Physics H7A, Fall 2012 Instructor: Professor Adrian Lee
Midterm Examination #2, Thursday, November 8, 2012

Please do work in your greenbooks. Show your reasoning carefully so that we can be sure that you derived the answer rather than guessing it or relying on memory; in addition, this enables us to give partial credit. You may use one double-sided 3.5 x 5 index cards of notes. Test duration is 90 minutes.

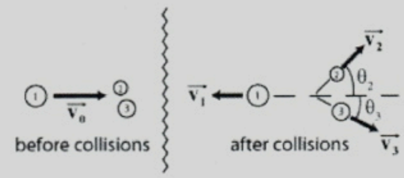
1 Three ball collision [25 pts. total]

Ball 1 traveling with velocity $\vec{v_0}$ collides with ball 2 and ball 3, both of which are initially at rest. The order of the collisions doesn't matter. All three balls have the same mass m, but their diameters are different. The collisions can be elastic or inelastic, and there is no friction. For each part below, explain your reasoning and be as quantitative as possible. After the collisions:

a) Could ball 1 end up with velocity $\vec{v_1} = -\vec{v_0}$? [5 pts.]

b) Could $|\theta_2| \neq |\theta_3|$ if ball 1 ends up at rest? [10 pts.]

c) If ball 1 ends up going in the opposite direction compared to its initial direction, could all three balls end up with the same speed but going in different directions? [10 pts.]



2 Billiard Balls [25 pts. total]

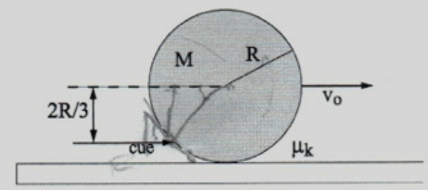
A billiard ball with radius R initially at rest is given a sharp blow by a cue stick. The force is horizontal and is applied at a distance $2/3\ R$ below the centerline of the ball. The initial speed of the ball after the blow is v_o, and the coefficient of kinetic friction is μ_k. The moment of inertia of a sphere around its center-of-mass is $I = 2/5MR^2$.

a) What is the initial angular speed ω_0? Is it clockwise or counterclockwise? [7 pts]

b) What is the speed of the ball once it begins rolling without slipping? [6 pts]

c) How far does the ball slide before it starts rolling without slipping? [6 pts]

d) What is the work done by friction on the ball? [6 pts]



3 Batter Up! [25 pts. total]

$\tau = \frac{4}{9}FR$

When a baseball or softball batter hits the ball, the force on the batter's hands depends on where along the bat the ball is struck. The minimum force occurs for one location called the sweet spot. Consider the approximation where the bat is rotating around one fixed point at very end of the bat where the batter is holding the bat. Show that there is a distance L_s from the pivot point (batter's hands) to the location where the ball hits such that the force perpendicular to the bat's long axis at the batter's hands is zero. You can approximate the bat as a rod of length L_B. If you require a moment of inertia, please calculate it from scratch.

Regrade #3

EXAMINATION BOOK

Name Kunal Marwaha

Class H7A Section 101

Instructor Adrian Lee

Date 11/8/12 Grade

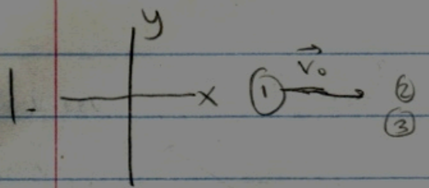
23436487

1	20
2	21
3	8 → 24
T	49 → 65

8 SHEETS
16 PAGES
11 x 8.5

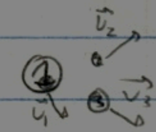
0 70972 77517 6

1.



Initially : ball 1, $\vec{v}_0$

final: ball 1, $\vec{v}_1$. ball 2, $\vec{v}_2$. ball 3, $\vec{v}_3$.



a) In an elastic collision, both energy and linear momentum are conserved.
In an inelastic collision, only linear momentum is conserved. Energy is lost.

$$\left.\begin{aligned} m.v_0 &= mv_1 + mv_2 + mv_3 \\ v_0 &= v_1 + v_2 + v_3 \end{aligned}\right\} \text{linear momentum conservation}$$

If $v_1 = -v_0$, then:

$$v_0 = -v_0 + v_2 + v_3$$

$$v_0 = \frac{v_2 + v_3}{2}$$

regardless of elasticity,
~~momentum~~ $E_{final} \leq E_{initial}$
(no potential energy change)
(the collision is not hyperelastic)

$$\frac{1}{2}mv_0^2 \geq \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2$$

$$\frac{1}{2}mv_0^2 - \frac{1}{2}m(v_0)^2 \geq \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2$$

$$0 \geq \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2$$

$$v_2^2 + v_3^2 \leq 0$$

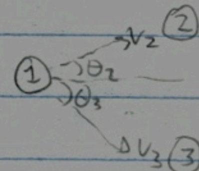
This is not possible,
as v_2^2 and v_3^2 are
both positive #s → Corner case: $v_2, v_3 = 0$, then $v_2^2 + v_3^2 = 0$ ✓
however, this implies
$v_0 = \frac{v_2 + v_3}{2} = \frac{0}{2} = 0$, but v_0 is nonzero.

