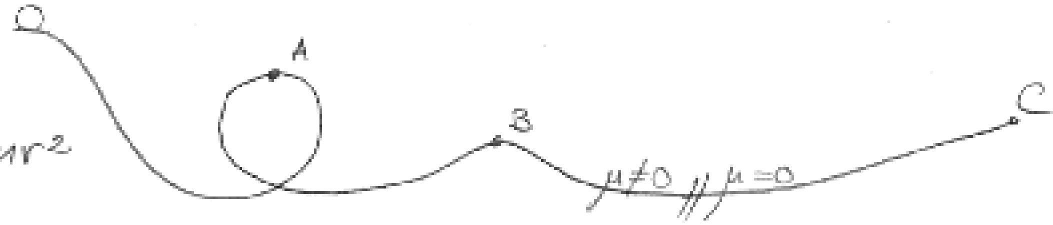
Problem 1.

m, r

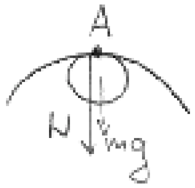
$I = \frac{2}{5} m r^2$



A] Conservative forces only acting And work of $f_s = 0$ $\Big\} \rightarrow$ Energy is conserved

@A $\quad mgH = mg2R_A + K_{trans} + K_{rot}$

$$= mg2R_A + \frac{1}{2}mv_A^2 + \frac{1}{2}I\omega_A^2$$



$$N + mg = m\frac{v_A^2}{R_A}$$ R_A max for $N \rightarrow 0$

Largest $R_A = \frac{v_A^2}{g}$ or $\boxed{v_A^2 = R_A g}$

Rolling w/o slipping condition $\boxed{\omega_A = v_A / r}$

$$\cancel{m}gH = \cancel{m}g2R_A + \frac{1}{2}\cancel{m}R_A g + \frac{1}{2}\frac{2}{5}\cancel{m}\cancel{r^2}\frac{1}{\cancel{r^2}}R_A g$$

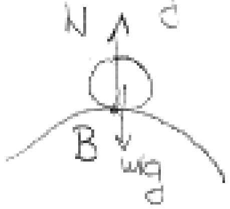
$$H = 2R_A + \frac{1}{2}R_A + \frac{1}{5}R_A = \frac{27}{10}R_A \qquad \boxed{R_A = \frac{10}{27}H}$$

B]

$$mgH = mgR_B + \frac{1}{2}mv_B^2 + \frac{1}{2}I\omega_B^2$$ Still rolling w/o slipping $\omega_B = v_B/r$

$$\cancel{m}gH = \cancel{m}gR_B + \frac{1}{2}\cancel{m}v_B^2 + \frac{1}{2}\frac{2}{5}\cancel{m}\cancel{r^2}\frac{v_B^2}{\cancel{r^2}}$$

$$gH = gR_B + \left(\frac{1}{2} + \frac{1}{5}\right)v_B^2$$



$$m\frac{v_B^2}{R_B} = mg - N$$ R_B min for $N \rightarrow 0$

$v_B^2 = gR_B$ for smallest R_B.

$$\cancel{g}H = \cancel{g}R_B + \frac{7}{10}\cancel{g}R_B = \frac{17}{10}\cancel{g}R_B \qquad \boxed{R_B = \frac{10}{17}H}$$

C] No more $f_s \longrightarrow \omega$ does not change as there is no torque on the way up.

$$\boxed{\omega = \omega_0 = v_0/r = \text{const}}$$

@ ground $\quad \cancel{m}gH = \frac{1}{2}\cancel{m}v_0^2 + \frac{1}{\cancel{2}}\frac{\cancel{2}}{5}\cancel{m}\cancel{r^2}\frac{v_0^2}{\cancel{r^2}}$

$$gH = \left(\frac{1}{2}+\frac{1}{5}\right)v_0^2 = \frac{7}{10}v_0^2 = \frac{7}{10}\omega_0^2 r^2$$

