the "internal" medium. The energy density in the transmitting medium does not vanish even in the case of total internal reflection. However, the so-called evanescent wave falls off exponentially with distance from the interface, so that no energy propagates to infinity.

d) Circularly polarized light is the sum of equal components linearly polarized in orthogonal directions and offset by a phase shift. If one could see the electric field vector at a point while looking in the direction of propagation, one would see the vector rotate at some fixed angular speed while keeping its constant magnitude. The portion of the field that makes it through the polarizer is

$$\vec{E} \cdot \vec{P} = E \cos \omega t \tag{1}$$

where $\vec{P}$ is the direction of the polarizing grid. Taking the square and the average over a period, we find that half the power makes it through the grid.

2 Periscope

a) The relation is

$$f = \frac{d}{4} \tag{2}$$

as you can convince yourself with a ray diagram or by putting $s_o = s_i = d/2$ into the thin lens equation:

$$\frac{1}{d/2} + \frac{1}{d/2} = \frac{1}{f}. \tag{3}$$

b) The position of the object certainly matters, as we can see from a simple counterexample. Setting $s_0 = d$ with $f = d/4$ as above we find that the image of the first lens is at $s_1 = d/3$. So the proposition is clearly false for $N = 1$ and there's no way to get back to an image at infinity or the focal length with a finite number of such steps.

c) The "relaying" lenses have no effect on the magnification for an image at infinity provided we place the objective and eyepiece as shown in figure 1, so for the calculation we can ignore everything but the objective and eyepiece lenses (we can also ignore the flat mirrors). The figure shows the basic setup and the angular magnification is generally

$$M \equiv \frac{\alpha_i}{\alpha_o} = \frac{h}{f_e}\frac{f_o}{h} = \frac{f_o}{f_e} \tag{4}$$

and $M = f_o/4$ for this periscope.

d) First, notice that the flat mirrors do not invert the image. For the relaying lenses, all object and image distances are positive and finite, so the image is inverted if there are an odd number of lenses, that is, if N is odd. This logic doesn't apply to the telescoping lenses, however, with the initial object and final image (in principle) at infinity. In fact, we know the telescope inverts the image and so for the whole apparatus to produce an upright image we want N to be odd.

3 Reimaging system

a) See Figure 2.

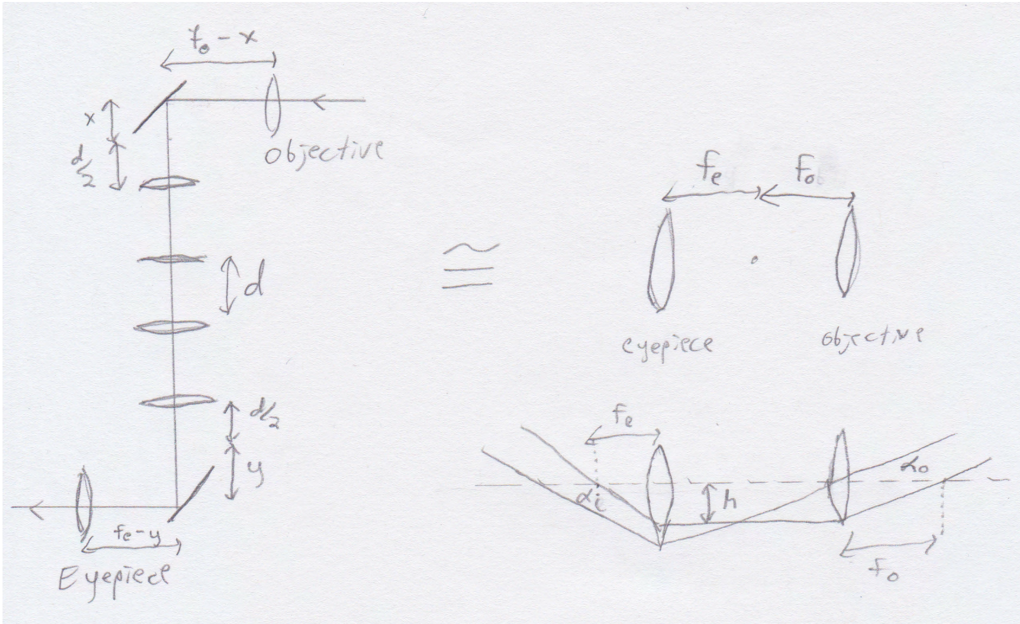

Figure 1: finding the angular magnification of a periscope. All the x and y business is trying to show is that there should be a distance of $d/2$ plus the relevant focal length between each outer lens and the first relaying lens.

b) A few quick applications of the thin lens formula will tell us the location of the final image. For the first lens we have

$$\frac{1}{2f_1} + \frac{1}{s_1} = \frac{1}{f_1} \quad \Longrightarrow \quad s_1 = 2f_1. \tag{5}$$

Putting this into the equation for the second lens gives

$$\frac{1}{2(f_1+f_2) - s_1} + \frac{1}{s_2} = \frac{1}{f_2} \quad \Longrightarrow \quad s_2 = 2f_2. \tag{6}$$