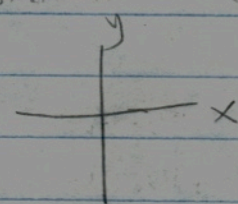
$\boxed{\text{No, } \vec{v}_1 \text{ cannot be } = -\vec{v}_0 .}$

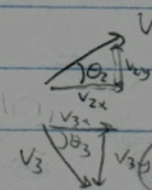
1.

b) if ball 1 ends up at rest:

final:







$V_{2x} = V_2 \cos\theta_2$

$V_{2y} = V_2 \sin\theta_2$

$V_{3y} = -V_3 \sin\theta_3$

$V_{3x} = V_3 \cos\theta_3$

In this collision, total linear momentum is conserved.

$(mv_0 = mv_1 + mv_2 + mv_3)$

In the y direction, there is no initial momentum (we define that ball 1 travels horizontally)

$(V_{0x} = V_0 \quad V_{0y} = 0 \quad V_{0x}^2 + V_{0y}^2 = V_0^2)$

$m(0) = mV_{1y} + mV_{2y} + mV_{3y}$

$0 = V_{1y} + V_{2y} + V_{3y}$. (if ball 1 at rest, $\vec{V}_1 = 0$)

$0 = V_{2y} + V_{3y}$

$V_{2y} = -V_{3y}$

$V_2 \sin\theta_2 = -(-V_3 \sin\theta_3)$

$V_2 \sin\theta_2 = V_3 \sin\theta_3$

$\frac{V_2}{V_3} = \frac{\sin\theta_3}{\sin\theta_2}$

total momentum conserved

$mv_0 = \cancel{mv_1} + mv_2 + mv_3$

$V_0 = V_2 + V_3$

x-direction

$mV_{0x} = mv_{1x} + mv_{2x} + mv_{3x}$

$V_{0x} = V_{2x} + V_{3x}$

$V_0 = V_2 \cos\theta_2 + V_3 \cos\theta_3$

$V_2 + V_3 = V_2 \cos\theta_2 + V_3 \cos\theta_3$

$V_2(1-\cos\theta_2) = V_3(1-\cos\theta_3)$

$\frac{V_2}{V_3} = \frac{1-\cos\theta_3}{1-\cos\theta_2}$

$\frac{1-\cos\theta_3}{1-\cos\theta_2} = \frac{\sin\theta_3}{\sin\theta_2}$

$\sin\theta_2 - \sin\theta_2\cos\theta_3 = \sin\theta_3 - \sin\theta_3\cos\theta_2$

$\sin\theta_2 - \sin\theta_3 = \sin(\theta_2 - \theta_3)$

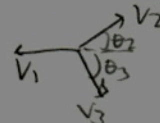
if $\theta_2 = \frac{\pi}{2}$

if $\theta_3 = 0$

solution works ✓ $|\theta_3|$ need not $= |\theta_2|$

Yes, $|\theta_2| \neq |\theta_3|$ is possible

1c.



$v_{2x} = v_2 \cos\theta \quad v_{2y} = v_2 \sin\theta$

$v_{3x} = v_3 \cos\theta \quad v_{3y} = -v_3 \sin\theta$

If $v_1 < 0$, all balls same speed, different directions?

Let $-v_1 = v_2 = v_3 = -v_{1x} =$ "v_n"

x direction

$m v_{0x} = m v_0 = m v_{1x} + m v_{2x} + m v_{3x}$ $\quad$ ∵ $v_{3y} = (-\sin\theta_3) v_3$

$v_0 = -v_n + v_n \cos\theta_2 + v_n \cos\theta_3$

y direction

$0 = \cancel{m v_{1y}} + m v_{2y} + m v_{3y}$

$v_{2y} = -v_{3y}$

$v_n \sin\theta_2 = v_n \sin\theta_3$

$\sin\theta_2 = \sin\theta_3$

$\frac{1}{2} m v_0^2 \geq \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \frac{1}{2} m v_3^2$

$v_0^2 \geq v_1^2 + v_2^2 + v_3^2$

$v_n^2 (\cos\theta_2 + \cos\theta_3 - 1)^2 \geq 3 v_n^2$

$(\cos\theta_2 + \cos\theta_3 - 1)^2 \geq 3$

(Energy is not gained, as per professor)

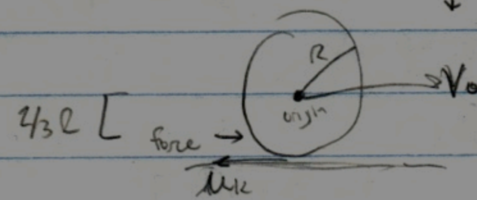
This is impossible.

thus, contradiction.