@ C $\quad \cancel{m}gH = \cancel{m}gh + \cancel{K_{trans}}^{\,0} + \frac{1}{\cancel{2}}\frac{\cancel{2}}{5}\cancel{m}r^2\frac{v_0^2}{r^2}$

$$gH = gh + \frac{1}{5}\,\frac{\overset{2}{\cancel{10}}}{7}gH \qquad \boxed{h = \frac{5}{7}H}$$

Problem 2.

mass of rocket $\quad \boxed{m(t) = m_e + m_f - Kt}$

burnout time $\quad m_f = KT \rightarrow \boxed{T = m_f/K}$

A] $\quad m\frac{dv}{dt} = -u\frac{dm}{dt} - mg \quad \leftarrow$ 2nd Newtons Law

$$dv = -u\frac{dm}{m(t)} - g\,dt \qquad \frac{dm}{dt} = -K$$

$$v(t) = \int_0^t dv(t) = -u\int_0^t \frac{dm}{m(t)} - gt = -u\ln\frac{m(t)}{m(0)} - gt$$

$$\boxed{v(t) = -u\ln\left(\frac{m_e+m_f-Kt}{m_e+m_f}\right) - gt}$$

B] @ $T = m_f/K \quad \boxed{v(T) = -u\ln\left(\frac{m_e}{m_e+m_f}\right) - g\frac{m_f}{K}}$

C] $\quad h = \int_0^T v(t)\,dt = -u\int_0^T\left[\ln(m_e+m_f-Kt) - \ln(m_e+m_f)\right]dt - \int_0^T gt\,dt$

$$= \frac{-u}{K}\int_0^T \ln(m_e+m_f-Kt)\,d(Kt) + u\ln(m_e+m_f)T - \frac{1}{2}gT^2$$

let $\quad x = m_e + m_f - Kt \quad$ then $\quad \int \ln x\,dx = \ln x\cdot x\Big] - \int dx$

$$\int \ln x \, dx = x \cdot \ln x - x \Big]_{m_e+m_f}^{m_e}$$

$$= m_e \ln m_e - m_e - (m_e + m_f)\ln(m_e + m_f) + m_e + m_f$$

$$h = \frac{+u}{K}\left(m_e \ln m_e \cancel{- m_e} - (m_e + \cancel{m_f})\ln(m_e + m_f) \cancel{+ m_e} + m_f \right)$$

$$+ u \ln(m_e \cancel{+} m_f)\frac{m_f}{K} - \frac{1}{2} g \left(\frac{m_f}{K}\right)^2$$

$$\boxed{h = -\frac{u}{K} m_e \ln\left(\frac{m_e + m_f}{m_e}\right) + \frac{u m_f}{K} - \frac{1}{2} g \left(\frac{m_f}{K}\right)^2}$$

D] $0 = m_e v_0 - m_f u \qquad v_0 = \frac{m_f u}{m_e}$

$$v(t) = v_0 - gt = \frac{m_f u}{m_e} - gt$$

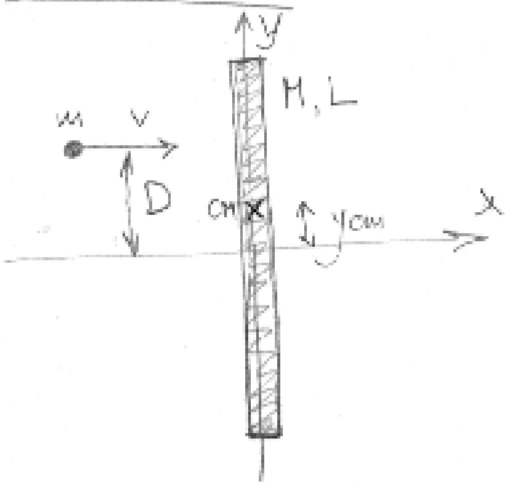
@ $T = m_f / K$

$$\boxed{v(T) = \frac{m_f u}{m_e} - g\frac{m_f}{K}}$$

E] $h = v_0 T - \frac{1}{2} g T^2 = \frac{m_f u}{m_e}\frac{m_f}{K} - \frac{1}{2} g \left(\frac{m_f}{K}\right)^2$

$$\boxed{h = \frac{u}{K}\frac{m_f^2}{m_e} - \frac{1}{2} g \left(\frac{m_f}{K}\right)^2}$$

Problem 3.



