PHYSICS 7B, Section 2 - Spring 2013
Second midterm, C. Bordel
Tuesday, April 9, 2013
7 - 9 pm

Gradient operator and infinitesimal displacement in spherical coordinates :

$$\vec{\nabla} f = \overrightarrow{grad}\, f = \left(\frac{\partial f}{\partial r}\right)\vec{u_r} + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right)\vec{u_\theta} + \left(\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\right)\vec{u_\varphi}$$

$$\vec{dl} = dr\,\vec{u_r} + r d\theta\,\vec{u_\theta} + r\sin\theta d\varphi\,\vec{u_\varphi}$$

Gradient operator and infinitesimal displacement in cylindrical coordinates :

$$\vec{\nabla} f = \overrightarrow{grad}\, f = \left(\frac{\partial f}{\partial r}\right)\vec{u_r} + \left(\frac{1}{r}\frac{\partial f}{\partial \theta}\right)\vec{u_\theta} + \left(\frac{\partial f}{\partial z}\right)\vec{u_z}$$

$$\vec{dl} = dr\,\vec{u_r} + r d\theta\,\vec{u_\theta} + dz\,\vec{u_z}$$

Series expansions, trigonometric formulae and integrals :

$$\frac{1}{(x+a)} \xrightarrow[x\to\infty]{} \frac{1}{x} - \frac{a^2}{x^2} + \frac{a^3}{x^3} - \frac{a^4}{x^4};\ \frac{1}{Ax^4 + Bx^3 + Cx^2 + Dx + E} \xrightarrow[x\to\infty]{} \frac{1}{Ax^4};$$

$$\cos x \xrightarrow[x\to 0]{} 1 - \tfrac{x^2}{2};\ \sin x \xrightarrow[x\to 0]{} x;\ \tan x \xrightarrow[x\to 0]{} x;$$

$$\sin 2x = 2\sin x\cos x;\ \cos 2x = 2\cos^2 x - 1;\ \int\frac{dx}{x^n} = -\frac{1}{(n-1)x^{n-1}}\ .$$

Problem 1 - Dielectrics & Capacitors (20 pts)

A parallel plate capacitor, initially uncharged and whose inter-plate space is filled with plexiglas of dielectric constant K, is connected to a battery delivering a voltage V and charged up to its maximum charge Q. The battery is then disconnected.

a- How does the electric field E_D within the insulator compare with the electric field E_0 after you remove the insulator from between the plates? Explain your answer.

b- What is the ratio of the capacitance C in the presence of the dielectric and the capacitance C_0 with vacuum between the plates?

Now we form an array of capacitors as shown below (Fig.1).

c- Calculate the capacitance C_{eq} of the equivalent capacitor.

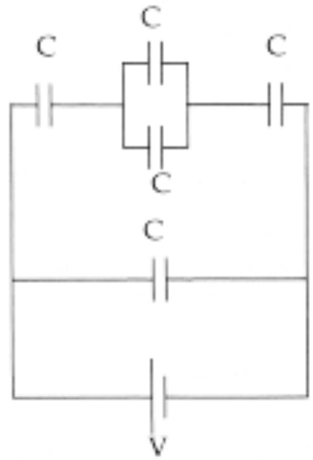

Figure 1

<u>Problem 2</u> - Electron speed in a wire (20 pts)

Based on the number density of free electrons in a Cu wire, a typical value of the magnitude of the drift velocity is 0.05 mm/s. It would therefore take several hours for an electron to travel 1 m!

a- Explain why the light goes on almost instantaneously when you flip a light switch. *Hint: remember what causes the electron motion.*
b- Explain what causes the drift velocity to be so small.
c- Using your understanding of the electrical resistivity at the microscopic scale, explain how the resistivity of a metal is expected to vary as a function of temperature.

<u>Problem 3</u> - Model of an atomic charge distribution (20 pts)

The charge distribution within an atom, whose total electric charge is zero, can be modeled by considering a spherical negative charge distribution $\rho(r)$ around the nucleus of charge $q>0$ occupying a small volume of radius R. At distance $r>R$ from the center O of the atom, the negative charge distribution $\rho(r)$ is given by:

$$\rho(r)=\frac{A}{r^5}, \text{ where } A \text{ is a constant.}$$

Determine the constant A, as well as the electric field $\vec{E}(r)$ and electric potential $V(r)$ created by the total charge distribution for $r>R$.

<u>Problem 4</u> - Non-uniformly charged ring (20 pts)

A non-uniformly charged ring of radius R carries a linear charge density $\lambda(\theta)=\lambda_0\cos(\theta)$, with $\lambda_0>0$. The ring lies in the (x,y) plane centered at the origin. See Figure 4(a).

a- Determine the direction of the electric field created at (0,0,z) and explain your reasoning.
b- Calculate the magnitude of this field.
c- A similar ring of linear charge density $\lambda(\theta) = -\lambda_0 \cos(\theta)$ is placed at distance $2R$ from the first one along the symmetry axis (Fig. 4(b)). Determine the new field on the symmetry axis.
d- Make a qualitative plot of the magnitude on axis of the electric field between the two rings.

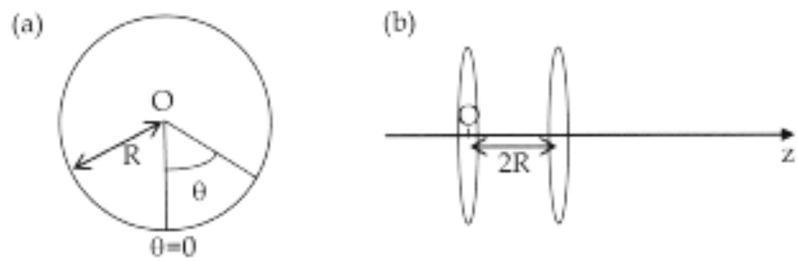

Figure 4

<u>Problem 5</u> - Combination of two electric dipoles (20 pts)

We consider the following point charge distribution, {A(-q), B(+3q), C(-3q), D(+q)}, where all the point charges are on the same axis with equal spacing $2a$ between them (Figure 5). We set V=0 at infinity for the electric potential.

a- Determine the electric potential created by such a charge distribution on the x axis.
b- Determine the asymptotic dependence of the electric potential as the distance goes to infinity ($x \to \infty$). *Note that it may be easier to simplify your expression before expanding.*
c- Calculate the electric field along the x axis for large distances ($x \to \infty$). Why is it acceptable to use the potential along one particular axis to calculate the electric field in this case?
d- Calculate the electric potential on the y axis and explain why the electric field is not zero on this axis. Give its direction.

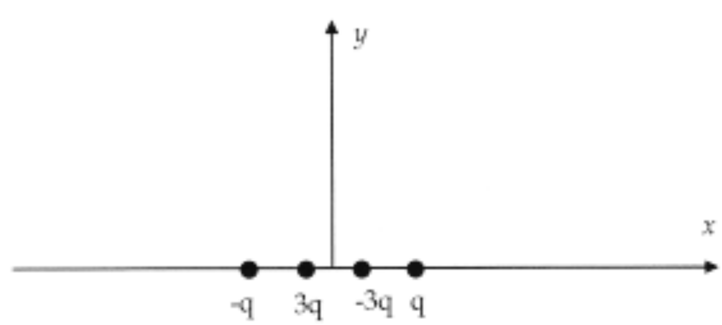

Figure 5