A similar calculation yields the final image distance $s_i = 2f_1$. We find the image in the same place as the object, as indicated by the ray diagram.

c) The magnification of the system is the product of the magnifications at each stage:

$$\begin{aligned} M &= \left(-\frac{s_1}{s_o}\right)\left(-\frac{s_2}{2(f_1+f_2) - s_1}\right)\left(-\frac{s_i}{2(f_1+f_2) - s_2}\right) & (7) \\ &= -\left(\frac{2f_1}{2f_1}\right)\left(\frac{2f_2}{2f_2}\right)\left(\frac{2f_1}{2f_1}\right) = -1 & (8) \end{aligned}$$

and from part (b) we know that the image is real.

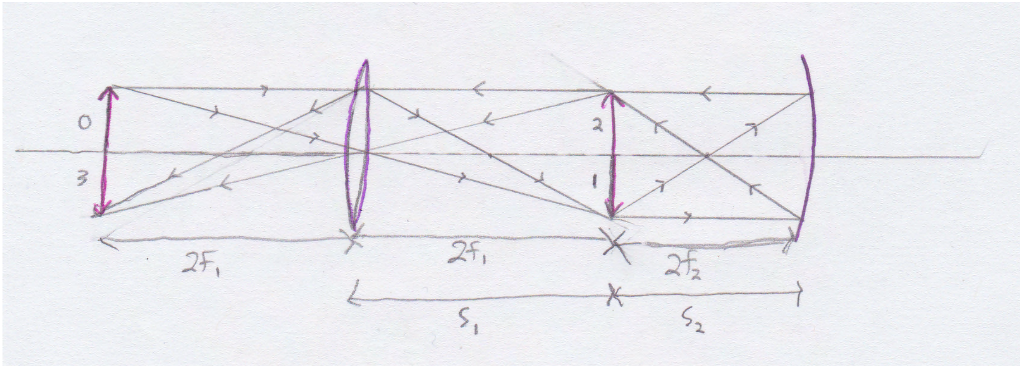

Figure 2: ray diagram for the reimaging system. I drew the diagram knowing exactly where the images were; I don't expect that sort of precision from people taking a timed exam.

d) We repeat the same calculations as in part (b) but substituting the new distances for the old and $f = f_1 = f_2$. For the first image,

$$\frac{1}{2f} + \frac{1}{s_1} = \frac{1}{f} \implies s_1 = 2f, \tag{9}$$

which is past the mirror. So the object for the mirror is virtual. For the second image,

$$\frac{1}{f - s_1} + \frac{1}{f - s_1} = \frac{1}{f} \implies s_2 = \frac{f}{2}. \tag{10}$$

This is the image Professor Lee meant by "the final image," although it is not in fact the final image produced by the system. (It is the final *real* image, though.) So I accepted answers for either this image or the really final one. The penultimate image is real and has magnification

$$M = \left(-\frac{2f}{2f}\right)\left(-\frac{f/2}{-f}\right) = \frac{-1}{2} \tag{11}$$

while the final image is $s_i = -f$ to the left of the lens (that is, f to the right of the lens) and is virtual with magnification 1.

4 Newton's Rings

Most of this problem is done in Hecht, so I'll be brief.

a) The center is bright for transmission and dark for reflection.

b) Let x_m be the radius of the m_{th} interference maximum for reflection and d_m the corresponding distance between the surfaces perpendicular to the flat surface. A bit of geometry gives the relation

$$x_m^2 = R^2 - (R - d_m)^2 = 2Rd_m - d_m^2. \tag{12}$$

We assume that $R \gg d$ so that we can neglect the second term compared to the first. Accounting for the phase shift at the air-glass interface, we can write the optical path difference as[1]

$$OPD_m = 2d_m + \frac{\lambda}{2}. \tag{13}$$

Since this is supposed to be an interference maxima, the OPD must be an integer multiple of the wavelength:

$$2d_m - \frac{\lambda}{2} = m\lambda. \tag{14}$$

Combining the last two equations and rearranging things gives an expression for x_m in terms of known quantities and m,

$$x_m = \sqrt{R\lambda(m - 1/2)}, \qquad m = 1, 2, 3, \ldots \tag{15}$$

and the dark rings' radii are given by the same expression without the $-1/2$ and including $m = 0$.

c) The liquid makes no difference to the relative phase shift: there is a new phase shift at the glass-liquid interface but the other phase shift is gone so that the relative phase shift is still π. But the optical path length within the liquid gets a factor of n_{liq} so that all the rings are in slightly different places:

$$OPD = 2n_{\text{liq}}d + \lambda/2 \Longrightarrow x_m = \sqrt{R\lambda(\frac{m}{n_{\text{liq}}} - 1/2)}, \qquad m = 1, 2, 3, \ldots \tag{16}$$

[1] The sign of the offset to account for the phase difference is arbitrary. I chose a plus sign so that equation (15) gives the first bright ring with $m = 1$.