Thus, NO, the balls cannot travel in same speed @ different directions

10

2.



$\overset{+}{\circlearrowleft}\ \circlearrowright +$ $\quad \circlearrowright -$

$I = \frac{2}{5} m R^2$

force $\rightarrow$ $2R/3$ $|\tau| = \frac{2R}{3}(F)$

$|\text{torque}| = |\vec{R} \times \vec{F}| = \left(\frac{2}{3}R\right)(F)$ $\left(\text{in COM frame: } \tau = \frac{dL}{dt}\right)$

$V_0 = \int_0^{t_0} a\, dt$ $\quad V_0 = \int_0^{t_0} \frac{F}{m} dt = V_0$

initially @ rest

$\frac{2}{3} R F = I \alpha$

$\alpha = \frac{\frac{2}{3}RF}{I} = \frac{\frac{2}{3}RF}{\frac{2}{5}mR^2} = \frac{5}{3}\frac{F}{m}\frac{1}{R}$

$\omega_0 = \int_0^{t_0} \frac{5}{3}\frac{F}{m}\frac{1}{R} dt = \frac{5}{3}\frac{1}{R}\int_0^{t_0}\frac{F}{m} dt = \frac{5}{3}\frac{1}{R}(V_0)$

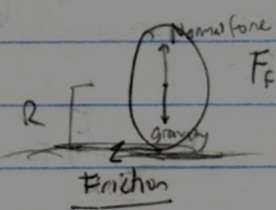
initially at rest $\quad \omega_0 = \int_0^{t} \alpha\, dt$

a) $\omega_0 = \frac{5}{3}\frac{V_0}{R}$ $\circlearrowright$ counter-clockwise

+4

b) When rolling without slipping: $V_{cm} = -R\omega$

$F_N = F_g$



$F_{fr} = \mu_k F_N$
$= \mu_k Mg$

$V = V_0 + \int_{t_0}^{t_1} \frac{F}{m} dt$
$= V_0 - \int_{t_0}^{t_1} \mu_k g\, dt$
$= V_0 - \mu_k g(\Delta t)$

$|\text{torque}| = |F_{fr} R| = I\alpha$

$\mu_k M g R = -\frac{2}{5} M R^2 \alpha$

$\alpha = -\frac{5}{2}\frac{\mu_k g}{R}$ (clockwise)

$\omega = \omega_0 + \int_{t_0}^{t_1} \alpha\, dt$
$= \omega_0 - \int_{t_0}^{t_1} \frac{5}{2}\frac{\mu_k g}{R} dt$
$= \omega_0 - \frac{5}{2}\frac{\mu_k g}{R}(\Delta t)$

$\omega_{ball} = \omega_0 - \frac{5}{2}\frac{\mu_k g}{R}(\Delta t)$
$= \omega_0 - \frac{5}{7}\left(\frac{V_0}{R} + \omega_0\right)$
$= \frac{2}{7}\omega_0 - \frac{5}{7}\frac{V_0}{R}$

$V = -R\omega$

$(V_0 - \mu_k g(\Delta t)) = -R\left(\omega_0 - \frac{5}{2}\frac{\mu_k g}{R}(\Delta t)\right)$
$= \frac{5}{2}\mu_k g(\Delta t) - R\omega_0$

$V_{ball} = V_0 - \mu_k g(\Delta t)$
$= V_0 - \frac{2}{7}(V_0 + R\omega_0)$
$= \frac{5}{7}V_0 - \frac{2}{7}R\omega_0$

$\omega_0 = \frac{5}{3}\frac{V_0}{R}$

+6

$V_0 + R\omega_0 = \frac{7}{2}\mu_k g(\Delta t)$ $\quad \Delta t = \frac{2}{7}\frac{(V_0 + R\omega_0)}{\mu_k g}$

c) Δd in Δt $\quad \Delta t = \frac{2}{7}\left(\frac{V_0 + R\omega_0}{\mu_k g}\right)$

$$V = V_0 - \mu_k g \Delta t \qquad a = \frac{F}{m} = -\mu_k g$$

$$\Delta d = \frac{V_f^2 - V_0^2}{2a} = \frac{\left(\frac{5}{7}V_0 - \frac{2}{7}R\omega_0\right)^2 - V_0^2}{2(-\mu_k g)} = \frac{\frac{25}{49}V_0^2 - \frac{20}{49}V_0 R\omega_0 + \frac{4}{49}R^2\omega_0^2 - V_0^2}{2(-\mu_k g)}$$

