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Discussion Section: [redacted]
6SI: [redacted]

Math 54 First Midterm Fall 2012 Instructor: D.-V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name , SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide the answer to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book). The Requirement is 20 points.

Problem I (5 + 3 + 2 + 4 pts) Let A and B be the matrices:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{pmatrix}$$

a) Compute a row - echelon form of B by elementary row operations and write each time the elementary matrix for the row operation you carry out.

b) Find bases of the column space Col B and of the nullspace Nul B of B.

c) Compute det A.

d) Compute the inverse of the matrix A .

[illegible]

/

Problem II (6 pts, each question 1 pt). Check True or False .

		True	False
a)	In R^n if X, Y, Z are linearly independent vectors and Y, Z, T are linearly independent then X, Y, Z, T are linearly independent .		✓
b)	In a vector space , the intersection of 2 subspaces is a subspace.	✓	
c)	In a vector space, the union of 2 subspaces is a subspace.		✓
d)	The set of rational numbers Q is a subspace of R.		✓
e)	There exist linear maps $S : R^3 \to R$ and $T : R \to R^2$ such that their composition $T \circ S : R^3 \to R^2$ is one- to -one and onto		✓
f)	There exist linear maps $S: R^3 \to R^2$ and $T: R^2 \to R^3$ such that their composition $S \circ T : R^2 \to R^2$ is one-to-one and onto.	✓	

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Math 54 Midterm 1

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Elementary Matrix, E

1. a) $B = \begin{bmatrix} 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix}$ $\longleftarrow$ $\begin{bmatrix} 1 & 0 & 0 & \cancel{0} \\ 1 & 1 & 0 & \cancel{0} \\ 0 & 0 & 1 & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} \end{bmatrix}$ ✓

+5/5

$$\boxed{\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}}$$

a row echelon form

✓ $\longleftrightarrow$ $\begin{bmatrix} 1 & 0 & 0 & \cancel{0} \\ 0 & 1 & 0 & \cancel{0} \\ 0 & 1 & 1 & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} \end{bmatrix}$ ✓

another echelon form of B $\left\{ \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right.$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ not needed but correct

b) on back side ——— △

b) $B \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{matrix} \uparrow & \uparrow & & \uparrow \end{matrix} \end{array}$

pivot Col 1, 2, 4

$$\boxed{\text{Basis Col } B = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}}$$ ✓

+3/3

Augment B w/ $\vec{0}$ and solve $A\vec{x} = \vec{0}$

$[B\ \vec{0}] \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$

$x_1 = x_3$

$x_2 = x_3$

x_3 is free

$x_4 = 0$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \\ 0 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\text{Basis Nul } B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}}$$ ✓

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D13 2B

c) $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

+2/2

$$\det A = 1 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - 0 \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= (2) - 0 + (1)$$

$$\boxed{\det A = 3}$$

✓

d) $[A \quad I_3] \sim [\overset{I_3}{\cancel{A^{-1}}} \quad \overset{A^{-1}}{\cancel{I}}]$ $\quad$ det A ≠ 0 ⇒ invertible (Thm for invertible matrices)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \sim$$

+4/4

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}}$$

✓

Check: $AA^{-1} = I$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$