University of California, Berkeley
Department of Mathematics
5^{th} November, 2012, 12:10-12:55 pm

MATH 53 - Test #2

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Discussion Section: ______________________________

Name of GSI: ______________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

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6	/4
Total	/32

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[6] 1. Evaluate the integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} \sqrt{1-x^2}\,dx\,dy$ by changing the order of integration.

[5] 2. Evaluate the integral $\iiint_E x\,dV$, where $E \subseteq \mathbb{R}^3$ is bounded by $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

3. Find the maximum and minimum of $f(x, y) = 2y^2 - 4x^2$ subject to the constraint $x^2 + \dfrac{y^2}{4} =$ 1, if they exist. [7]

(Note: this problem can be solved either with algebra and calculus, or by drawing a suitable picture, as long as it's properly explained.)

4. Let $D \subseteq \mathbb{R}^2$ be the region bounded by the curves $y = 2x - 1$, $y = 2x - 4$, $2x + 3y = -1$, and $2x + 3y = 3$. Find a rectangle R and transformation T that maps R onto D, and compute
[6] the Jacobian of T.

5. Show that the Jacobian of the spherical coordinate transformation is given by $J_T(\rho, \phi, \theta) =$
[4] $\rho^2 \sin \phi$.

6. Let f be a continuous function on a closed, bounded set $D \subseteq \mathbb{R}^2$, and let m and M denote the absolute minimum and maximum of f on D. The *Intermediate Value Theorem* in two variables states that if f is continuous and D is *connected* (consists of one solid piece), then f attains every value between m and M (i.e. the range of f is $[m, M]$). Use these facts to
[4] prove that there exists a point $(x_0, y_0) \in D$ such that

$$\iint_D f(x,y)\,dA = f(x_0,y_0)\,\text{Area}(D).$$