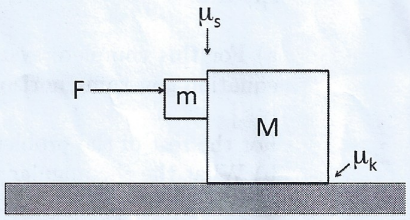
Physics H7A, Fall 2012 Instructor: Professor Adrian Lee
Final Examination, Tuesday, December 11, 2012

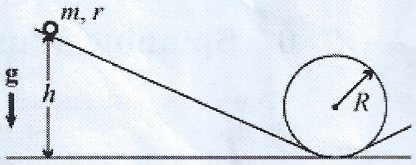
Please do work in your blue/greenbooks. Show your reasoning carefully so that we can be sure that you derived the answer rather than guessing it or relying on memory; in addition, this enables us to give partial credit. You may use three double-sided 3.5 x 5 index cards of notes. Test duration is three hours. Calculators are not allowed.

1 Blocks [30 pts. total]



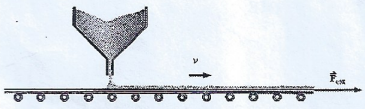
Two blocks with masses m and M are free to move. The coefficient of static friction between the two blocks is μ_s and the coefficient of kinetic friction between M and the surface it is sitting on is μ_k. Calculate the minimal horizontal force F required such that m does not slide down the side of M. Assume that the blocks are accelerating rather than standing still. [30 pts.]

2 Loop the loop [30 pts. total]



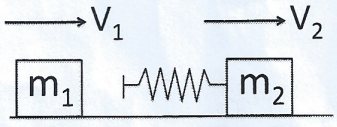
A solid sphere of mass m and radius r starts at rest at height h and rolls without slipping down the straight track and into a circular loop of radius R. Assume $r << h$ and $r << R$.
a) Find the minimum starting height h so the sphere will not lose contact at the top of the loop. (The moment of inertia for a solid sphere is $2/5\ mr^2$.) [20 pts]
b) Identify all the forces acting on the sphere and find the work done by each force in reaching the bottom of the straight track. [10 pts]

3 Conveyer Belt [30 pts. total]



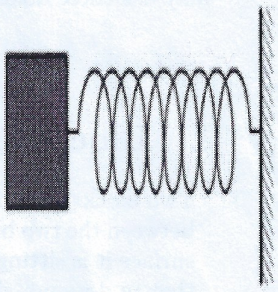
In a gravel yard, a hopper drops gravel at a rate of dm/dt onto a conveyer belt that moves at a constant speed of v.
(a) Determine the force needed to keep the conveyor belt moving. [20 pts]
(b) Calculate the power output of the motor that drives the conveyor belt. [5 pts]
(c) Does all this power go into kinetic energy of the gravel? If not, where does the energy go? [5 pts]

4 Sliding Blocks [30 pts. total]



Consider two blocks with masses m_1 and m_2 respectively sliding on a horizontal flat surface. The coefficient of kinetic friction between the blocks and the surface is μ_k. The leading block has a massless spring with spring constant k attached to its trailing end (see drawing). When m_1 contacts the spring attached to m_2 (t = 0), the velocities of m_1 and m_2 are v_1 and v_2 respectively.
a) Find the center-of-mass velocity of the system $v_{cm}(t)$. You should assume that neither block changes the direction of its motion or comes to rest for any time of interest. [15 pts.]
b) Find the maximum compression of the spring. [10 pts.]
c) Find the work done by friction while the spring is being compressed (that is until the spring reaches maximum compression). [5 pts.]

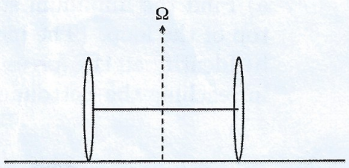
5 Damped Oscillator hit by bullet! [30 pts. total]



A block of mass m is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system:
1) If the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.
2) The viscous resistive force is equal to mg if the block moves with a certain known speed u.

a) For this complete system (including spring and damper) write the differential equation governing horizontal oscillations of the mass in terms of m, g, h and u. [10 pts.]
For the rest of the problem, assume $u = 3\sqrt{gh}$.
b) What the the angular frequency of the damped oscillations? [5 pts.]
c) What is the Q of this oscillator? [5 pts.]
d) If a bullet of negligible mass but non-negligible momentum traveling horizontally (in the positive x direction) hits the mass at $t = 0$, find the value of the phase angle δ in the subsequent motion, and sketch x vs. t for the first few cycles. [10 pts.]

6 Spinning your wheels [30 pts. total]



Two wheels with moment of inertia I are connected by a rod of length l, as shown in figure. The system rests on a frictionless surface, and the wheels rotate in the same direction with frequency ω around the axis of the rod. Each wheel has mass M. Additionally, the whole system rotates with frequency Ω around the vertical axis through the center of the rod.
a) What is the magnitude and relative direction of the component of angular momentum along the axis of the rod? Make a sketch of the angular momentum of each wheel along the axis of the rod. [10 pts.]
b) What is the largest value of Ω for which both wheels stay on the ground? [20 pts]