The center of mass of rod with monkey stuck to it is going to move in a straight line with $\vec{V}_{cm} = V_{cm}\hat{i}$ while the rod (with monkey) rotates about it with speed ω.

$$y_{cm} = \frac{mD + M\cdot 0}{m+M} = \boxed{\frac{m}{m+M} D = y_{CM}}$$

$\vec{P}$ is conserved: $mv = (m+M)V_{CM}$ $\boxed{V_{CM} = \frac{m}{m+M} v}$

$\boxed{x_{CM} = V_{CM}t = \frac{m}{m+M} vt}$ where $t=0$ @ the collision

$\vec{L}$ is conserved about CM: $mv(D - y_{CM}) = I\omega$

$$I = \frac{1}{12}ML^2 + My_{cm}^2 + m(D - y_{cm})^2$$

$$= \frac{1}{12}ML^2 + M\left(\frac{m}{m+M}\right)^2 D^2 + mD^2\left(\frac{M}{m+M}\right)^2$$

$$= \boxed{\frac{1}{12}ML^2 + \frac{mM}{m+M}D^2 = I}$$

$$\omega = \frac{\frac{m\cancel{M}}{m+M} vD}{\frac{1}{12}\cancel{M}L^2 + \frac{m\cancel{M}}{m+M}D^2}$$

$$\boxed{\omega = \frac{mvD}{\frac{1}{12}(m+M)L^2 + mD^2}}$$

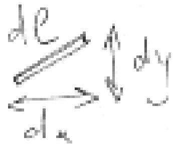
Problem 4.

A] Forces on a piece of rope:

$$T_x(x) = T_x(x+dx) = T_0 = \text{const}$$

$$T_y(x) + dmg = T_y(x+dx)$$

$$dmg = w\,dl = w\sqrt{dx^2+dy^2} = w\,dx\sqrt{1+(dy/dx)^2}$$



$$\frac{dy}{dx} = y' \qquad dmg = w\sqrt{1+y'^2}\,dx$$

$$\frac{T_y(x+dx) - T_y(x)}{dx} = \left.\frac{dT_y}{dx}\right|_x = w\sqrt{1+y'^2}$$

$$\frac{T_y}{T_0} = \left.\frac{T_y}{T_x}\right|_x = \tan\theta\Big|_x = \left.\frac{dy}{dx}\right|_x = y'(x) \qquad \boxed{T_y = T_0 y'}$$

$$\frac{dT_y}{dx} = T_0\frac{d^2y}{dx^2} = \boxed{T_0 y'' = w\sqrt{1+y'^2}}$$

B]
$$y(x) = a\cosh\left(\frac{x}{a}\right)$$
$$y'(x) = \sinh\left(\frac{x}{a}\right)$$
$$y''(x) = \frac{1}{a}\cosh\left(\frac{x}{a}\right)$$

$$\frac{1}{a}\cosh\left(\frac{x}{a}\right) = \frac{w}{T_0}\underbrace{\sqrt{1+\sinh^2\left(\frac{x}{a}\right)}}_{\cosh\left(\frac{x}{a}\right)}$$

$$\frac{1}{a} = \frac{w}{T_0} \qquad \boxed{a = \frac{T_0}{w}}$$

C] @ the endpoint $x = D$

$$2T_y = mg$$

$$T_y\Big|_D = \frac{w\,2L}{2}$$

$$\tan\theta\Big|_D = \frac{T_y}{T_x} = \frac{T_y}{T_0} = \boxed{\frac{wL}{T_0} = \tan\theta\Big|_D}$$

D] $\tan\theta\big|_D = y'(D) = \sinh\left(\frac{D}{a}\right)$

$$\boxed{\sinh\frac{wD}{T_0} = \frac{wL}{T_0}}$$