$$= \frac{-\frac{24}{49}V_0^2 - \frac{20}{49}V_0 R\omega_0 + \frac{4}{49}R^2\omega_0^2}{-2\mu_k g}$$

$$\boxed{\Delta d = \frac{24V_0^2 + 20V_0 R\omega_0 - 4R^2\omega_0^2}{98\mu_k g}}$$

$$\omega_0 = \frac{5}{3}\frac{V_0}{R}$$

$$\Delta d = \frac{24V_0^2 + 20\left(\frac{5}{3}\right)V_0^2 + 4\left(\frac{5}{3}\right)^2 V_0^2}{98\mu_k g} = \left(\frac{V_0^2}{98\mu_k g}\right)\left(24 + \frac{100}{3} - \frac{100}{9}\right)$$

+6

d) work by friction $= \cancel{\Delta KE} = \cancel{KE_{initial} - KE_{final}} = F(\Delta d)$

not quite, ball is rolling as well.

$$F = -\mu_k M g \qquad \Delta d = \text{(see above)}$$

+2

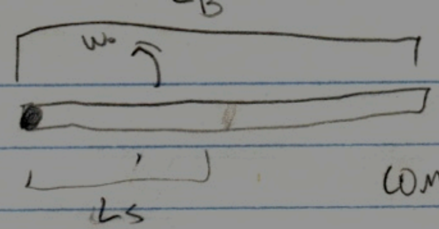
$$\text{work} = -\frac{M\left(24V_0^2 + 20V_0 R\omega_0 - 4R^2\omega_0^2\right)}{98}$$

(work is negative because $\vec{F}$ is negative and $\Delta \vec{d}$ is positive)

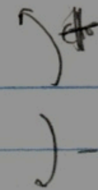
$$\text{work} = -M\left(\frac{V_0^2}{98}\right)\left(24 + \frac{100}{3} - \frac{100}{9}\right)$$

21/25

3.



Initially: constant ω_0



COM: $\frac{L_B}{2}$

Moment of inertia of a bat

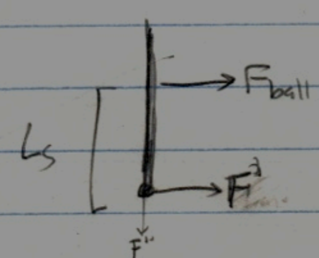
$$|L| = |R \times P| = RmV = \int_0^{L_B} mR^2\omega \, dR = \frac{1}{3}mR^2\omega \Big|_0^{L_B} = \frac{1}{3}mL_B^2\omega = I\omega$$

$$\frac{1}{3}mL_B^2\omega = I\omega$$

$$I = \frac{1}{3}mL_B^2$$

traveling @ constant ω_0

then:



$$P = \int dP = \int_{t_0}^{t_f} F\,dt = \int_{t_0}^{t_f} (F' + F_{ball})\,dt$$

$V_f = 0$ $\quad mV_f - mV_0 = -mV_0 = \int_{t_0}^{t_f} F'(dt) - \int_{t_0}^{t_f} F_{ball}\,dt$

$V_i = V_0$

If hinge (hands) is origin, $|\tau| = |R \times F| = L_s F_{ball}$ (clockwise)

24

also, @ fixed parts:

$\tau = \frac{dL}{dt}$ (fixed) pts

$$\tau = I\alpha \qquad \alpha = \frac{\tau}{I} = \frac{L_s F_{ball}}{\frac{1}{3}mL_B^2}$$

$\omega_f = 0$

$\omega_i = \omega_0$

$$\omega_f = 0 = \omega_0 - \int \frac{3L_s}{mL_B^2} F_{ball}\,dt$$

$$\omega_f = \omega_0 \int \alpha\,dt$$

$$-\int F_{ball}\,dt + mV_0 = \int_{t_0}^{t_f} F'(dt)$$

$$mV_0 - mV_0\left(\frac{2m L_B}{3 L_s}\right) = \int_{t_0}^{t_f} F'(dt)$$

For $F' = 0$ then $\boxed{\frac{L_B}{L_s} = \frac{3}{2}}$ $L_s = \frac{2}{3}L_B$

$$\omega_0 = \frac{3L_s}{mL_B^2} \int F_{ball}\,dt$$

$$\int F_{ball}\,dt = \frac{mL_B^2}{3L_s}\omega_0 = \frac{2m}{3}\frac{L_B}{L_s}V_0$$

$V_{cm} = \omega\left(\frac{L_B}{2}\right)$

$V_0 = \omega_0\left(\frac{L_B}{2}\right)$

When $L_s = \frac{2}{3}L_B$ $\quad F' = 0$ (no force on batter's hands)